What do Banks do? Distinguishing right from the left

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Abstract

We propose novel techniques to examine whether loans, deposits and other banking variables can be classified as inputs or outputs. This allows for formal discrimination between the intermediation and alternative approaches on a bank-specific basis. First, we show that the problem of distinguishing "right" from the "left", viz. endogenous versus predetermined variables, is not trivial and requires new techniques. This approach is fully non-parametric and does not need a specific model to be estimated. Second, by adopting an endogenous directions model in the context of directional technology distance functions, we show that the sample distribution of directions is characteristically bimodal. This suggests that the endogeneity issue must be considered on a bank - specific basis. We use MCMC for numerical Bayesian analysis.

Keywords: banking; endogeneity; production approach; intermediation approach; Bayesian analysis; Markov Chain Monte Carlo.

JEL codes: G21, C11, C13.

Acknowledgement: The authors are grateful to Professor Mike G. Tsionas for insightful discussions that contributed to the early development of this research. Professor Tsionas passed away in 2024/01, during this project, and the authors wish to acknowledge his intellectual influence and guidance.

1. Introduction

To bring efficiency concepts to bear in investigating the optimal structure of the banking firm, one must begin with a theory of the banking firm. That is, what do banks do? Most of the literature applies traditional microeconomic theory of firm production to banking firms—a bank is a factory producing financial services. The recent literature takes seriously the bank as a financial intermediary that differs from other types of firms. Factors important for banks that have generally been ignored in much of the literature include the bank's choice of risk and diversification of assets, asset quality and its feedback on the bank's input prices, and the bank's financial capital structure.

The problem of inputs and outputs in banking is not new. It is well known that there is no consensus regarding the definition of bank output (Triplett, 1991; Berger and Humphrey, 1992). The production approach developed by Benston (1965) and Bell and Murphy (1968), supports claims that banks produce many categories of loans and deposits, using labor and capital as inputs. Benston et al. (1982) argues that output should be measured in terms of what banks do that cause operating expenses to be incurred (see Benston and Smith, 1976). The intermediate approach (Sealy and Lindley, 1997; Murray and White, 1983; Favero and Papi, 1995) emphasizes the intermediating role of banks, i.e. the fact that they buy capital and collect deposits, which they convert into the production of loans and other assets. Deposits along with labor and capital are the inputs and loans are used to measure output. Akhavein et al. (1997) use the profit function in which both inputs and outputs are endogenously determined. Mester et al. (2007) find empirical evidence in favour of a synergy between the liability and asset sides of a commercial bank's balance sheet. What is synergy and how it can be modelled in the input - output space is, of course, an important question. The problem is, of course, whether banks act indeed as if they maximize their profits (Hughes et al., 2001 develop the relevant argument). Hughes, Mester, and Moon (2001) argued that performance is likely to reflect agency problems or costs with top performers being subject to less agency costs. In this sense, performance is not simply a residual but rather an integral part of the banking production process; a fact that we capture in this paper by letting performance determine directional measures in the input - output space. Synergy does not, certainly, exclude a specification where movement in the input - output space or directions, depend on observed characteristics. If performance is a determinant of directions, agency problems are likely to be important for banks. In the opposite case, performance should not have explanatory power for directional measures as it reflects agency problems.

The user cost approach relies on the user cost of money to determine whether a bank asset or liability is an input or output. The approach was suggested by Donovan (1978), Barnett (1980) and Hancock (1985), who developed a production theory for financial firms. The user cost of money is defined as the difference between a benchmark rate (representing the opportunity cost of the bank) and the interest rate (rate of return) associated with holding this asset (Guarda and Rouabah, 2007). For a bank liability, the user cost of money is the difference between the interest rate of this liability and a benchmark rate. If the user cost of money is positive (negative), then the asset or liability in question is considered an input (output). Morttinen (2002) measured the output of banks, based on the user cost of money approach, and she considered the opportunity cost of deposits and loans. She used data from banks' financial statements and payment transactions and calculated Tornqvist-type indices for the output and labour productivity of six European countries.

The fundamental problem is the multiple nature of the services as the existence of a consistent output aggregate is questionable (Kim, 1986). For instance, Black (1975) and Fama (1985) argue that banks have an advantage in providing loans to depositors, whose past behavior offers credit information. Other authors assume that banks produce loans because they can reduce both the signaling cost needed for resolving the ex-ante information asymmetry between entrepreneurs and investors (see Leland and Pyle, 1977) and the monitoring cost for mitigating the ex post moral hazard problem (see Diamond, 1984; Wang, 2003; Kane and Malkiel, 1965; Bernanke, 1983; Goodhart, 1989; for a literature review). Relating to Mester's (2008) statement in the opening paragraph of this section, a strand of the literature combines the theory of financial intermediation with the microeconomics of banking production (as in Hughes et al. 2000; Hughes, 1999; Hughes et al., 1999; Hughes et al., 2001).

Therefore, distinguishing "right from the left" (as y and X in a linear regression) is a non-trivial issue in banking studies and the theory of banking itself. Complications arising from the fact that the bank may have multiple objectives, or conflicts arise between different interest groups and the fact that the bank is essentially a producer of information, complicate the fundamental issue of discriminating between inputs and outputs. Issues of quality are also likely to arise. Berger and Mester (1997) propose the alternative profit function which substitutes output levels for output prices in the specification of the profit function. This function is estimated to provide additional information when the assumptions underlying standard profit maximization do not hold. It may provide useful information if there are unobserved differences in output qualities across banks, outputs are not completely variable, output markets are not perfectly competitive, or output prices are measured with error.

In this paper we argue that the issues can, to certain extent, be resolved using a data-based approach. Although this point of view is reasonable it has not been pursued so far as there are considerable technical difficulties in model comparison with different endogenous variables. For example, if D, L, X denotes deposits, loans and traditional inputs, it is not clear how to compare the conditional distributions $[D|L,X]$ and $[L|D,X]$ or $[D,L|X]$. We solve this problem using relatively straightforward econometrics. We should not overlook another important possibility. Given that the conditional distributions can be compared in the light of the data, *it does not necessarily follow that "one size fits all"*. For some banks the intermediation approach may be relevant but for others the production approach could be the right structure. The data can be quite informative in this instance, as we show using a directional distance function with endogenous directions.

Our objective is not very different from Berger and Mester (1997), viz. opening the black box of banks and seeing what determines their decisions. Whereas Berger and Mester (1997) propose an alternative profit function, which has the same variables as the cost function (input prices and outputs) we rely on two different procedures. The first is completely non-parametric and can be implemented easily using widely available kernel density techniques. It relies on comparing conditional distributions to examine which one fits the data best. As the test statistics are new and we do not know its behavior in finite samples, we use the bootstrap to approximate its finite-sample properties and p-values. Our second approach relies on the Directional Technology Distance Function (DTDF) to study the technology. In recent years this approach has been found quite useful in empirical research. As the DTDF depends on certain directions it can effectively separate inputs from outputs given observed "movements" of banks in the input - output space. Its drawback is that the directions have been thought so far as fixed parameters for the entire sample. We remove this deficiency by making the directions bank - specific. Again, from this approach, *it does not necessarily follow that "one size fits all"* and directional measures can yield valuable information about different banks doing different things and / or behaving differently in the input - output space. In addition, we complement the DTDF approach with two important novel techniques. First, we check for robustness via "reality" or "counter-factual" tests based on the *available* sample. Second, we take the endogenous directional measures one step further by making them functions of environmental variables to understand, precisely, *why, if and to what extent different banks do indeed do different things and / or indeed they behave differently*.

The paper is organized as follows. The models are presented in section 2. Discrimination between inputs and outputs is taken up in section 3. Data and empirical results are presented in section 4. In section 5 we consider discrimination based on the directional technology distance function. In section 6 we take up the important issue of robustness / reality checks using feasible Bayesian procedures through MCMC. An outline and additional remarks are discussed in the final section.

2. Sample and Variables

The dataset includes U.S. financial institutions classified as *banks* in the LSEG-EIKON database, supplemented with hand-collected information from banks' annual reports for the period 2012 to 2025. The sample period coincides with a phase of significant regulatory and political developments in the U.S. banking sector, while enhancing the economic relevance of the data.[1] The United States was chosen as

[1] Notable examples include the Equator Principles, the relaxation of Dodd-Frank Act, adjustments aligned with the new accounting rules by the Sustainability Accounting Standards Board, and the guidelines of Bank for International

empirical setting, motivated by its single-country structure with common regulatory framework, mitigating the influence of confounding or country-specific factors, and its status as the largest economy with a globally significant banking sector. Thus, there is a reasonable degree of homogeneity across data, operations, and the regulatory, risk, and macroeconomic environment in which these banks operate. Due to data availability, the final sample comprises 255 U.S. financial institutions with bank-year observations that include a comprehensive set of balance sheet variables.

As main variables, we include the prices and quantity values of *labour*, *capital*, *deposits*, *funds*, *loans*, *other funds*, and *other earning assets*. To calculate the prices and the quantities of the main variables, we first draw the raw variables from LSEG-EIKON database in millions of $US and we calculate their log values, drawing from the standard banking literature (Sealey and Lindley, 1977; Berger and Mester, 1997). We construct unit prices as implicit deflators. The price of labour is defined as personnel expenses divided by the number of employees. The price of capital is measured as other operating expenses (excluding personnel) with deprecation relative to fixed assets. The price of deposits is calculated as interest expenses on deposits over total deposits, capturing the average interest rate paid. The price of funds is given by interest expenses on borrowed funds to total borrowed funds, reflecting the unit cost of wholesale funding. The price of loans is defined as interest income on loans to total gross loans. The price of other earning assets is calculated as income from other assets to total other assets, and the price of other earning funds as income from other earning assets over total other earning assets.

We also consider the quantities of the variables of interest (Berger and Humphrey, 1992; Fries and Taci, 2005). Labor quantity is obtained as personnel expenses divided by the price of labour. Capital quantity is defined as total fixed assets divided by the price of capital. Quantities for loans, deposits and other earning assets are obtained from total (book) values divided by respective implicit prices. To capture concerns about lending risk we include *non-performing loans* (Fukuyama and Matousek, 2017) while total assets and equity (in logs) are included as environmental variables in all models. The total number of observations is 3,570. Detailed variable definitions are reported in Table A1 in the Appendix. The price and quantity variables are winsorized at the 1st and 99th percentiles, within each year.

[Table A1 here]

Table A2 in Appendix reports the summary statistics of the prices for the full sample.[2] Two main arguments extracted from Table A2 that motivate further our analysis. When the price of deposits p_D is large relative to the price of loans p_{Ln}, particurarly for large banks, this is consistent with deposits being costly inputs. As reported in Table A2 (and the corresponding Table S1in the Supplement), the mean of p_D

Settlements.

[2] Details for summary statistics are provided in Table S1, in Supplement.

exceeds the mean of p_{Ln} across the full sample, and the gap between the two prices widens monotonically across different banks' size groups. This is exactly the pattern that the user of cost approach would predict as a signal of deposit input status: when the interest cost of funding (i.e. deposit price) exceeds the return on assets (i.e. loan price), the user cost of deposits is positive, thereby classifying deposits as inputs. Moreover, the coefficient of variation (CV) of p_D is notably large, substantially exceeding those of p_L (price of labor) and p_K (price of capital), indicating that the unit cost of deposit funding varies considerably across institutions, within different size groups. This cross-sectional dispersion in deposit prices constitutes a foundational empirical justification for our bank-specific approach: if all banks faced the same deposit price, the user cost criterion would classify deposits uniformly, and bank-specific direction estimation would be worthless. The large CV of p_D demonstrates that this uniformity does not hold in the U.S. sample.

[Table A2 here]

3. Cost Functions Models

3.1 Cost Functions

Suppose $x \in \Re_+^k$ represents a set of traditional inputs (i.e. capital, labour, etc.) whose prices are $w \in \Re_+^K$. Denote by $L \in \Re_+^l$ a vector of loans and by $D \in \Re_+^d$ the vector of deposits. The question is whether we can consider loans and deposits as input or output. Denote the prices of loans by $p_L \in \Re_+^l$ and the prices of deposits by $p_D \in \Re_+^d$. We distinguish the following hypotheses along with the corresponding cost functions.

A. **Hypothesis I (***financial intermediation approach, Sealy and Lindley, 1977*): If deposits are inputs and loans are outputs, the cost function is:

$$C_I(w, p_D; L) = \min_{x \in \Re_+^k, D \in \Re_+^d} : \{w'x + p'_D D | F(x, D, L) \geq 1\} \tag{1}$$

where $F(x, D, L)$ denotes a transformation function which describes the process of production as taking x and deposits to produce given amounts of loans.

B. **Hypothesis II** (*reverse approach*): If loans are inputs and deposits, are outputs, then the cost function is:

$$C_{II}(w, p_L; D) = \min_{x \in \Re_+^k, L \in \Re_+^l} : \{w'x + p'_L L | F(x, D, L) \geq 1\} \tag{2}$$

In both cases, the function $F(x, D, L)$ describes feasible combinations in the technology set:

$$\mathcal{T} = \{(x, L, D) \in \Re^{k+l+d} : (x, L, D) \text{ can be produced}\}$$

However, the cost functions in two hypotheses are different as they depend on different arguments. From Eq. (1), we can obtain optimal quantities x^* and D^* as functions of w, p_D, and L, while from Eq. (2), we obtain optimal quantities $x^*$and L^* as functions of w, p_L, and D. Therefore, x is always endogenous but which variable, L or D is endogenous depends on whether we adopt Hypothesis I or II.

C. **Hypothesis III** (*the production approach, Bell and Murphy, 1968; Benston and Smith, 1976*): This approach views the bank as using traditional outputs to produce both loans and deposits, with the cost function, given as:

$$C_{III}(w, p_L, p_D; D, L) = \min_{x \in \Re_+^k} : \{w'x | F(x, D, L) \geq 1\} \tag{3}$$

In the production approach, only x is endogenous and the predetermined variables are all prices as well as loans and deposits.

Summarising, we therefore have the following possibilities:

$$H_I: \quad \begin{bmatrix} x^* \\ D^* \end{bmatrix} = f_I(w, p_D; L) + u_I \tag{4}$$

$$H_{II}: \quad \begin{bmatrix} x^* \\ L^* \end{bmatrix} = f_{II}(w, p_L; D) + u_{II} \tag{5}$$

$$H_{III}: \quad x^* = f_{III}(w, p_L, p_D; D, L) + u_{III} \tag{6}$$

where f denotes a vector function.

3.2 Discrimination based on the demand functions

The first possible strategy is to rely on information related to the traditional inputs x^* from Eq. (4) through Eq. (6). In each of the three cases, x^* is a function of different arguments so a choice could be made based on a fit criterion about x^*. The problem with this approach is that if H_I, for example, is true, then ignoring the endogeneity of D in Eq. (5) would result in wrong estimates. On the other hand, if we allow for endogeneity then, implicitly, we are undermining the validity of H_I in the strict sense, whose testing is the very objective. In our context, suppose L the vector of loans and D the vector of deposits for two alternative competing models, H_I and H_{II}. From Eq. (4), we obtain the conditional distribution $p(D|L, W)$ where W denotes the vector of prices, while from Eq. (5) we obtain $p(L|D, W)$. With an estimate of the joint distribution, say $\tilde{p}$, we have:

$$p^I(L|W) = \frac{\tilde{p}(L, D|W)}{p(D|L, W)}, \quad p^{II}(D|W) = \frac{\tilde{p}(L, D|W)}{p(L|D, W)} \tag{7}$$

where $p^I(L|W)$ is model's I prediction about the marginal distribution of loans, $p^{II}(D|W)$ is model's II prediction about the marginal distribution of deposits, and $\tilde{p}(L, D|W)$ is an estimate of the joint distribution conditional on prices. From Eq. (4) and (5), the conditional distributions are multivariate normal in $\Re^d$ and $\Re^l$ respectively. Given an estimate of the joint distribution, $p^I(L|W)$ and $p^{II}(D|W)$ can be compared to their observed counterparts and decide which model is best based on the closeness of fit between the implied and observed marginal distributions.

The vector (demand) functions f_I and f_{II} in Eq. (4) and (5) are derived by applying the *Shephard's* *lemma*[3] to the quadratic cost function. The choice of quadratic lies on the advantages of quadratic form. Since $u \sim N(0, \Sigma)$, the quadratic specification ensures that the conditional distributions are closed-form multivariate normal. Moreover, the quadratic form operates directly on the level of prices and quantities without requiring logarithmic transformation, which avoids complications from near-zero quantity values that may arise after winsorization. Under the quadratic specification, the cost function takes the form:

$$C^{(k)}(z^{(k)}) = a_0 + a' z^{(k)} + \frac{1}{2} z'^{(k)} \mathbf{B} z^{(k)},\ B_{ij} = B_{ji} \tag{8}$$

where $z^{(k)}$ denotes the prices and quantities under the *Hypothesis* k, $\boldsymbol{a}$ is a parameter vector and $\mathbf{B}$ is a symmetric parameter matrix. Differentiating with respect to the input price of each endogenous variable yields a system of linear demand equations, which takes the form:

$$\boldsymbol{y}^{(k)} = \boldsymbol{X}^{(k)} \boldsymbol{\beta}^{(k)} + \boldsymbol{u}^{(k)} \text{ with } \boldsymbol{u}^{(k)} \sim N(0, \boldsymbol{\Sigma} \otimes \mathbf{I}_n) \tag{9}$$

Under H_I, the system in Eq. (4) the endogenous vector is $[x_L^*, x_K^*, D^*]$ with regressors $\boldsymbol{X}_I = [1, w_1, w_2, p_D, Q_L, Q_O]$. The system in H_I (Eq. (5) is similar with H_I, where the endogenous vector is $[x_L^*, x_K^*, L^*]$ with regressors $\boldsymbol{X}_{II} = [1, w_1, w_2, p_L, Q_D, Q_O]$. Under H_I in Eq. (6), we have $\boldsymbol{X}_{III} = [1, w_1, w_2, p_L, p_D, Q_D, Q_L, Q_O]$ and only $[x_L^*, x_K^*]$ is endogenous. All three systems are estimated jointly with a standard *Seemingly Unrelated Regression* (SUR) model with two-step feasible GLS:

$$\hat{\beta}^{GLS} = \left(\boldsymbol{X}'\left(\widehat{\boldsymbol{\Sigma}}^{-1} \otimes \mathbf{I}_n\right)\boldsymbol{X}\right)^{-1} \boldsymbol{X}'\left(\widehat{\boldsymbol{\Sigma}}^{-1} \otimes \mathbf{I}_n\right)\boldsymbol{y} \tag{10}$$

The actual joint density of (log) deposits and loans, conditional on prices W, is estimated non parametrically using a bivariate variation of the Kolmogorov-Smirnov statistic but uses the estimated density values of cumulative distribution, given by:

$$\tilde{p}(d, l) = \frac{1}{n} \sum_{1}^{n} K_{h_1}(d - logD_i) \cdot K_{h_2}(l - logL_i) \tag{11}$$

where K_h is the Gaussian kernel and the bandwidths h_1 and h_2 follow Silverman's (1986) rule of thumb.

The implied joint density $\hat{\tilde{p}}^{(k)}(d, l)$ differs by hypothesis. Under H_I (financial intermediation), the conditional density from the quadratic cost function estimated via SUR has the form:

$$p^I(D_i \mid L_i, W_i) = \mathcal{N}\left(D_i\, ;\, \boldsymbol{X}_{\boldsymbol{I},\boldsymbol{i}} \widehat{\boldsymbol{\beta}}_D, \hat{\sigma}_D^2\right)$$

The implied joint density is then $\tilde{p}^I(d, l) = \bar{p}^I(d) \widehat{m}_L(l)$ with $\bar{p}^I(d) = \frac{1}{n} \sum_{i=1}^{n} \mathcal{N}\left(d\, ;\, \log\hat{\mu}_{D,i}, \hat{\sigma}_D^2\right)$ is the mixture of fitted normals evaluated on the deposit grid, and $\widehat{m}_L(l)$ is the univariate kernel density estimation of log loans. Under H_{II} (loans input deposits output), the conditional density is given by

$$p^{II}(L_i \mid D_i, W_i) = \mathcal{N}\left(L_i\, ;\, \boldsymbol{X}_{\boldsymbol{II},\boldsymbol{i}} \widehat{\boldsymbol{\beta}}_L, \hat{\sigma}_L^2\right)$$

[3] Shephard's lemma states that the conditional factor demand for input i is obtained by differentiating the cost function with respect to the price of that input.

The implied joint density now is $\tilde{p}^{II}(d,l) = \hat{m}_D(d)\bar{p}^{II}(l)$ where $\bar{p}^{II}(l) = \frac{1}{n}\sum_{i=1}^{n} \mathcal{N}\left(l\,;\ \log\hat{\mu}_{L,i}, \hat{\sigma}_L^2\right)$ and $\hat{m}_D(d)$ is the univariate kernel density estimation of log deposits. Under H_{III} (production approach), the D and L are predetermined so the model places no restriction on the joint distribution, and the implied density is the independent product univariate kernel density estimations of logs: $\tilde{p}^{III}(d,l) = \hat{m}_L(l)\,\hat{m}_D(d)$.

For each hypothesis $k \in (I, II, III)$, we develop a *deviance test statistic* to compare the actual multivariate densities to the implied ones given by:

$$\mathcal{D}^{(k)} = \max_{d,l} |\,\tilde{p}(d,l) - \tilde{p}^{(k)}(d,l)| \tag{12}$$

This is a bivariate analogue of the Kolmogorov–Smirnov statistic, operating on estimated density values rather than on the empirical cumulative distribution function. We obtain p-values of the test statistic; we bootstrap the data using 10,000 replications with resampling and replacement. For each hypothesis k, we have:

1. Draw a bootstrap sample of size n with replacement from the initial data: $\{(\log D_i^*, \log L_i^*)\}_{i=1}^{n}$.
2. Recompute $\mathcal{D}^{(k)*}$on the bootstrap sample using the same procedure.
3. Repeat for B = 10,000 replications.

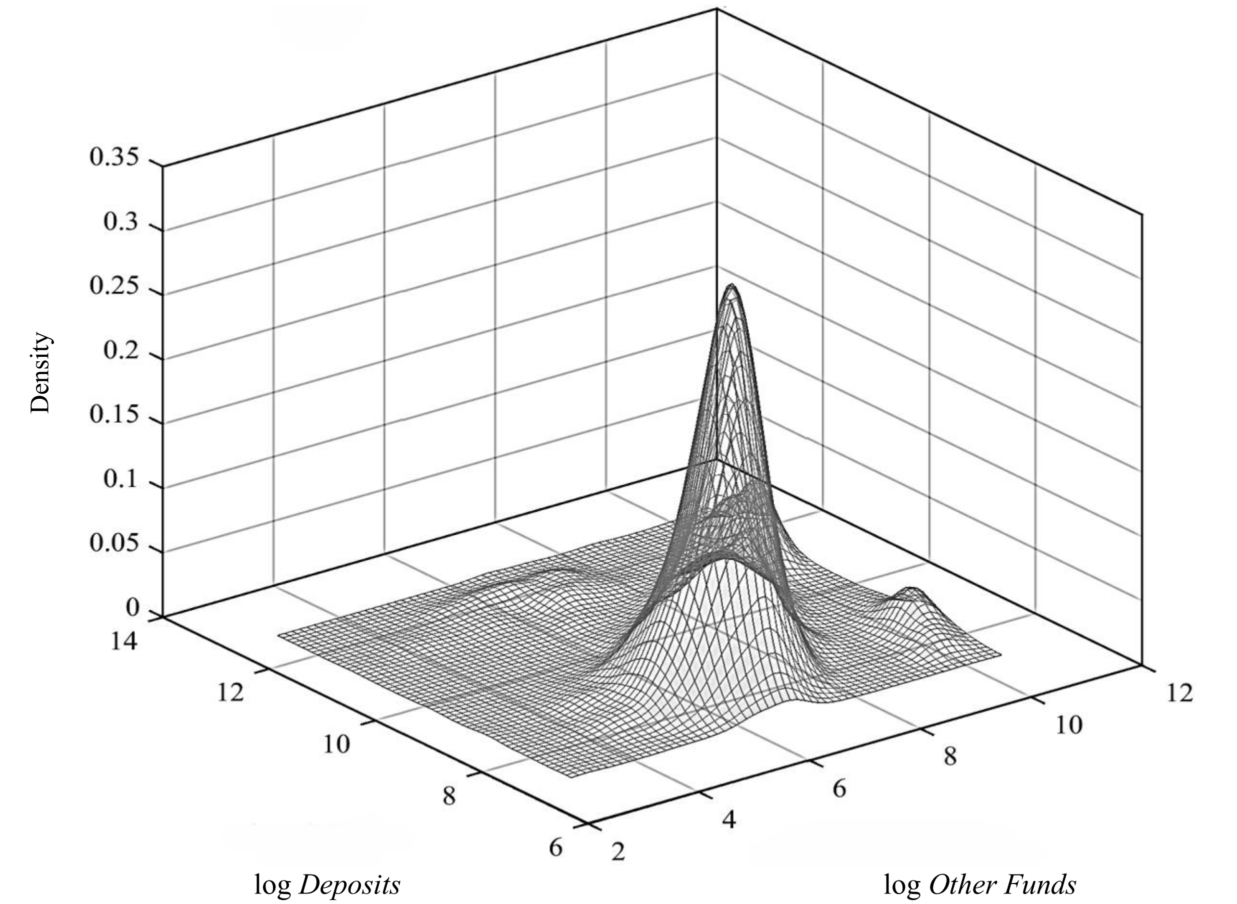
The bootstrap p-value is:

$$p\text{-}value = \frac{1}{B}\sum_{b=1}^{B} \mathbf{1}\left(\mathcal{D}^{(b)} \geq \mathcal{D}^{\text{obs}}\right)$$

where $\mathcal{D}^{(b)}$ is the deviance statistic computed on the b-th bootstrap sample. A *p-value* > 0.05 indicates failure to reject the null hypothesis that the implied density is statistically identical with the actual density, so the hypothesis is supported by the data.

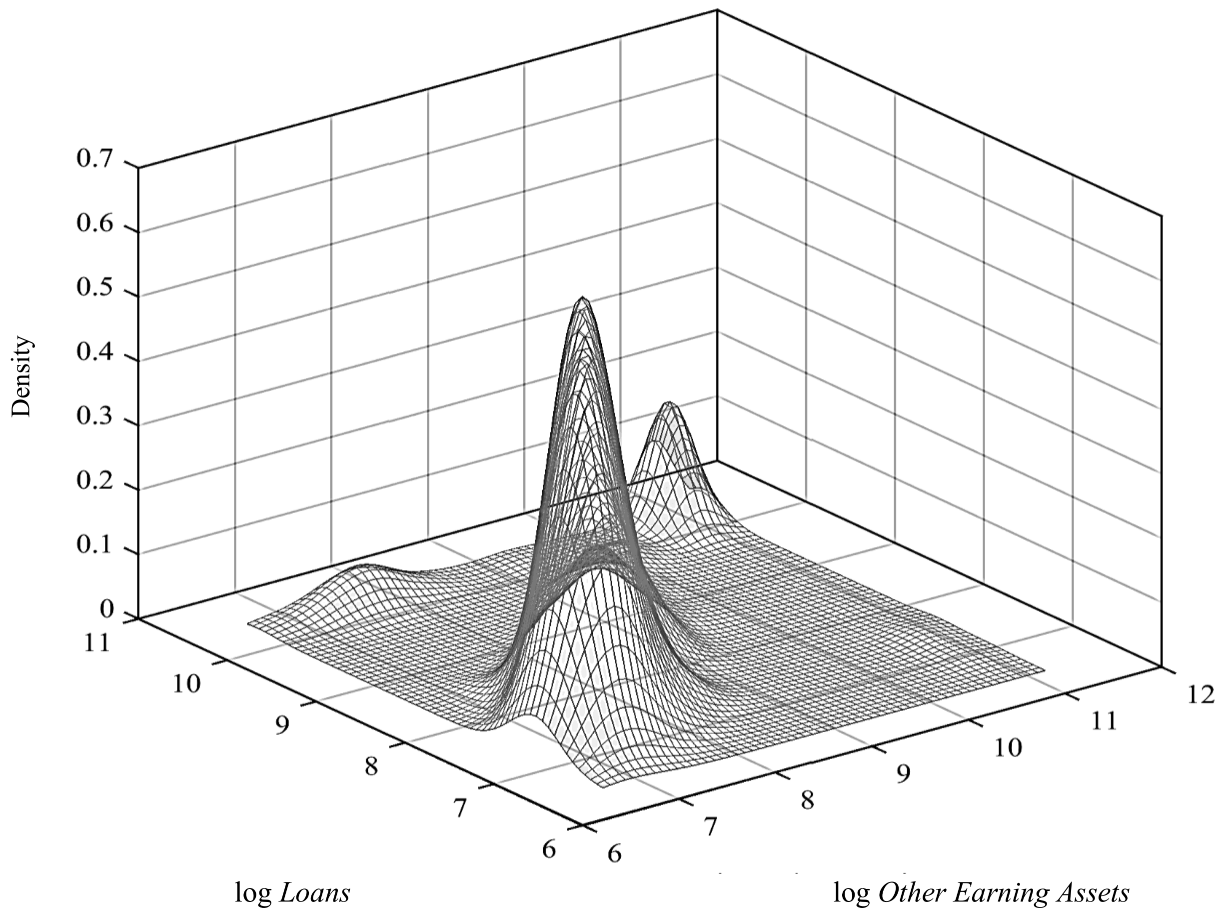
In Figures 1 and 2, we report the bivariate densities for the potential outputs.

Figure 1. Joint distributions of Deposits and Other Funds



The bivariate kernel density estimate of (log) *deposits* and *other funds* reveals a concentrated unimodal joint distribution, with the mass of the density clustered around a single dominant peak. This pattern shows a strong comovement between the two variables across the sample of U.S. commercial banks over the period 2012–2025. It suggests that deposits and other funds are not independently distributed but rather jointly determined within the liability side of the banks. The tight concentration of probability mass is consistent with the view that U.S. banks maintain relatively homogeneous funding sources, reflecting the common post Dodd-Frank regulatory and macroeconomic environment and the Basel III liquidity requirements that govern both retail deposit-taking and wholesale funding activity. The absence of multimodality in this joint distribution implies that we don't see evidence of distinct subgroups of banks pursuing fundamentally different funding strategies with respect to deposits and other funds, and thereby diverse business models.

Figure 2. Joint distributions of Loans and Other Earning Assets



Relative to Figure 1, the bivariate kernel density estimate of (log) *loans* and *other earning assets* shows a more dispersed joint distribution, with the probability mass spread across a broader region with less peaks. This high variation reflects the cross-sectional heterogeneity in the asset side of U.S. banks. The distribution suggests that U.S. banks substantially differ in how they allocate earning assets between traditional loan portfolios and marketable assets. This heterogeneity is driven by differences in business model, size, and risk appetite across banks. The broader spread of the joint density of loans and other earning assets relative to deposits and other funds in Figure 1, provides initial evidence that the asset side of the balance sheet shows greater structural diversity than the liability side, motivating of treating the input-output classification as a bank-specific rather than a sample determined.

In Figure 3 we report the actual and implied densities from H_I (Figure 3a), H_{II} (Figure 3b), and H_{III} (Figure 3c) while in Table 1, we report the results of the $\mathcal{D}$ statistic.[4]

[4] We report the analogues of Figures 1-2 for all possible cases reported in Figure S1 in the Supplement. We have tried to test the production approach as well but the number of observations (3,570) is too small to permit accurate estimation of the four-dimensional joint distribution of outputs across the various subsamples. Even for the whole

Figure 3. Actual and Implied Densities

Notes: Figure 3 is organised into three panels, Figures 3a, 3b, and 3c, corresponding respectively to Hypothesis I (Financial Intermediation), Hypothesis II (Reverse Approach), and Hypothesis III (Production Approach). The black line is the actual line, while the red dot line is the implied line for the corresponding H_k. The horizontal axes are the log of quantities.

Figure 3a. Hypothesis I – Financial Intermediation (deposits inputs, loans outputs)

log *Deposits* | log *Other Funds* | log *Loans* | log *OEA*

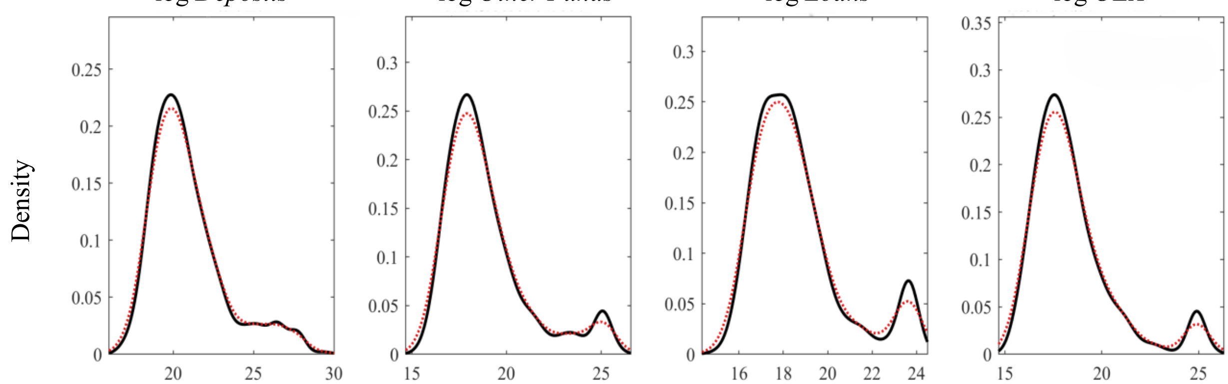


Figure 3b. Hypothesis II – Reverse Approach (loans inputs and deposits outputs)

log *Deposits* | log *Other Funds* | log *Loans* | log *OEA*

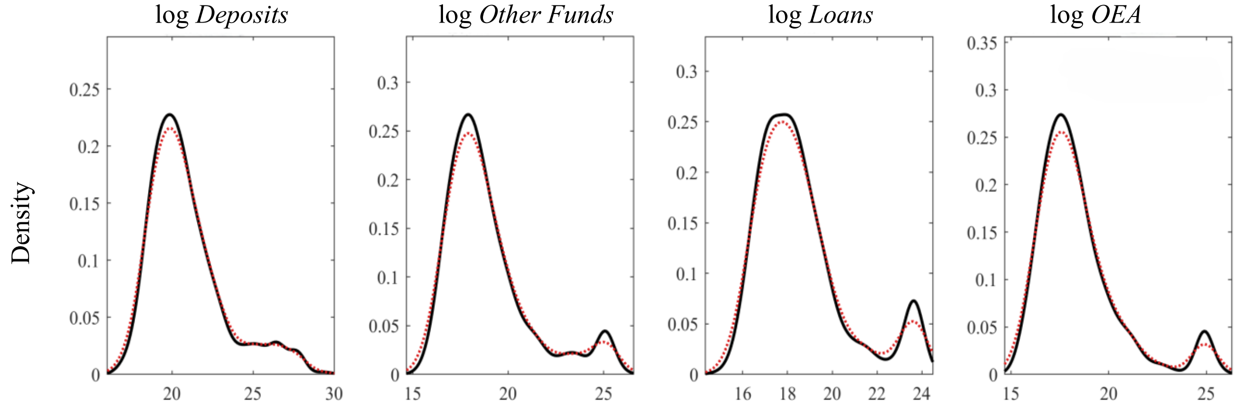


Figure 3c. Hypothesis III – Production Approach (deposits and loans both outputs)

log *Deposits* | log *Other Funds* | log *Loans* | log *OEA*

sample the estimation of this density is challenging due to the curse of dimensionality.

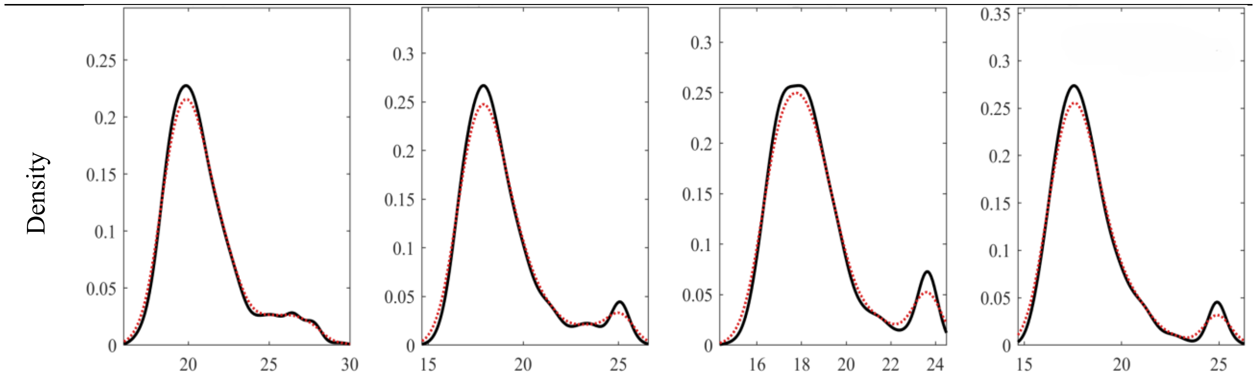


Table 1. Empirical results for deviance statistic $\mathcal{D}$

Notes: Large (small) banks were defined as those in the upper (lower) third of the distribution of log total assets. This partition is chosen so that the number of observations is equal and p-values of $\mathcal{D}$ do not depend so much on sample size. Therefore, we have: *Small:* $log\ (TA) \leq q_{1/3}$, *Medium:* $q_{1/3} < log\ (TA) \leq q_{2/3}$, *Large:* $q_{2/3} < log\ (TA)$

	Hypothesis I: Financial Intermediation		*Hypothesis II: Reverse Approach*		*Hypothesis III: Production Approach*	
	$\mathcal{D}^{(I)}$	p-*value*	$\mathcal{D}^{(II)}$	p-*value*	$\mathcal{D}^{(III)}$	p-value
Entire sample	0.081	0.511	0.083	0.448	0.045	0.649
Small banks	0.276	0.390	0.281	0.389	0.118	0.409
Medium banks	0.279	0.551	0.287	0.532	0.153	0.383
Large banks	0.128	0.411	0.127	0.539	0.127	0.277

Figure 3 presents the actual marginal density (solid black line) against the implied marginal density derived from the fitted quadratic cost function under each hypothesis (dotted red line) for each variable of interest. Under H_I (Figure 3a), the implied densities for log loans and log other assets closely track the actual distributions across the central mass, though they exhibit a broader spread reflecting the parametric smoothing imposed by the fitted conditional normal $p(D|L,W)$. The implied density of log deposits departs slightly more from the actual, particularly in the left tail, where the mixture of fitted normals from the SUR system underestimates the density of smaller deposit-holding institutions. This pattern is consistent with the intermediation view under which deposits are endogenously treated as inputs. Under H_{II}, the implied density is most informative for log deposits, where the fitted conditional normal $p(L|D,W)$ generates an implied marginal that is smoother and more symmetric than the actual distribution. indicating that the reverse approach imposes distributional restrictions that the data only partially support. For log loans and log other earning asset, the implied and actual lines are closely aligned, which is expected but offers no discriminatory power.

Under H_{III} (Figure 3c), the implied densities are derived under the independence assumption; that is, the implied joint density factorises into the product of marginal Kernel densities estimated with a wider bandwidth. The resulting implied densities are smoother and more symmetric than the actual distributions, and the departure between actual and implied is more pronounced for log loans and log other earning assets than for log deposits. Economically, this divergence for loan-side variables suggests that the production approach induces a degree of smoothness that the data do not support, particularly for larger institutions with concentrated loan portfolios. The closer alignment between actual and implied densities for log deposits under H_{III} is consistent with the finding reported in Table 1, where Hypothesis III fits small banks adequately, as deposit distributions among small banks tend to be tighter and closer to smooth implied density.

Turning to Table 1, for the full sample of 3,570 bank-year observations, all three hypotheses yield p-values above conventional significance thresholds [$p^{III} = 0.649$, $p^{I} = 0.511$, $p^{II} = 0.448$), indicating that none can be rejected on the basis of the $\mathcal{D}$ statistic alone. The corresponding deviance values are small, with H_{III} producing the lowest observed statistic ($\mathcal{D}^{III} = 0.045$), suggesting that the production approach imposes the least departure between actual and implied densities at the aggregate level. This is consistent with Figure 3b, reflecting the absence of cross-restrictions and the distributional smoothing induced across all asset-side variables.

Among the three size groups, small banks yield the highest $\mathcal{D}$ statistics across all hypotheses ($\mathcal{D}^{(III)} = 0.118$, $\mathcal{D}^{(I)} = 0.276$, $\mathcal{D}^{(II)} = 0.281$), yet all p-values remain comfortably above conventional rejection thresholds ($p^{III} = 0.409$, $p^{I} = 0.390$, $p^{II} = 0.389$). The relatively higher p-value for H_{III} indicates that the production approach fits small banks best among the three hypotheses, consistent with the relationship-banking model that characterises small U.S. community banks. The nearly identical *p-values* for $H_I$and $H_{II}$for this subgroup suggest that the data cannot discriminate between the intermediation and reverse configurations for smaller institutions, a pattern that is visible in Figure 3a where the implied density of log deposits under H_I does not depart dramatically from the actual even for the left tail of the distribution, reflecting the tighter and more symmetric deposit distributions among smaller banks.

For medium banks, H_I achieves the highest *p-value* ($p^{I} = 0.551$) while H_{III} declines ($p^{III} = 0.383$). The observed deviance under H_I ($\mathcal{D}^{(I)} = 0.279$) is substantial but the bootstrap distribution of $\mathcal{D}^{(I)}$ is sufficiently dispersed to keep the hypothesis acceptable. This reversal is economically meaningful: medium-sized U.S. regional banks, operating beyond purely local lending relationships, are more naturally characterised as classical financial intermediaries channelling deposit funding into diversified loan portfolios. The widening gap between the H_{III} and H_I *p-values* for medium banks is consistent with Figure 3c.

The size gradient ends among large banks, where H_{III} records its lowest *p-value* across all subsamples

($p^{III} = 0.277$) while H_{II} reaches its highest ($p^{II} = 0.539$) and H_I remains stable ($p^I = 0.411$). The fact that H_{III} may be marginally discriminatory for large banks (its p-value almost 20% below that of the entire sample) shows a novel empirical finding: the production approach becomes progressively less compatible with the data as bank size increases. Large banks are most naturally characterised by the reverse configuration, consistent with the FMC or wholesale-balance-sheet interpretation. This is directly shown in Figure 3b, where the implied deposit density under H_{II} is smoother and more symmetric than the actual, a divergence that is more pronounced for large banks than for the other subgroups, reflecting the concentrated and asymmetric deposit structures that characterise large institutions.

4. Discrimination based on the Directional Technology Distance Functions

The cost function-based discrimination approach developed in Section 3 provides a solid method for comparing our competing hypotheses about the input-output classification of loans and deposits. However, it is subject to an important limitation: the conditional distribution framework relies on the assumption that the endogenous variable in each hypothesis is determined by the cost-minimization problem, and thus, by construction, the choice variables are treated as inputs. This creates a circularity where the classification we investigate is embedded in the structure of the estimating equations. Moreover, the approach operates at the full-sample level, implicitly imposing a homogeneous input-output structure across all banks. The preferred hypothesis differs across bank size groups, indicating meaningful heterogeneity in banking technology that a single sample-wide classification cannot capture. To address both limitations at the same time, we employ the *Directional Technology Distance Function* (DTDF), which offers two key advantages. First, it separates the endogeneity problem from the classification problem by treating the direction vector, whether a variable behaves as an input or an output, as an object of inference rather than a main assumption. Second, by estimating directions at the bank level, it allows the data to determine whether the intermediation approach, the production approach, or an intermediate configuration best describes each institution.

Following Chambers et al. (1998), the directional distance function is:

$$\vec{D}\left(x, y; g_x, g_y\right) = \max\left\{\beta: (x, y) + \beta\left(-g_x, g_y\right) \in \mathcal{T}\right\} \tag{13}$$

if there exists $\beta \in \Re$ such that $(x, y) + \beta\left(-g_x, g_y\right) \in \mathcal{T}$, otherwise $\vec{D}\left(x, y; g_x, g_y\right) = -\infty$. In this context, g_x and g_y denote specific directions in the input and output spaces. The following properties can be established:

D1. Translation property: $\vec{D}\left(x - \alpha g_x, y + \alpha g_y; g_x, g_y\right) = \vec{D}\left(x, y; g_x, g_y\right) - \alpha, \quad \forall \alpha \in \Re$

D2. g-homogeneity of degree minus one: $\vec{D}\left(x, y; \lambda g_x, \lambda g_y\right) = \lambda^{-1} \vec{D}\left(x, y; g_x, g_y\right), \quad \forall \lambda > 0$

D3. Input monotonicity: $x' \geq x \Rightarrow \vec{D}\left(x', y; g_x, g_y\right) \geq \vec{D}\left(x, y; g_x, g_y\right)$

D4. Output monotonicity: $y' \geq y \Rightarrow \vec{D}\left(x', y; g_x, g_y\right) \leq \vec{D}\left(x, y; g_x, g_y\right)$

D5. Concavity: $\vec{D}\left(x, y; \lambda g_x, \lambda g_y\right)$ is concave in (x, y).

We also have that $(x, y) \in \mathcal{T} \Leftrightarrow \vec{D}\left(x, y; \lambda g_x, \lambda g_y\right) \geq 0$ and thus, the profit function has the following representation:

$$\Pi(p, w) = \max_{x,y}: \left\{p'y - w'x + \vec{D}\left(x, y; \lambda g_x, \lambda g_y\right)\left(p'g_y + w'g_x\right)\right\} \tag{14}$$

From this representation, we derive the following first order conditions (Fare and Grosskopf, 2000):

$$\frac{w}{p'g_y + w'g_x} = \nabla_x \vec{D}\left(x, y; g_x, g_y\right) \tag{15}$$

and,

$$\frac{p}{p'g_y + w'g_x} = -\nabla_y \vec{D}\left(x, y; g_x, g_y\right) \tag{16}$$

where the sign of a direction vector implies economically that a positive means output orientation, while negative is an input orientation.

Suppose $z = (x, y) \in \Re^q$ and let $z = (x, L, D) \in \mathbb{R}^{k+l+d}$ denote the full vector of traditional inputs, loans, and deposits, where k,l, and d denote the dimensions of x, L, and D, respectively, while we set $q = k + l + d$. Let also $g = (g_x, g_L, g_D) \in \mathbb{R}^q$ denote the full direction vector, where $g_x \in \mathbb{R}^k$, $g_L \in \mathbb{R}^l$, and $g_D \in \mathbb{R}^d$ are the directions assigned to traditional inputs, loans and deposits, respectively. The quadratic form to DTDF is:

$$T(z) = \sum_{i=1}^{q} \alpha_i\,(z_i + g_i) + \sum_{i=1}^{q}\sum_{j=1}^{q} \beta_{ij}\,(z_i + g_i)(z_j + g_j) \tag{17}$$

subject to the following restrictions which impose translation:

$$\sum_{i=1}^{q} \alpha_i\, g_i + 1 = 0, \quad \sum_{j=1}^{q} \beta_{ij}\, g_j = 0,\ i = 1, \dots, q \tag{18}$$

along with symmetry condition:

$$\beta_{ij} = \beta_{ji}, \text{when } i, j = 1, \dots, q. \tag{19}$$

In theory, the traditional inputs x should be classified as inputs, which under the directional-distance representation corresponds to $g_x < 0$. We do not impose this as restriction; instead, we let the data verify ex post that the marginal posterior of g_x concentrates on negative values. Thus, we set $g_x = -\mathbf{1}_k$ where $\mathbf{1}_k$ is the unit vector in $\Re^k$. If we set, $g_L = \mathbf{1}_l, g_D = -\mathbf{1}_d$ then loans are outputs and deposits are inputs. If we set, $g_L = -\mathbf{1}_l, g_D = \mathbf{1}_d$ then loans are inputs and deposits are outputs. Loans and deposits can be both outputs if we set $g_L = \mathbf{1}_l, g_D = \mathbf{1}_d$. We, therefore, set the following direction assignment table of g_L, and g_D assignments:

Hypothesis	g_x	g_L	g_D
H_I: *Intermediation*	$-\mathbf{1}_k$	$+\mathbf{1}_l$	$-\mathbf{1}_d$
H_{II}: *Reverse*	$-\mathbf{1}_k$	$-\mathbf{1}_l$	$+\mathbf{1}_d$
H_{III}: *Production*	$-\mathbf{1}_k$	$+\mathbf{1}_l$	$+\mathbf{1}_d$

Therefore, Eq. (16) can be estimated, along with the first order conditions in Eq. (14) and (15) by imposing a variety of directional vectors, which implies different views about what constitutes inputs and what constitutes outputs. The imposition of restrictions allows formal model selection based on marginal likelihood and Bayes factors[5] (Kass and Raftery, 1995). The advantage of this approach lies in its ability to disentangle effectively the endogeneity problem from the classification problem of inputs and outputs. Under the first order conditions, all variables are treated as endogenous; but because of the different direction vectors this does not, by itself, imply any assumption about whether a variable is an input or an output. This limitation arises in cost function-based discrimination approaches using conditional distributions, where choice variables are inputs by construction. Importantly, when prices are not available, Eq. (17-18) can still be estimated by allowing for the endogeneity of z and imposing alternative direction vectors. The estimation then proceeds using Eq. (17) along with Eq. (15-16), subject to the constraints in Eq. (18) and (19).

The direction assignment table above defines the three configurations against which Bayes factors can be computed. As a complement to this fixed-restriction comparison, our empirical exercise treats the entire direction vector $g \in \mathbb{R}^q$, with $q = k + l + d = 6$, as an object of inference. That is, we estimate $g = (g_x, g_L, g_D)$ jointly with the technology parameters $(a, \beta, \mathbf{\Sigma})$ without imposing $g_x = -\mathbf{1}_k$. The input or output orientation of each variable, including the traditional inputs labour and capital, then emerges from the sign of the marginal posterior of the corresponding component of g, rather than from the sign restriction. This unrestricted formulation is what allows us to detect substantive heterogeneity across banks in the input-output classification of loans and deposits.

Empirically, we extend the input-output classification to include *other earning assets*, and *other funds*, so that $z = (x_L, x_K, q_D, q_{OEA}, q_{Lo}, q_F) \in \mathbb{R}^6$. The corresponding directions g_{OEA} and g_{OF} are estimated

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[5] Given two competing models M_A and M_B, defined by their respective direction vector assignments, the Bayes factor is given by: $BF_{AB} = \frac{p(\text{data}|M_A)}{p(\text{data}|M_B)}$, where $p(\text{data} \mid M_k) = \int p(\text{data} \mid \theta_k, M_k)\, p(\theta_k \mid M_k)\, d\theta_k$ is the marginal likelihood of model M_k, obtained by integrating out all parameters θ_k with respect to their prior. The values of $BF_{AB} > 3$ constitute substantial evidence in favor of M_A, while $BF_{AB} > 20$ constitutes strong evidence. The marginal likelihoods are computed numerically from the MCMC output using the harmonic mean estimator as a baseline, complemented by the Savage–Dickey density ratio for nested model comparisons.

freely, on the same footing as g_L and g_D. The Hypotheses in the table above remain unsure about the orientation of these two aggregates, and we therefore do not include them in the Bayes-factor comparison among hypotheses. We also assume that directions are fixed for a given bank over time, but *directions can be different for every bank*, even in the same country.[6] Let $i = 1, \dots, N$ index individual banks and $t = 1, \dots, T_i$ index time periods for bank i. To formalize what "*bank-specific fixed directions*" per bank means, the direction vector for bank i is denoted as:

$$g^{(i)} = (g_x^{(i)}, g_L^{(i)}, g_D^{(i)}) \in \mathbb{R}^q \tag{20}$$

where $g_x^{(i)} \in \mathbb{R}^k$, $g_L^{(i)} \in \mathbb{R}^l$, and $g_D^{(i)} \in \mathbb{R}^d$ are the directions assigned to traditional inputs, loans, and deposits, respectively for bank i. The assumption of time-invariance within banks implies $g_t^{(i)} = g^{(i)}$ for all t, so that directions constitute bank-specific structural parameters rather than time-varying quantities. The full parameter vector for bank i is therefore $\Theta^{(i)} = (\alpha, \beta, g^{(i)}, \Sigma)$, where α and β are the technology parameters of Eq. (13) that are common across banks, and $\mathbf{\Sigma}$ is the covariance matrix of the disturbances. The joint posterior distribution is:

$$p\left(g^{(i)}, \alpha, \beta, \mathbf{\Sigma} \mid \text{data}\right) \propto p\left(\text{data} \mid g^{(i)}, \alpha, \beta, \Sigma\right) \cdot p(g^{(i)}) \cdot p(\alpha, \beta) \cdot p(\Sigma)$$

where all priors are flat over $(-M, M)^{\dim(\theta)}$ with $M = 10^7$, and $p(\Sigma) \propto (\det\, \Sigma)^{-(d+1)/2}$. The sign of each element of $g^{(i)}$ carries direct economic content: a positive direction for loans, $g_L^{(i)} > 0$, indicates that loans behave as outputs for bank i, whereas $g_L^{(i)} < 0$ indicates an input orientation. The same interpretation applies to $g_D^{(i)}$ for deposits. The ross-sectional distribution of these estimated directions (its multimodality) constitutes a main empirical finding of this section.

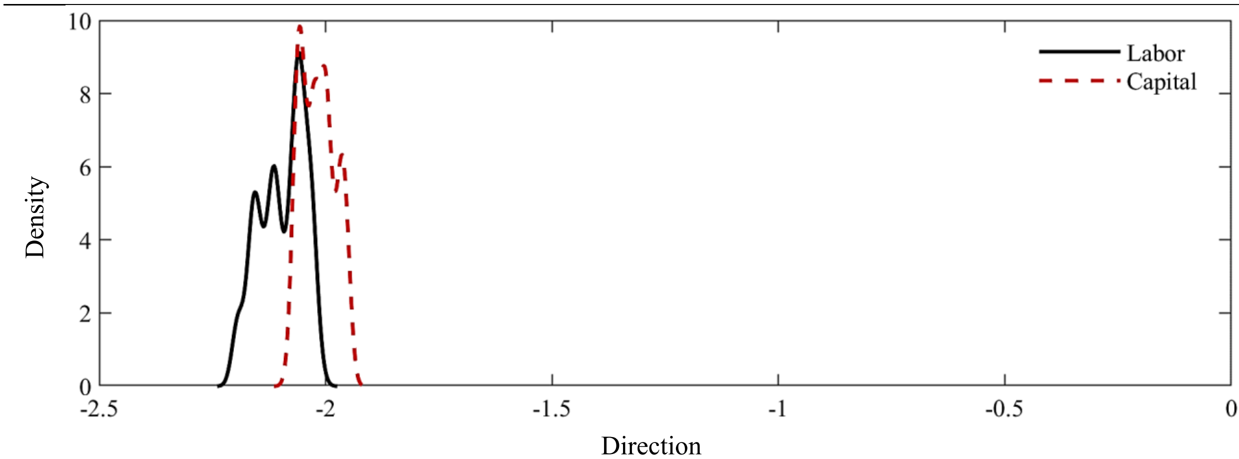
For estimation, we use a *Bayesian MCMC* procedure of Girolami and Calderhead (2011) under full Manifold MALA sampler, using information from the gradient and Hessian of the log-posterior (see Technical Appendix A1).[7] All our priors are flat over intervals of the form $(-M, M)$ where $M = 10^7$. Disturbances are added to the system, assuming to follow a multivariate normal distribution with covariance matrix Σ. Using the benchmark prior $p(\Sigma) \propto (\det \Sigma)^{-(d+1)/2}$ where d equals the number of equations in Eq. (17) and Eq. (15-16), integration with respect to the different elements of Σ is possible yielding an expression.

[6] In principle, we may allow for time-varying directions as well (see conclusion remarks). Here, we restrict attention to the time-invariant but bank-specific case mostly in order to allow for more data-based information in the determination of directions.

[7] The Girolami and Calderhead (2011) method explores the posterior much faster compared to traditional *Metropolis-Hastings* algorithms. A robustness exercise to MCMC sampler with *Adaptive Random Walk Metropolis-Hastings* sampler with economic interpretation is provided in the Supplement.

We proceed under two alternative assumptions: (i) *Directions are the same for all banks*. (ii) *Directions can be different for each bank*. The results are presented in Tables 4 and 5 as well as in Figures 4 and 5. Table 4 reports the posterior summaries treating the direction vector as common across all banks (left panel) and then allowing $g^{(i)}$ to vary by institution (right panel). Table 5 decomposes the bank-specific posteriors by directional sign.

Figure 4. Sample distributions of directions with same directions



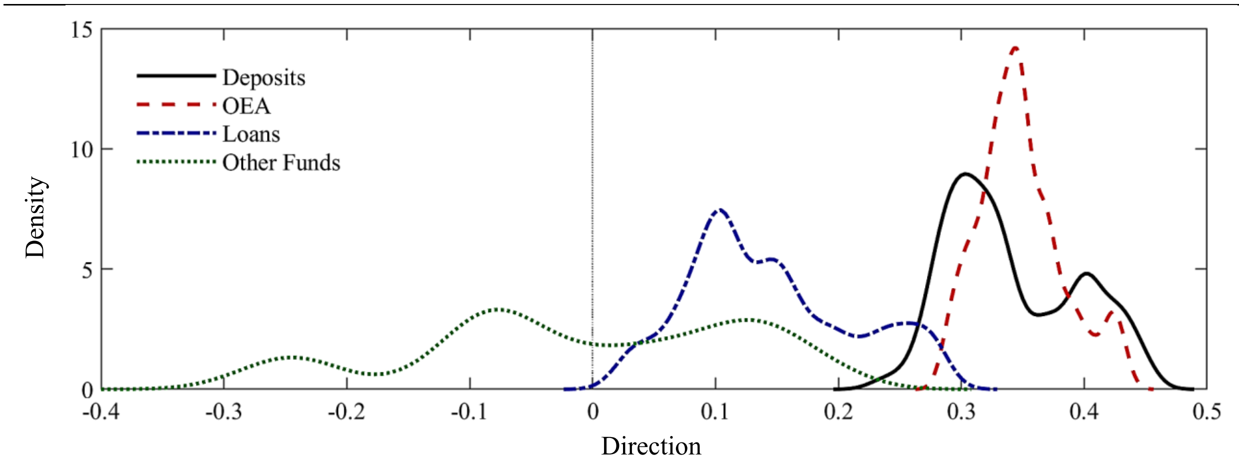


Figure 4 presents the posterior densities of the common direction vector g, under the full Manifold MALA sampler with the assumption that all banks share a single directional configuration. The posterior densities of g_L and g_K are concentrated and located constantly on the negative side of the directional space, with $g_L = -2.10$ (95% credible interval at $[-2.19, -2.02]$) and $g_K = -2.02$ (95% credible interval at $[-2.07, -1.95]$). The two distributions show a substantial mass overlap, indicating that the data identify labour and capital as occupying the same region of the input space (i.e. banks do not distinguish or optimize differently when employing capital versus labour). The Fisher-preconditioned sampler estimates these directions with high precision (posterior standard deviations of 0.050 and 0.037 respectively in Table 4), and the credible intervals lie strictly below zero by approximately forty posterior standard deviations.

Under the DTDF, this constitutes empirical validation of the input role traditionally assigned to physical capital and labour in the cost-function and production approach. Three out of four balance-sheet aggregates fall in the strictly positive directional region, i.e. $g_L = 0.341$ (95% credible interval at $[0.264, 0.443]$), $g_{OEA} = 0.349$ (95% credible interval at $[0.292, 0.427]$), and $g_{Ln} = 0.144$ (95% credible interval at$[0.033, 0.278]$). Thus, the aggregate-level posterior classifies deposits, OEA and loans as outputs, with deposits and OEA occupying almost similar positions in the directional space (posterior means within 0.008 of each other) and loans occupying a lower but still credibly positive position. This proximity between the deposit and *OEA* directions suggests that, at the level of the average banking firm, the directional contribution of deposit-taking and asset-market activity to the technology is comparable in magnitude. The posterior of $g_F = -0.011$ with a 95% credible interval of $[-0.268, 0.192]$, and therefore, *other funds* is the only directional component for which the common-direction posterior does not tell us whether wholesale funding behaves as an input (consistent with the financial-intermediation interpretation) or as an output (consistent with the interpretation that wholesale-funding management generates value for the bank).

Table 4. Posterior Summaries of Directional Measures

Notes: The table reports posterior summaries of directional measures. In the left panel, the table displays the common direction vector $g \in \mathbb{R}^6$ obtained from the MCMC chain in which all banks share a single direction vector. We report the posterior mean, posterior standard deviation, and the 2.5% and 97.5% quantiles of the posterior distribution, computed from 3,000 post-burn-in draws (4,000 iterations, 1,000 burn-in). In the right panel, we report the cross-sectional summary of bank-specific posterior means $\hat{g}^{(i)} = E[g^{(i)} \mid \text{data}]$ across the 255 U.S. banks in the sample, computed from 3,000 post-burn-in iterations (4,000 iterations, 1,000 burn-in, no thinning) per bank. Cross-bank standard deviations represent the cross-sectional heterogeneity in posterior point estimates, and they are not directly comparable to the posterior standard deviations of the common-direction model. Both chains are estimated under the full Riemann Manifold MALA sampler of Girolami and Calderhead (2011) using the per-observation-averaged empirical Fisher metric and the curvature-correction terms (Appendix 1). Both chains target the same posterior under flat technology priors on each component of the standardised direction vector.

	Common-Direction Posterior, $g^{(i)}$				**Cross-bank Distribution of $\hat{g}^{(i)}$**				
	Mean	*SD*	*2.5%*	*97.5%*	*Mean*	*SD*	*Median*	*2.5%*	*97.5%*
Traditional Inputs									
Labour	-2.096	0.050	-2.194	-2.024	-1.957	1.189	-1.999	-4.203	0.477
Capital	-2.016	0.037	-2.070	-1.951	-2.043	1.142	-2.090	-3.943	0.329
Balance-sheet Aggregates									
Deposits	0.341	0.053	0.264	0.443	-0.090	1.231	0.016	-2.621	2.190
Other earning assets	0.349	0.034	0.292	0.427	-0.249	1.124	-0.095	-2.581	1.869
Loans	0.144	0.067	0.033	0.278	-0.504	1.079	-0.471	-2.728	1.464

Other Funds	-0.011	0.133	-0.268	0.192	0.205	1.360	0.265	-2.360	2.515

Figure 5. Sample distributions of directions with different directions

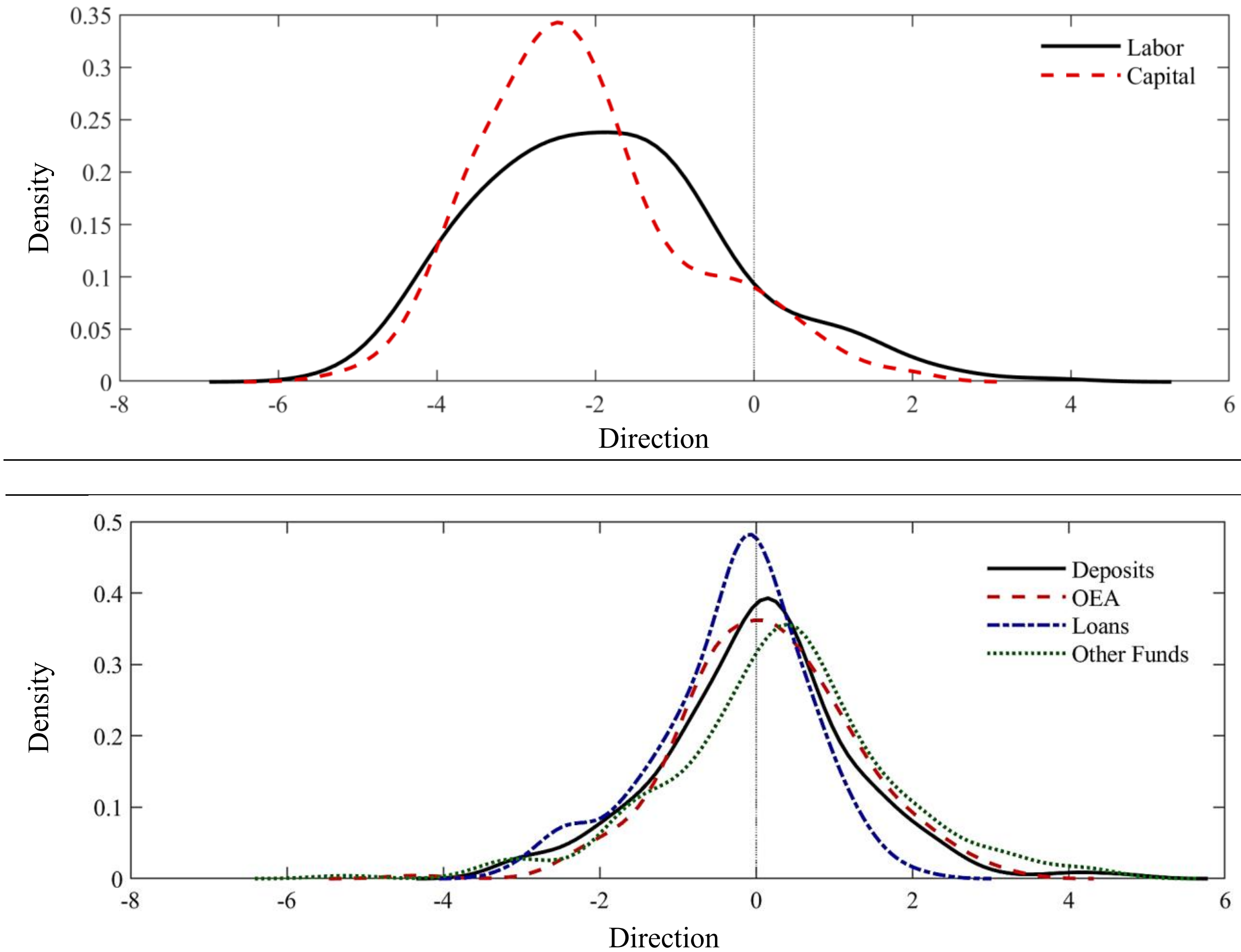


Table 5. Sign decomposition of bank-specific directions

Notes: For each variable in the DTDF, the table reports the percentage of banks for which the posterior mean of the bank-specific direction $\hat{g}_j^{(i)}$ is positive (output orientation) or negative (input orientation), along with the percentage of banks in which the 95% credible interval of $g_j^{(i)}$ contains zero (i.e. banks for which the directional orientation cannot be classified). Posterior summaries are based on 255 U.S. financial institutions for the period 2012-2025, with 3,000 post burn-in MCMC iterations per bank obtained under full Riemann Manifold MALA sampler of Girolami and Calderhead (2011), implementing the natural-gradient drift, the metric-derivative correction, and the Christoffel-symbol trace-correction terms detailed in Appendix 1. The empirical Fisher information $G(\theta) = N^{-1}\sum_n s_n(\theta)s_n^{\top}(\theta)'$ is computed as the per-observation average of the score outer product.

	% Banks Output	*% Banks Inputs*	*% Zero*	*% Clearly Output*	*% Clearly Input*
	(sign of $\hat{g}^{(i)}$)		*(95% CI =0)*	*(95% CI >0)*	*(95% CI <0)*

Traditional Inputs					
Labour	5.1	94.6	44.7	0.4	54.9
Capital	3.9	96.1	29.0	0.4	70.6
Balance-sheet Aggregates					
Deposits	50.6	49.4	74.1	17.3	8.6
Other earning assets	45.5	54.5	76.9	11.8	11.4
Loans	34.1	65.9	80.0	8.2	11.8
Other Funds	59.6	40.4	71.0	21.6	7.5

Figure 5 displays the cross-bank densities of the posterior means $\hat{g}^{(i)}$, obtained when the direction vector is allowed to vary across banks under the manifold sampler. The cross-bank distributions of $\hat{g}_L^{(i)}$ and $\hat{g}_K^{(i)}$ are unimodal, symmetric, and concentrated on the negative side, with cross-bank means of -1.96 and -2.04, respectively; and cross-bank standard deviations of 1.19 and 1.14. The two densities are similar in shape and location and show that the directional role of labour and capital is identified as input for the majority of banks, where 94.9% of institutions exhibit negative-mean labour directions and 96.1% exhibit negative-mean capital directions, although the per-bank credible interval coverage is wider (54.9% of banks display credibly negative labour directions and 70.6% display credibly negative capital directions, with the residual share predominantly zero, 44.7% and 29.0%, rather than oppositely signed). Only 0.4% of banks exhibit a credibly positive direction for either variable. The data therefore support for the input role of traditional production factors at the institution level.

The four balance-sheet aggregates exhibit unimodal cross-bank distributions centred near zero, with peak densities approximately 0.33 (*deposits*), 0.37 (*OEA*), 0.34 (*loans*) and 0.30 (*other funds*). Cross-bank standard deviations range from 1.08 (loans) to 1.36 (other funds). The cross-bank distributions place substantial mass on both sides of zero, with the means of $\hat{g}_{DK}^{(i)}$ $(= -0.090)$, $\hat{g}_{OEA}^{(i)}$ $(= -0.249)$, $\hat{g}_{Ln}^{(i)}$ $(= -0.504)$ on the negative side, in contrast to the positive common-direction posteriors of Figure 4 for the same three variables. This pattern of a positive aggregate direction co-existing with a negatively skewed cross-bank distribution of bank-specific direction means, indicates that the sample-level positive classification of these three aggregates is driven by a sub-population of banks with strong positive bank-specific directions, while the typical bank in the sample exhibits a weak input-orientation.

Finally, the directional role of loans exhibits the highest indeterminate share among the four balance-sheet aggregates (80.0%), the most negative cross-bank mean (-0.504), and the most negatively skewed cross-bank distribution. Loans are classified as input for 11.8% of banks and as output for 8.2% of banks. Economically, we argue that, at the institution level, the data identify loans as input more often than output, with a substantial subset of banks exhibiting input directional behaviour for their loan portfolio (i.e. financial intermediation). The contrast with the common-direction posterior is therefore particularly highlighted for loans: the positive common-direction estimate of 0.144 coexists with a negatively skewed bank-specific distribution centred at -0.504. The data therefore reject the 'one size fits all' framing: at the institutional level, banks differ structurally in the directional role they assign to lending, and the average-bank classification under a homogeneous direction restriction is not representative of the typical institution. To our knowledge, this outcome is novel and puts forward a new perspective on banking technology.

5. Reality checks

In this section, we test whether the bank-specific posterior means $\hat{g}^{(i)}$, reported in the right panel of Table 4, and the cross-bank dispersion illustrated in Figure 5 are robust to misspecification in the partitioning of environmental variables across banks, i.e. sub-sample stability with respect to environmental, price, and quantity variables. We develop a 'reality check' based on Athey and Imbens (2015) measure of robustness to misspecification in linear regression models, in order to examine the sensitivity of our DTDF approach. The Athey-Imbens procedure assesses whether posterior point estimates of $\hat{g}^{(i)}$remain stable across data partitions; however, it does not address misspecification of the DDF form itself.

Briefly, our 'reality check' proceeds as follows. Suppose the model is:

$$Y = \beta X + \gamma' Z + \varepsilon$$

where X is the regressor of interest with β is the scalar coefficient parameter of interest, Z is a vector of other (control) variables and γ is the corresponding coefficient vector. Athey and Imbens (2015) propose to estimate the model based on different classifications of the Zs. The standard deviation of $\hat{\beta}$ across all classifications quantifies how much the parameter of interest moves when the controls Z are partitioned in different ways and, thereby, serves then as a measure of robustness of correct specification. If the estimates $\hat{\beta}$ are relatively robust across different ranges of the Z variables, one is relatively confident that specification errors are likely to be small.

For the case of DTDF, the parameter of interest is $g^{(i)}$ (the bank-specific direction vector), while the control variables are those that can be partitioned to form sub-samples for robustness: prices of labour, capital, deposits, funds and prices of loans, other assets and other earning funds (collectively denoted by P); quantity values for loans, deposits, other funds and other earning assets as well as other funds (Q), plus labor and capital (L, K). For each variable, we consider values above and below the median as the separating values. Our interest focuses on estimates (posterior means) of the direction vector g. Suppose $\hat{g}^{(j)}$ denotes the posterior mean of g in a sub-sample indexed by $j = 1, \dots, J$. Our measure of robustness is:

$$R^{(c)} = (J^{(c)})^{-1} \sum_{j=1}^{J^{(c)}} \parallel \hat{g}^{(j,c)} - \bar{g} \parallel_1 \tag{21}$$

where $c \in \{1, \dots, 7\}$ indexes the separator configuration, $J^{(c)} = 2^{s^{(c)}}$is the number of sub-samples induced by configuration c with $s^{(c)}$separator variables, $\hat{g}^{(j,c)}$is the posterior mean of g in sub-sample j of configuration c, $\bar{g}$ is the posterior mean of g in the full sample, and $\|\cdot\|_1$is the L_1norm summed across the six directional components. The sub-samples are constructed by median splits applied independently to each separator variable, producing a binary partition, so s separator variables generate 2^s disjoint cells. For separator configuration c involving $s^{(c)}$ separator variables indexed by the set $\mathcal{S}^{(c)} \subseteq \{V_1, \dots, V_K\}$, let $b_j \in \{0,1\}^{s^{(c)}}$be a binary vector of length $s^{(c)}$ indexed by $j = 1, \dots, 2^{s^{(c)}}$. Sub-sample j of configuration c is defined as the set of bank-year observations:

$$\mathcal{D}_j^{(c)} = \{(i,t) \colon \mathbf{1}\{V_{i,t,k} > \text{median}(V_k)\} = b_{j,k},\ \forall k \in \mathcal{S}^{(c)}\} \tag{22}$$

where $V_{i,t,k}$ is the value of separator variable k for bank i in year t, $\text{median}(V_k)$ is its sample median, and $b_{j,k} \in \{0,1\}$ encodes which side of the median sub-sample j corresponds to.

Due to the large number of estimated models, it is necessary to follow a shortcut in MCMC. Specifically, the posterior of each model is approximated using the sampling-importance-resampling (SIR algorithm) procedure of Rubin (1988). The original MCMC sample is re-used or re-shuffled using the ratio of the posteriors to determine the probability of inclusion with replacement. As this is not particularly computationally intensive, the original MCMC sample (consisting of 100,000 draws from the posterior) is re-shuffled to produce new MCMC samples of length 10,000.[8]

For sub-sample $\mathcal{D}_j^{(c)}$, the posterior of the directional parameter under the sub-sample likelihood is approximated by re-weighting the original full-sample MCMC chain $\{g^{(s)}\}_{s=1}^{S}$. The importance weight assigned to MCMC draw $s$for sub-sample j of configuration c is:

$$w_{s,j}^{(c)} = \exp\Big(\sum_{(i,t)\in\mathcal{D}_j^{(c)}} \ell_{i,t}(\theta_s) - \sum_{(i,t)\in\mathcal{D}_{\text{full}}} \ell_{i,t}(\theta_s)\Big) \tag{23}$$

where $\ell_{i,t}$ is the standard log-sum-exp normalisation for numerical stability, with the first term restricting the likelihood product to the sub-sample $\mathcal{D}_j^{(c)}$ and the second term runs over the full sample. The sub-sample posterior mean is given by:[9]

$$\hat{g}^{(j,c)} = \frac{\sum_{s=1}^{S} w_{s,j}^{(c)}\, g^{(s)}}{\sum_{s=1}^{S} w_{s,j}^{(c)}} \tag{24}$$

---

[8] For the relevant details that justify the procedure, the reader is referred to Smith and Gelfand (1992).

[9] We check if SIR weights become unstable by calculating the effective sample size (ESS) of the importance weights: $\text{ESS}_j^{(c)} = \frac{(\sum_s w_{s,j}^{(c)})^2}{\sum_s (w_{s,j}^{(c)})^2}$. If ESS falls below 10% of s for any sub-sample, that sub-sample's $\hat{g}^{(j,c)}$ is unreliable.

The critical values of R along with the p-values are obtained from a bootstrap (using 10,000 replications) to the entire data set. The bootstrap p-value is defined as:

$$p^{(c)} = \frac{1}{B}\sum_{b=1}^{B} \mathbf{1}\{ R^{(c,b)} \geq R_{\text{obs}}^{(c)}\} \tag{25}$$

where $B = 10{,}000$ is the number of bootstrap replications (clustered at the bank level), $R_{\text{obs}}^{(c)}$ is the observed R statistic on the full sample under configuration c, and $R^{(c,b)}$ is the R statistic computed on the $b-th$ bootstrap resample. We resample all banks with replacement, preserving the time-series structure within each bank and serial correlation, and recompute $\bar{g}$, all $\hat{g}^{(j,c)}$, and thereby, recompute $R^{(c,b)}$ on each resample.

Table 6. Specification / Reality tests

Notes: The p-value is estimated from the bootstrap in the whole data set. The sample is N=3750 bank-year observations, with 255 U.S. banks, quantities are q=6, and we consider the following separators: $|Q| = 4$, $|P|$=6, $|LK| = 2$. Total sub-sample models across all configurations are 5524. The full-sample GC posterior means of g used are labour = -2.095; capital = -2.016; deposits = +0.340; other earning assets = +0.348; loans = +0.144; and other funds = -0.011. The expected number of cells is $2^{s(c)}$, but only the number of empirically populated cells contribute to $R^{(c)}$.

Separators	Number of models	R statistic (via SIR posterior means)	p-value(1)	Mean ESS
Q	s = 4, J = 2^s = 16	0.420	0.573	25.2
P	s = 6, J = 2^s = 64	0.423	0.574	22.2
L, K	s = 2, J = 2^s = 4	0.405	0.556	40.9
Q, P	s = 10, J = 2^s = 1024	0.423	0.575	22.0
Q, (L, K)	s = 6, J = 2^s = 64	0.421	0.574	23.5
P, (L, K)	s = 8, J = 2^s = 256	0.423	0.575	22.0
Q, P, (L, K)	s = 12, J = 2^s = 4096	0.423	0.574	21.9

Table 6 reports the Athey–Imbens reality checks. Across all separator configurations, the R statistic remains tightly concentrated between 0.405 and 0.423. These values imply that the directional estimates move moderate when the sample is partitioned by quantities, prices, or labour-capital variables. The corresponding clustered-bootstrap *p-values* range from 0.556 to 0.575, indicating that the observed dispersion of sub-sample posterior means is not large relative to the clustered bootstrap distribution. The evidence supports the stability of the estimated direction vector. The reality checks support the argument that the classification of banks according to direction-based input-output orientations is not a product of a particular partition of prices, quantities, or labour-capital variables. While no finite-sample procedure can rule out all forms of misspecification, the evidence suggests that our specification is not sensitive to misspecifications that can be revealed *by the data* in the light of the Athey and Imbens (2015) test.

7. Balance-Sheet Centrality and Bank-Specific Orientations

The preceding evidence suggests that the input-output orientation is not homogeneous across banking institutions, as shown on the DTDF analysis in Section 4. This motivates a second-stage analysis that relates direction vectors to the position of banks based on their balance sheet and funding structure of the financial system. The rationale is not to simply repeat that directions are bank-specific, but to explain the determinants of those directions, which characteristics matter, and why some banks have different roles in the financial system. This is important because cross-bank heterogeneity in directions needs an economic interpretation beyond size and risk. Banks having more strong positions in the balance-sheet and funding structure of the banking system and operate as traditional intermediaries, treating deposits as inputs and loans as outputs, consistent with *Hypothesis I* in our framework, where other banks, instead, that are less central may adopt opposite or mixed production function.

In the baseline *intermediation* approach (*Hypothesis I*), banks collect deposits and other funding claims as inputs and transform them into loans as outputs, i.e. $g_{Ln}^{(i)} > 0$ and $g_{D}^{(i)} < 0$ in the DTDF framework. This view is consistent with the *intermediation-of-loanable-funds* (ILF) function (Diamond and Dybvig, 1983; Diamond, 1984; Holmström and Tirole, 1997), associated with *countercyclical leverage*, where banks must first obtain deposits before extending credit to borrowers. Deposits and loans are therefore slow-moving balance sheet variables, because lending is constrained by the prior accumulation of loanable funds. An alternative view emphasizes *financing through money creation* (FMC), associated with *pro-cyclical leverage*, where the extension of credit itself creates deposit-like liabilities and expands the balance sheet. Under the FMC function, particularly for large, leveraged or wholesale funded banks, loans behave as inputs while deposits or deposit-like liabilities adjust endogenously, i.e. $g_{Ln}^{(i)} < 0$ and $g_{D}^{(i)} > 0$. A key distinction between these two functions is leverage cyclicality. Under ILF function, leverage is typically countercyclical or acyclical because assets adjust slowly relative to equity. Under FMC function, leverage becomes procyclical: banks expand lending and assets faster than net worth increases during booms and deleverage by contracting credit and shrinking their balance sheets during downturns.

From the perspective of the intermediation approach, the early study of Sealey and Lindley (1977) argued that banks use deposits as inputs to generate loans as outputs and later studies extended this framework to delegated monitoring, liquidity insurance, moral hazard, and capital constraints (Tobin, 1969, Thakor, 1997). From the perspective of FCM model, Gu et al. (2013) and Donaldson et al. (2018) show that deposit-taking and lending are not separable activities, but arise together from frictions related to commitment, liquidity provision, monitoring, and private-money creation. These models explain why banks combine deposit services, lending, and inside-money claims within the same institution. A bridge between balance-sheet and wholesale-banking is provided by the studies of Borio and Disyatat (2011), Adrian and Shin (2014), and Gertler et al. (2016), who explained why large, leveraged, wholesale-funded institutions may not fit the simple ILF model. Adrian and Shin (2014) connect balance-sheet expansion, book leverage, equity, and credit supply, showing that core balance-sheet institutions actively adjust assets and leverage over the cycle. Gertler et al. (2016) further distinguish retail from wholesale banking and highlight why wholesale funding networks matter in crisis propagation.

The core empirical question we pose is different but complementary. Rather than asking why banks perform both functions, we ask *which function dominates for different banks*. The estimated direction vector allows this distinction to be identified directly: some banks exhibit a deposit-input/loan-output orientation, while others display a loan-input/deposit-output orientation. Our analysis therefore seeks to open the black box by relating these orientations to observable balance-sheet characteristics, such as size, leverage, wholesale funding intensity, and risk. Distinguishing the FMC and ILF functions is theoretical because the two views are sometimes conflated in empirical banking models (Jakab and Kumhof, 2015). In our framework, the bank-specific direction estimates map directly into an operational distinction (input/output orientation of loans and deposits) without needing to take a side on which model wins the leverage debate.

Hence, our empirical question is *whether core and wholesale-funded banks resemble more classical intermediaries, whether they are better described as money-creating balance sheet institutions, or whether they follow production-type or mixed configurations*. Rather than imposing ex ante which orientation should characterize banks occupying more central positions in the balance-sheet and funding structure of the banking system, we examine whether centrality is associated with any of the four directional configurations identified by the unrestricted DTDF model: intermediation, reverse/FMC, production, or mixed orientations. This distinction is therefore the key economic mechanism question in our framework and extends beyond the limited question “deposits first” versus “loans first”. It also concerns whether banks’ balance-sheet roles are better characterized by slow-moving against jump balance-sheet adjustment and countercyclical against procyclical leverage. We summarize this framework in two competing empirical hypotheses. The first is the classical intermediation hypothesis. If balance-sheet centrality reflects superior deposit-franchise capacity and premiered access to stable funding, then more core banks should be more likely to display the intermediation approach:

$$H_A\!: Centrality_i \uparrow \Rightarrow Pr(g_{D,i} < 0,\, g_{Ln,i} > 0) \uparrow.$$

Under H_A, core banks are systemically important and behave as classical intermediaries, with deposits treated as inputs and loans are outputs. Less core banks may instead exhibit production-type or mixed orientations, reflecting a more heterogeneous set of balance-sheet services.

The second is the financing-through-money-creation hypothesis. If centrality reflects the ability to expand credit through wholesale-funded balance sheets, then more systemically important banks should be more likely to display the reverse orientation:

$$H_B\colon Centrality_i \uparrow \Rightarrow Pr(g_{D,i} > 0,\ g_{Ln,i} < 0) \uparrow.$$

Under H_B, loans behave as the initiating balance-sheet item, while deposits or deposit-like claims adjust endogenously as part of the liquidity-creation process. In this case, centrality is associated with the FCM function.

7.1. Bank-Level Orientations in the Input-output Space

The DTDF analysis identifies substantial cross-sectional heterogeneity in the directional classification of bank balance-sheet aggregates. The unrestricted bank-specific model estimates a separate direction vector for each institution, under the assumption that the direction vector is constant over the sample period for a given bank, instead of explaining an annual transition in orientation. In the DTDF approach, let $g_{D,i}$ and $g_{Ln,i}$ denote the bank-specific directions for deposits and loans, respectively, with a positive direction indicates output orientation, and a negative direction indicates input orientation. Thus, we define the bank-level binary orientation indicators:

$$Y_{D,i} = \mathbf{1}(\, g_{D,i} > 0 \,), Y_{Ln,i} = \mathbf{1}(\, g_{Ln,i} > 0 \,) \tag{26}$$

where $Y_{D,i} = 1$ indicates that deposits are classified as outputs for bank i, and $Y_{Ln,i} = 1$ indicates that loans are classified as outputs.

The joint signs of $g_{D,i}$ and $g_{Ln,i}$ define the unrestricted DTDF estimates into the following configurations:

$$C_i = \begin{cases} H_I, & Y_{D,i} = 0,\ Y_{Ln,i} = 1, \\ H_{II}, & Y_{D,i} = 1,\ Y_{Ln,i} = 0, \\ H_{III}, & Y_{D,i} = 1,\ Y_{Ln,i} = 1, \\ H_{IV}, & Y_{D,i} = 0,\ Y_{Ln,i} = 0 \end{cases} \tag{27}$$

The H_I configuration is consistent with financial intermediation, where deposits are inputs and loans are outputs, i.e. $g_{D,i} < 0,\ g_{Ln,i} > 0$. The H_{II} configuration corresponds to the reverse orientation, where deposits are outputs and loans are inputs, i.e. $g_{D,i} > 0,\ g_{Ln,i} < 0$, that is of particular interest for the money-creation interpretation. Under H_{III}, both deposits and loans are outputs, i.e. $g_{D,i} > 0,\ g_{Ln,i} > 0$, consistent with the production approach. The fourth configuration H_{IV}, is not a theoretical benchmark, but it is admitted by the unrestricted directional model and must therefore be retained whenever it occurs in the posterior classifications. We have:

7.2. Balance-sheet Centrality and Explanatory Variables

In Table 1, we show that the preferred approach shifts by bank size group: from H_{III} for small banks, to H_I for medium banks, and to H_{II} for large banks. The earlier evidence suggests a size-related gradient:

small banks are relatively closer to the production configuration, medium banks to the classical intermediation configuration, and large banks to the reverse configuration. These findings motivate for treating our two empirical hypotheses (H_{7a} and H_{7b}) as competing rather than equivalent. In the empirical analysis of this Section, we therefore examine whether the estimated direction vectors are systematically related to determinants as the measures of balance-sheet centrality, wholesale funding intensity, leverage, size, and risk. Since we lack data for bilateral interbank exposures, we therefore construct a *continuous balance-sheet centrality index*, capturing the bank's position within the balance-sheet and funding structure of the banking system through size, leverage, and reliance on non-deposit funding. Because several components capture size scale, we report specifications using the following balance-sheet book values (instead of quantities since it is not a DTDF quantity-space centrality) for each component separately.

We use $Size = \log(TA_{it})$ as proxy for size centrality, where size is not only interpreted just as scale (i.e. "directions depend on size") but as a proxy for the operational technology of the bank within the FMC versus ILF function. Following Adrian and Shin (2014) who stress book leverage as total assets over book equity when studying lending and credit supply, we use $Leverage_{it} = log\left(\frac{TA_{it}}{Equity_{it}}\right)$. As wholesale-funding centrality, we use the *wholesale funding ratio* as a continuous proxy for wholesale-funding dependence, defined as, $WFR_{it} = \frac{OtherFunds_{it}}{Deposits_{it}+OtherFunds_{it}}$. Higher values indicate greater dependence on wholesale funding relative to retail-deposit funding. A positive relationship between WFR and H_{II} orientation would support the financing-through-money-creation hypothesis, while a positive relationship with H_I orientation would instead support the classical intermediation hypothesis. As deposit centrality, we use the deposit share defined as: $DepositShare_{it} = \frac{Deposits_{it}}{\sum_i Deposits_{it}}$, capturing whether a bank is a major deposit-taking node. As lending-market centrality, we use the loans share defined as, $LoanShare_{it} = \frac{Loans_{it}}{\sum_i Loans_{it}}$, capturing whether a bank is central on the asset originating side. Finally, we proxy market-based centrality, we use the other earning assets' centrality, defined as $OEAshare_{it} = \frac{OEA_{it}}{TA_{it}}$, which distinguishes traditional loan-deposit banks from market-based banks. Because centrality is relative, each component is standardized within year. Let $z_t(X_{it})$ denotes the within-year standardized value (z-score) of variable X_{it}. The annual *balance-sheet centrality index* is then defined as a composite indicator:

$$\begin{aligned} Centrality_{it} &= \frac{1}{6}\sum_{k=1}^{K} z_t(X_{k,it}) \\ &= \frac{1}{6}[z_t(\log TA_{it}) + z_t(Leverage_{it}) + z_t(WFR_{it}) + z_t(LoanShare_{it}) \\ &\quad + z_t(DepositShare_{it}) + z_t(OEAshare_{it})] \end{aligned} \tag{28}$$

Since the estimated directions are bank-specific and time-invariant in the current DTDF specification, the corresponding explanatory variables in the main model are in bank-level average centrality, as:

The bank-level average centrality is:

$$\overline{Centrality_i} = \frac{1}{T_i} \sum_{t \in T_i} Centrality_{it} \tag{29}$$

with analogous definitions for the bank-level averages of risk, size, leverage, wholesale funding intensity, deposit share, loan share, and other earning assets share.

Finally, we control bank credit risk using the average relative non-performing-loan ratio:

$$Risk_{it} = z_t \left(\frac{NPL_{it}}{GrossLoans_{it}} \right) \tag{30}$$

The NPL ratio captures deterioration in the loan portfolio, which is relevant when explaining whether loans and deposits occupy different input-output orientations.

Table 7. Summary for Orientations

Notes: The valid DTDF sample is 3570 observations, with unique banks reconstructed from valid sample to be 255.

Orientation	*N*	*Percent (%)*	*Mean Centrality*	*Mean Size*	*Mean Leverage*	*Mean WFR*	*Mean Risk*
H_I	36	14.1%	0.034	-0.066	0.051	0.112	0.029
H_{II}	73	28.6%	-0.105	-0.07	-0.029	-0.153	-0.000
H_{III}	63	24.7%	0.058	0.111	0.113	0.099	0.011
H_{IV}	83	32.5%	0.024	-0.056	-0.062	0.026	-0.019

7.3. The Joint Bank-Level Probit Model of Deposits and Loans

The economic hypotheses (H_A and H_B) concern the joint orientation of deposits and loans, rather than the sign of either direction alone. We therefore estimate a bivariate probit model for the two binary outcomes defined in Eq. (17). Let:

$$\begin{aligned} Y_{D,i}^* &= X_i' \beta_D + \varepsilon_{D,i}, \text{ with observed outcome: } Y_{D,i} = \mathbf{1}\left(Y_{D,i}^* > 0 \right) \\ Y_{Ln,i}^* &= X_i' \beta_{Ln} + \varepsilon_{Ln,i}, \text{ with observed outcome: } Y_{Ln,i} = \mathbf{1}\left(Y_{Ln,i}^* > 0 \right) \end{aligned} \tag{31}$$

where X_i is the vector of explanatory variables describing bank i's balance-sheet position, funding structure and risk. The error terms are assumed to follow a standard bivariate normal distribution:

$$\begin{pmatrix} \varepsilon_{D,i} \\ \varepsilon_{Ln,i} \end{pmatrix} \sim N \left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix} \right] \tag{32}$$

The parameter ρ allows unobserved bank characteristics which affect deposit orientation to be correlated with those that affect the loan orientation. This suggests that deposit-taking and lending are not independent functions of the bank, which enhances the joint nature of orientation that motivates the analysis in this

Section.[10]

We estimate two complementary specifications.[11] The first (*Model A*) provides a test of whether balance-sheet centrality explains orientation. The second (*Model B*) decomposes the centrality index to its underlying components, to determine which channel drives estimated relation. Thus, Eq. (22) becomes for Model A:

Model A: Balance-Sheet Centrality Specification

$$
\begin{aligned}
Y^*_{D,i} &= \alpha_D + \beta_{D,C}\overline{Centrality}_i + \beta_{D,R}\overline{Risk}_i + \varepsilon_{D,i}, \\
Y^*_{Ln,i} &= \alpha_{Ln} + \beta_{Ln,C}\overline{Centrality}_i + \beta_{Ln,R}\overline{Risk}_i + \varepsilon_{Ln,i}
\end{aligned} \tag{33}
$$

Model A tests whether banks with higher balance-sheet centrality are more likely to occupy the intermediation, reverse, or production orientations. A positive marginal effect of centrality on $Pr(H_I)$ is consistent with the intermediation hypothesis that more large central institutions are more likely to treat deposits as inputs and loans as outputs where a positive marginal effect on $Pr(H_{II})$ is consistent with money-creation hypothesis.

By decomposing centrality index, we have the second specification of Eq. (22):

Model B: Balance-Sheet Decomposed Specification

$$
\begin{aligned}
Y^*_{D,i} = \alpha_D &+ \beta_{D,S}\overline{Size}_i + \beta_{D,L}\overline{Leverage}_i + \beta_{D,W}\overline{WFR}_i + \beta_{D,DS}\overline{DepositShare}_i \\
&+ \beta_{D,LS}\overline{LoanShare}_i + \beta_{D,O}\overline{OEAshare}_i + \beta_{D,R}\overline{Risk}_i + \varepsilon_{D,i} \\
Y^*_{Ln,i} = \alpha_{Ln} &+ \beta_{Ln,S}\overline{Size}_i + \beta_{Ln,L}\overline{Leverage}_i + \beta_{Ln,W}\overline{WFR}_i + \beta_{Ln,DS}\overline{DepositShare}_i \\
&+ \beta_{Ln,LS}\overline{LoanShare}_i + \beta_{Ln,OEA}\overline{OEAshare}_i + \beta_{Ln,R}\overline{Risk}_i + \varepsilon_{Ln,i}
\end{aligned} \tag{34}
$$

Model B determines the degree of whether the components contribute independently to the directional classification. By comparing Model A with Model B, we assess whether the centrality result is simply a restatement of bank size. The decomposed specification allows us to examine whether wholesale-funding intensity and leverage explain orientation after controlling for scale. If they do not, the appropriate interpretation is a size-based orientation gradient. If they do, the evidence supports a richer interpretation in which funding structure and balance-sheet expansion capacity contribute independently to the orientation of banks in the input–output space.[12] Finally, both models are estimated without bank fixed effects. The dependent orientation is bank-specific and time-invariant under the DTDF specification, so bank fixed effects would absorb the relevant variation.

Table 8. Regression results

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[10] Orientation Probabilities are reported in Appendix A.

[11] The process of posterior uncertainty in orientation classification is described in Appendix B.

[12] Marginal effects and interpretation are described in Appendix C.

Notes: The table reports the results under the *Joint Bank-Level Probit Model of Deposits and Loans. z-statistics* are reported in brackets and standard errors in parentheses, whereas *, **, and *** denote statistical significance at the 10%, 5%, and 1% levels, respectively. The correlation parameter ρ between the two latent equations is also reported as *rho*.

	Model A		Model B	
Equation	**Deposits**	**Loans**	**Deposits**	**Loans**
constant	0.065	**-0.284*** **	0.070	**-0.287*** **
	(0.080)	(0.081)	(0.081)	(0.082)
	[0.816]	[-3.499]	[0.865]	[-3.469]
Centrality	**-**0.082	**0.168** **		
	(0.142)	(0.136)		
	[-0.576]	[1.240]		
Risk	0.033	0.024	0.015	-0.011
	(0.150)	(0.150)	(0.155)	(0.156)
	[0.225]	[0.164]	[0.097]	[-0.076]
Size			0.114	0.026
			(0.129)	(0.128)
			[0.886]	[0.208]
Leverage			0.081	0.170
			(0.104)	(0.107)
			[0.779]	[1.589]
WFR			-0.080	0.137
			(0.109)	(0.113)
			[-0.731]	[1.214]
Deposits Share			1.712	-1.495
			(1.473)	(1.103)
			[-1.162]	[-1.355]
Loans Share			1.561	1.473
			(1.458)	(1.146)
			[1.070]	[1.284]
OEA Share			0.00006	0.078
			(0.107)	(0.108)
			[0.005]	[0.724]
Observations	245	245	243	243
Correlation rho	**0.244*** ** (0.098) [2.497]		**0.234*** ** (0.100) [0.019]	
LR test H_0: rho=0	**5.858** *(p=0.015)*		**5.178*** ** *(p=0.022)*	

Table 9. Average Marginal Effects

Model	Variable	AME Pr(H_I)	AME Pr(H_{II})	AME Pr(H_{III})	AME Pr(H_{IV})
Model A	*Centrality*	0.041	-0.056	0.023	-0.008
	Risk	-0.001	0.003	0.010	-0.013

Model B	*Size*	-0.012	0.023	0.022	-0.032
	Leverage	0.016	-0.015	0.047	-0.048
	WFR	0.034	-0.048	0.017	-0.003
	Deposit Share	0.003	-0.109	-0.560	0.666
	Loan Share	0.016	0.076	0.533	-0.625
	OEA Share	0.013	-0.016	0.016	-0.013
	Risk	-0.004	0.006	0.000	-0.002

Table 10. Predicted Probabilities

Model	*g(D,L)*	*N*	*Average Pr(HI)*	*Average Pr(HII)*	*Average Pr(HIII)*	*Average Pr(HIV)*
Model A	HI (0.1)	36	0.148	0.283	0.241	0.325
	HII (1,0)	70	0.142	0.290	0.238	0.328
	HIII (1,1)	59	0.150	0.281	0.242	0.325
	HIV (0,0)	80	0.148	0.285	0.240	0.326
Model B	HI (0.1)	36	0.151	0.278	0.238	0.331
	HII (1,0)	70	0.137	0.296	0.234	0.330
	HIII (1,1)	59	0.148	0.283	0.258	0.308
	HIV (0,0)	80	0.144	0.285	0.228	0.341

Table 8 reports the estimates from the joint bank-level probit model for deposit and loan orientations. The first result is that the estimated correlation coefficient between the two equations is positive and statistically significant in both specifications ($\rho = 0.244,\ p < 0.01$ for Model A and $\rho = 0.234,\ p < 0.01$ for Model B) with the likelihood-ratio tests reject the null hypothesis of independent equations in both models. This result supports the use of a joint mode whether unobserved bank characteristics that affect whether deposits are classified as outputs are systematically related to those affecting whether loans are classified as outputs. In economic terms, the deposit and loan sides of the balance sheet are not independent orientation decisions.

Model A provides limited evidence that balance-sheet centrality explains the joint orientation of deposits and loans. The coefficient $\beta_{D,C}$ of centrality is negative and not statistically significant in the deposit equation but positive and significant in the loan equation ($\hat{\beta}_{Ln,C} = 0.597, p = 0.01$), indicating that more central banks are more likely to have a loan-output orientation, but not necessarily a deposit-output orientation. This can also be viewed from the average marginal effects in Table 9 where a higher centrality index increases the probability of the intermediation H_I and production H_{III} configurations ($AME_{Pr(HI)} = 0.102$ and $AME_{Pr(HIII)} = 0.101$) while reduces for H_{II} and H_{IV} ($AME_{Pr(HII)} = -0.081$ and $AME_{Pr(HIV)} = -0.122$). Thus, centrality shifts banks toward configurations in which loans behave as outputs but does not support the prediction that more central banks are more likely to display the reverse/FMC orientation. Model B shows that no individual component has a statistically significant effect on either the deposit-output or loan-output equation. Although all components enter with economically interpretable signs, the estimates do not support any evidence for a specific balance-sheet channel. In Table 9, the average marginal effects do not support the view that wholesale-funded or highly leveraged institutions are systematically more likely to display the reverse/FMC configuration. The risk variable is insignificant in both equations of Model A and Model B, suggesting that the bank-level NPL ratio does not explain whether deposits and loans are classified as inputs or outputs. This does not imply that risk is irrelevant for bank production but, rather, indicates that credit-risk differences do not drive the cross-sectional orientation classification once the direction vector has already been estimated from the technology model.

Table 10 shows the predicted probabilities for models A and B, which are relatively similar across the four observed orientation classes. In Model A, the average predicted probability of H_{II} ranges from 0.281 to 0.290 across the four observed groups, while the predicted probability of H_{IV} is between 0.325 and 0.328. Model B introduces slightly more separation, particularly for H_{III} and H_{IV}, but the differences are small. Therefore, our findings provide evidence that deposit and loan orientations are jointly determined, but they do not provide strong evidence that balance-sheet centrality, wholesale funding, or leverage independently identify a reverse/FMC orientation. The results are more consistent with a weak centrality-related shift toward loan-output configurations, especially H_I and H_{III}, than with a strong money-creation or wholesale-balance-sheet mechanism. Therefore, a second-stage probit model supports the relevance of joint balance-sheet orientation, but it does not by itself establish that large, leveraged, or wholesale-funded banks operate under an H_{II}-type technology.

8. Dynamic DTDF, Bank-Year Orientations, and Bayesian CRE Panel Bivariate Probit

8.1. Dynamic DTDF with Bank-Year Time-Varying Directions

The baseline DTDF model assumes that each bank has a time-invariant direction vector $g^{(i)}$. In that specification, the direction vector is bank-specific and fixed over time, satisfying the objective to classify each bank according to its average input-output orientation over the sample period. However, if the objective is to examine whether banks switch their input-output orientation over time, the direction vector must vary by bank and year, extending from $g_j^{(i)}$ to a dynamic extension, denoted as $g_{j,it}$. The *dynamic DTDF* estimates one latent direction vector for each bank-year observation. Thus, the research question becomes: *do banks switch their input-output orientation over time as their balance-sheet structure changes?*

Let the observed quantity vector for bank $i = 1, \ldots, N$ in year $t = 1, \ldots, T_i$ be:

$$z_{it} = [q_{L,it}, q_{K,it}, q_{D,it}, q_{OEA,it}, q_{Ln,it}, q_{F,it}]' \in \mathbb{R}^6 \quad (35)$$

where $q_{L,it}$ denotes labour, $q_{K,it}$ denotes capital, $q_{D,it}$ denotes deposits, $q_{OEA,it}$ denotes other earning assets, $q_{Ln,it}$ denotes loans, and $q_{F,it}$ denotes other funds. Let the corresponding bank-year direction vector be:

$$g_{it} = [g_{L,it}, g_{K,it}, g_{D,it}, g_{OEA,it}, g_{Ln,it}, g_{F,it}]' \in \mathbb{R}^6 \quad (36)$$

The sign of each component of g_{it} determines the annual input-output orientation of the corresponding balance-sheet variable. A positive direction indicates output orientation, while a negative direction indicates input orientation. Therefore, the estimated direction vector provides a classification of banking activities. Deposits are classified as outputs when $g_{D,it} > 0$ and as inputs when $g_{D,it} < 0$ for bank i in year t. Likewise, loans are classified as outputs when $g_{Ln,it} > 0$ and as inputs when $g_{Ln,it} < 0$. Hence, the sign of $g_{D,it}$ and $g_{Ln,it}$ determine where a bank's orientation is closer to an intermediation, production, or mixed banking function.

The quadratic directional technology distance function is written as:

$$D(z_{it}; g_{it}) = \alpha_0 + \sum_{j=1}^{6} \alpha_j \left(z_{j,it} + g_{j,it}\right) + \frac{1}{2}\sum_{j=1}^{6}\sum_{k=1}^{6} \beta_{jk} \left(z_{j,it} + g_{j,it}\right)\left(z_{k,it} + g_{k,it}\right) \quad (37)$$

with the symmetry condition $\beta_{jk} = \beta_{kj}$ for $j, k = 1,2, \dots ,6$, and translation restriction: $\sum_{j=1}^{6} \alpha_j \, g_{j,it} + 1 = 0$ and $\sum_{k=1}^{6} \beta_{jk} \, g_{k,it} = 0$ for $j = 1,2, \dots ,6$. Because $g_{j,it}$ now varies across both banks and years, these restrictions must hold conditional on each bank-year direction vector.[13]

Let the empirical DTDF system be written in the compact form:

$$m_{it}(\theta, g_{it}) = \varepsilon_{it} \quad (38)$$

where $m_{it}(\theta, g_{it})$ stacks the DTDF equation and the associated first-order conditions, where $\theta = (a, \beta)$ and the error vector satisfies $\varepsilon_{it} \sim N(0, \Sigma)$. Therefore, the likelihood contribution of bank i in year t is:

$$p(z_{it} | \theta, g_{it}, \Sigma) \propto |\Sigma|^{-1/2} exp\left[-\frac{1}{2} m_{it}(\theta, g_{it})' \Sigma^{-1} m_{it}(\theta, g_{it})\right] \quad (39)$$

The full likelihood is:

$$L(\theta, \Sigma, \{g_{it}\}) = \prod_{i=1}^{N} \prod_{t=1}^{T_i} p(z_{it} | \theta, g_{it}, \Sigma) \quad (40)$$

The dynamic DTDF imposes persistence in the direction vector through a smoothing prior, avoiding economically implausible jumps in annual orientation when estimating g_{it}. For each bank i, the direction vector follows:

[13] In practice, this can be implemented either by constrained estimation or by projecting the technology parameters into the translation-consistent space at each posterior draw.

$$g_{i,t} = \mu_i + \Phi(g_{i,t-1} - \mu_i) + u_{it} \tag{41}$$

where μ_i is the bank-specific long-run direction vector, Φ is the diagonal persistence matrix where $\Phi = diag(\phi_L, \phi_K, \phi_D, \phi_{OEA}, \phi_{Ln}, \phi_F)$ for tractability reasons with $|\phi| < 1$, and $u_{it} \sim N(0, Q_g)$ where u_{it} is the innovation term, and Q_g is the innovation covariance matrix of the direction vector. The model allows a bank's orientation to evolve over time but prevents it from changing arbitrarily from one year to the next.

The initial direction vector is drawn from the stationary distribution implied by the dynamic process: $g_{i,1} \sim N(\mu_i, P_0)$, where $P_0 = \Phi P_0 \Phi' + Q_g$. Under the diagonal specification, this becomes: $P_0 = \frac{\sigma_{g,j}^2}{1-\phi_j^2}$. To allow for cross-bank heterogeneity, the bank-specific long-run directions are drawn from a common population distribution:

$$\mu_i \sim N(\mu_0, \Omega_\mu) \tag{42}$$

where μ_0 is the population mean and Ω_μ captures cross-bank heterogeneity in long-run input-output orientation. The complete dynamic DTDF model is defined by the observation equation (Eq. 29), the state equation (Eq. 32), and the cross-bank heterogeneity equation (Eq. 33).

The posterior distribution is proportional to the likelihood multiplied by the dynamic state density, the initial state density, the heterogeneity density and the prior distribution,[14] given as:

$$p(\theta, \Sigma, \{g_{it}\}, \{\mu_i\}, \mu_0, \Phi, Q_g, \Omega_\mu \mid \mathcal{D}) \propto L_z \, L_g \, L_\mu \, p(\theta) p(\Sigma) p(\mu_0) p(\Omega_\mu) p(\Phi) p(Q_g) \tag{43}$$

The observation likelihood is

$$L_z = \left[\prod_{i=1}^{N} \prod_{t=1}^{T_i} p\,(z_{it} \mid \theta, g_{it}, \Sigma) \right]$$

the dynamic state density is

$$L_g = \left[\prod_{i=1}^{N} p\left(g_{i1} \mid \mu_i, \Phi, Q_g\right) \prod_{t=2}^{T_i} p\left(g_{it} \mid g_{i,t-1}, \mu_i, \Phi, Q_g\right) \right]$$

and the cross-bank heterogeneity is

$$L_\mu = \left[\prod_{i=1}^{N} p\left(\mu_i \mid \mu_0, \Omega_\mu\right) \right]$$

Diffuse priors may be imposed on the technology parameters,

$$p(\theta) \propto 1.$$

For the DTDF disturbance covariance matrix,

[14] The Bayesian MCMC algorithm for estimating the dynamic DTDF, including the updating of the latent direction paths g_{it}, the persistence parameters, covariance matrices, and technology parameters, is described in Appendix D.

$$\Sigma \sim IW(\nu_\Sigma, S_\Sigma)$$

or the Jeffreys-type prior as in the static DTDF specification. The population mean direction vector follows

$$\mu_0 \sim N(m_0, V_0),$$

while the cross-bank covariance matrix follows

$$\Omega_\mu \sim IW(\nu_\mu, S_\mu).$$

Assuming

$$Q_g = diag(\sigma_{g,1}^2, \dots, \sigma_{g,6}^2)$$

the innovation variances are assigned

$$\sigma_{g,j}^2 \sim IG(a_g, b_g), \text{ where } j = 1, \dots, 6.$$

The persistence parameters satisfy

$$\phi_j \sim N(\bar{\phi}, V_\phi)\mathbf{1}|\phi_j| < 1.$$

The posterior output of the dynamic DTDF stage consists of the posterior draws

$$g_{it}^{(s)}: i = 1, \dots, N, t = 1, \dots, T_i, s = 1, \dots, S$$

from which posterior means, credible intervals, and bank-year classifications can be constructed.

8.2. Annual Orientation Indicators

For each posterior draw s, the annual deposit and loan orientation indicators are:

$$\begin{aligned} &\textit{Deposits: } Y_{D,it}^{(s)} = \mathbf{1}\left(g_{D,it}^{(s)} > 0\right) \\ &\textit{Loans: } Y_{Ln,it}^{(s)} = \mathbf{1}\left(g_{Ln,it}^{(s)} > 0\right) \end{aligned} \tag{44}$$

Accordingly, the posterior-mean hard classifications are:

$$\begin{aligned} &\hat{Y}_{D,it} = \mathbf{1}\left(E\left[g_{D,it} \mid \mathcal{D}\right] > 0\right) \\ &\hat{Y}_{Ln,it} = \mathbf{1}\left(E\left[g_{Ln,it} \mid \mathcal{D}\right] > 0\right) \end{aligned} \tag{45}$$

The joint signs of $g_{D,it}^{(s)}$ and $g_{Ln,it}^{(s)}$ define now the dynamic unrestricted DTDF estimates into the following annual orientation configurations:

$$C_i = \begin{cases} H_I, & \hat{Y}_{D,it} = 0,\ \hat{Y}_{Ln,it} = 1, \\ H_{II}, & \hat{Y}_{D,it} = 1,\ \hat{Y}_{Ln,it} = 0, \\ H_{III}, & \hat{Y}_{D,it} = 1,\ \hat{Y}_{Ln,it} = 1, \\ H_{IV}, & \hat{Y}_{D,it} = 0,\ \hat{Y}_{Ln,it} = 0 \end{cases} \tag{46}$$

Equivalently, posterior orientation probabilities are computed as:

$$\pi_{it,r} = Pr(C_{it} = r \mid \mathcal{D}) \text{ for } r \in \{H_I, H_{II}, H_{III}, H_{IV}\} \tag{47}$$

These probabilities are approximated from the posterior draws as:

$$\pi_{it,r} \approx \frac{1}{S}\sum_{s=1}^{S} \mathbf{1}\left(C_{it}^{(s)} = r\right)$$

Therefore, the dynamic DTDF produces bank-year input-output orientations, allowing banks to transition over time across alternative orientation configurations. At the same time, the autoregressive state process imposes persistence on the latent direction vectors, ensuring that changes in estimated orientations occur gradually rather than suddenly. Economically, our framework captures whether banks' positions in input-output space evolve in response to changes in their balance-sheet structure, leverage, funding composition, risk profile, and market-based activities. Consequently, the model captures both heterogeneity across institutions and dynamic adjustment within institutions over time.

8.3. Correlated Random Effects Panel Bivariate Probit

Let X_{it} denote the vector of time-varying bank-level explanatory variables, i.e. size, leverage, wholesale funding ratio, deposit share, loan share, other earning asset share, and risk. After estimating the dynamic DTDF, the dependent variables are the annual indicators $Y_{D,it} = \mathbf{1}(g_{D,it} > 0)$ and $Y_{Ln,it} = \mathbf{1}(g_{Ln,it} > 0)$. We moreover include the bank-level *Mundlak averages*, denote by $\bar{X}_i = \frac{1}{T_i}\sum_{t=1}^{T_i} X_{it}$, allowing the bank random effects to be correlated with the regressors in a nonlinear probit, and avoiding the need of bank fixed effects.

Thus, the *correlated-random effects* (CRE) *panel bivariate probit* is given as:

$$\begin{aligned} Y_{D,it}^{*} &= X_{it}'\beta_D + \bar{X}_i'\pi_D + \delta_{D,t} + a_{D,i} + \varepsilon_{D,it} \\ Y_{Ln,it}^{*} &= X_{it}'\beta_{Ln} + \bar{X}_i'\pi_{Ln} + \delta_{Ln,t} + a_{Ln,i} + \varepsilon_{Ln,it} \end{aligned} \tag{48}$$

The observed binary outcomes are:

$$\begin{aligned} Y_{D,it} &= \mathbf{1}(Y_{D,it}^{*} > 0) \\ Y_{Ln,it} &= \mathbf{1}(Y_{Ln,it}^{*} > 0) \end{aligned} \tag{49}$$

The bank random effects are allowed to be correlated across deposit and lending equations:

$$\begin{pmatrix} a_{D,i} \\ a_{Ln,i} \end{pmatrix} \sim N\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_{a,D}^2 & \rho_a\sigma_{a,D}\sigma_{a,Ln} \\ \rho_a\sigma_{a,D}\sigma_{a,Ln} & \sigma_{a,Ln}^2 \end{pmatrix}\right] \tag{50}$$

where Σ_a is the covariance matrix of the bank-level random effects. The idiosyncratic errors are also allowed to be correlated across the two latent equations:

$$\begin{pmatrix} \varepsilon_{D,it} \\ \varepsilon_{Ln,it} \end{pmatrix} \sim N\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \rho_\varepsilon \\ \rho_\varepsilon & 1 \end{pmatrix}\right] \tag{51}$$

where Σ_ε is the covariance matrix of the idiosyncratic shocks, normalized to unit variances in the probit model:

$$\Sigma_\varepsilon = \begin{pmatrix} 1 & \rho_\varepsilon \\ \rho_\varepsilon & 1 \end{pmatrix}$$

The year fixed effects are denoted as $\delta_{D,t}$ and $\delta_{Ln,t}$ for deposits and loans respectively, and absorb common macroeconomic, regulatory, and accounting annual shocks.

The marginal likelihood contribution for bank i, after integrating out the random effects, is:

$$L_i(\psi) = \int \left[\prod_{t=1}^{T_i} Pr\left(Y_{D,it}, Y_{Ln,it} \mid X_{it}, \bar{X}_i, \delta_t, a_i, \psi\right)\right] \phi_2(a_i; 0, \Sigma_a)\, da_i \qquad (52)$$

where ψ collects all parameters of the panel bivariate probit model. The full likelihood is given by:

$$L(\psi) = \prod_{i=1}^{N} L_i\,(\psi)$$

We estimate the correlated-random-effects panel bivariate probit in a Bayesian framework.[15]

The results are reported in Table 11.

Table 11. Posterior Means

Pr_gD_output	**Mean**	**Std. dev.**	**MCSE**	**Median**	**[95% cred. interval]**	
Risk	-0.007	0.050	0.002	-0.010	-0.097	0.102
bar_Centrality	-0.307	0.215	0.011	-0.307	-0.723	0.097
bar_Risk	0.201	0.096	0.004	0.204	0.004	0.388
Centrality	0.187	0.211	0.011	0.182	-0.220	0.589
_cons	1.612	0.038	0.002	1.612	1.538	1.687
Pr_gLn_output						
Risk	0.024	0.044	0.002	0.023	-0.058	0.113
bar_Centrality	0.304	0.198	0.010	0.292	-0.085	0.728
bar_Risk	0.185	0.074	0.003	0.186	0.037	0.335
Centrality	0.057	0.174	0.009	0.061	-0.296	0.381
_cons	1.158	0.030	0.001	1.159	1.098	1.217
athrho	-0.002	0.061	0.001	-0.002	-0.126	0.119

9. Concluding remarks

The issue of inputs and outputs in the context of banking has a long history and many different approaches have been proposed and used in the literature. Although all approaches have an underlying *raison d' etre*,

[15] The Bayesian MCMC algorithm for the correlated-random-effects panel bivariate probit, including latent-variable augmentation, updates of regression coefficients, year effects, bank random effects, covariance parameters, and the computation of predicted probabilities and marginal effects, is described in Appendix D.

it would be preferable to have procedures that can discriminate inputs from outputs from a data-based approach. The fundamental problem is that if D, L, X denotes deposits, loans and traditional inputs, it is not clear how to compare the conditional distributions $[D|L, X]$ and $[L|D, X]$ or $[D, L|X]$. In this paper, we address this problem using relatively straightforward empirical procedures. However, we account for another important possibility: For some banks the intermediation approach may be relevant but for others the production approach could be the right structure. In other words, the "one size fits all" approach may not be relevant; a point that has been overlooked in the banking literature. The data can be quite informative in this instance, as we show using a directional technology distance function with endogenous directions.

The techniques developed and applied in this study, can be useful in many respects. First, researchers do not have to commit themselves to a single approach to banking production as they can formally compare different approaches in the light of the data. Second, they can classify banks, in a second stage, to homogeneous groups that seem statistically similar in terms of the structure of procedure. Third, this classification can provide more accurate measures of returns to scale, technical change, scope economies etc. Mixing banks with different structures, due to their different behavioural objectives can, of course, be detrimental to accurate measurement of these quantities. An important contribution of this paper is to examine why directions are what they are based on the directional technology distance function. Although directions are endogenous in this study, they are not functions of environmental variables that can answer the question why certain banks are different from others. To capitalize on this idea in future research, it is likely to necessitate accurate data on size, managerial incentives, risk etc. It is also likely to require more theory in terms of behavioural objectives of banks and the structure of banking activities by bank type as opposed to the universe of all banks as homogeneous firms where "one size fits all".

In future research, time-varying directions can be also examined through of a modification of the new techniques in this paper. Persistence of directions is, however, likely to be of much value, so dynamic models can prove useful in this line of research. For Bayesian inference, particle filtering or Sequential Monte Carlo techniques, as in the present paper, can be used. The important aspect would be why directions change over time, if at all. In this context we develop a "reality check" to examine the sensitivity of our approach. As the results are promising, it is not an exaggeration to claim that the new approaches can be of interest to a wide audience of applied and theoretical research on banking. As new tools are available from distinguishing "right from the left" it is not inconceivable that these developments will facilitate the consideration of other dimensions such as input and output quality, conflicting objectives due to principal-agent or moral hazard problems, the role of the bank as a producer of information arising from screening various categories of loans and past histories of customers etc. This idea is formalized using performance as a determinant of directions.

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Appendix. Table of Results

Table A1. Variable Definitions

Notes: The table presents the definitions of all variables used in the empirical analysis.

Variables	Definition
Total Gross Loans	A bank's total gross loan portfolio refers to the total value of loans and advances extended to customers before deducting loan-loss provisions.
Net (banking related) Loans	It refers to the total value of loans and advances extended to customers after the deduction of loan-loss provisions and write-offs. Net loans equal to gross loans minus loan loss provisions.
Other Earning Assets	It refers to investment securities, financial assets at fair value through profit or loss, and any revenue-generating entities other than loans.
Total Customer Deposits	It refers to the ratio of total deposits (demand deposits, savings accounts, and term deposits held by households, corporations, and other non-bank clients).
Non-Performing Loan Ratio	Non-performing loans refer to credit exposures for which the borrower has failed to meet contractual debt service obligations.
Total Deposits	Total deposits refer to the aggregate amount of deposits within a bank, including both retail and institutional sources of funds.
Fixed Assets (Property, plant, and equipment)	Fixed assets (or property, plant, and equipment) represent the tangible long-term assets held by a bank for use in its operational activities rather than for sale in the ordinary course of business.
Other Funds	It refers to non-deposit funding sources, i.e., the liability side of the balance sheet excluding customer deposits, which typically includes interbank borrowings (i.e. wholesale funding from other banks), certificates of deposit issued to institutional investors, subordinated debt and long-term bonds issued and repurchase agreements (repos).
Depreciation & amortization expense	Depreciation and amortization expense refers to the systematic and periodic allocation of the cost of a bank's tangible and intangible assets over their respective estimated useful lives.
Returns on Equity (ROE)	It measures a bank's net profitability (after taxes and all expenses) relative to the capital its shareholders provide.
Prices	**Definition**
Price of labour	Cost of one unit of labour: w_L = *Personnel Expenses/Number of Employees*
Price of physical capital	Cost of using one euro of fixed capital (including depreciation): w_K = *Other Operating Expenses (excluding personnel)/Fixed Assets*
Price of deposits	Average interest rate paid on deposits: w_D = *Interest Expenses on Deposits / Total Deposits*
Price of funds	Unit cost of wholesale funding, interbank loans, or market debt: w_F = *Interest Expenses on Other Funds / Other Borrowed Funds*
Price of loans	The unit price of a bank's lending activity: w_{Ln} = *Interest Expenses on Loans / Gross Loans*
Price of other earning assets	The unit return on a bank's non-loan interest-bearing assets: w_{OEA} = *Income from other assets / Total other assets*
Quantities	**Definition**
Labor quantity	q_L = *Personnel Expenses* / w_L
Capital quantity (with deracialisation)	q_K = *Fixed Assets* / w_K
Deposit quantity	q_D = *Total Deposits* / w_D
Fund quantity	q_F = *Other Borrowed Funds* / w_F
Loan quantity	q_{Ln} = *Total Loans* / w_{Ln}
Other earning asset quantity	q_{OEA} = *Other Earning Assets* / w_{OEA}

Table A2. Summary Statistics for Prices

Notes: The table presents the summary statistics (mean, standard deviation, min and max values, median, and observations) for prices in the full sample. A detailed summary statistics for quantities and the main variables along with some empirical review and analysis is provided in the Supplement.

	Mean	Std Dev.	Min	P25	Median	P75	Max	CV
Price of Labor	108.965	39.322	38.961	79.417	100.847	134.233	260.615	0.361
Price of Capital	0.624	0.817	0.058	0.257	0.381	0.577	5.560	1.309
Price of Deposits	0.008	0.008	0.000	0.003	0.005	0.011	0.039	0.971
Price of Other Funds	0.041	0.061	0.001	0.015	0.025	0.043	0.517	1.482
Price of Loans	0.051	0.015	0.028	0.042	0.047	0.056	0.107	0.286
Price of OEA	0.020	0.009	0.001	0.014	0.019	0.024	0.070	0.456

Technical Appendix

Appendix 1. Girolami-Calderhead update

The algorithm uses local information about both the gradient and the Hessian of the log-posterior conditional of $\boldsymbol{\theta}$ at the existing draw. Suppose $\mathcal{L}(\boldsymbol{\theta}) = \log\, p\,(\boldsymbol{\theta}|\boldsymbol{X})$ is used to denote for simplicity the log posterior of $\boldsymbol{\theta}$. Moreover, define:

$$\boldsymbol{G}(\boldsymbol{\theta}) = \text{est.}\,\text{cov}\frac{\partial}{\partial\boldsymbol{\theta}}\log\, p\,(\boldsymbol{X}|\boldsymbol{\theta}), \tag{A.1}$$

the empirical counterpart of

$$\boldsymbol{G}_o(\boldsymbol{\theta}) = -E_{y|\boldsymbol{\theta}}\frac{\partial^2}{\partial\boldsymbol{\theta}\partial\boldsymbol{\theta}'}\log\, p\,(\boldsymbol{X}|\boldsymbol{\theta}). \tag{A.2}$$

The Langevin diffusion is given by the following stochastic differential equation:

$$d\boldsymbol{\theta}(t) = \frac{1}{2}\widetilde{\boldsymbol{\nabla}}_{\boldsymbol{\theta}}\mathcal{L}\{\boldsymbol{\theta}(t)\}dt + d\mathbf{B}(t) \tag{A.3}$$

where

$$\widetilde{\boldsymbol{\nabla}}_{\boldsymbol{\theta}}\mathcal{L}\{\boldsymbol{\theta}(t)\} = -\boldsymbol{G}^{-1}\{\boldsymbol{\theta}(t)\}\cdot\widetilde{\boldsymbol{\nabla}}_{\boldsymbol{\theta}}\mathcal{L}\{\boldsymbol{\theta}(t)\} \tag{A.4}$$

is the so called "natural gradient" of the Riemann manifold generated by the log posterior.

The elements of the Brownian motion are:

$$\boldsymbol{G}^{-1}\{\boldsymbol{\theta}(t)\}d\mathrm{B}_i(t) = |\boldsymbol{G}\{\boldsymbol{\theta}(t)\}|^{-1/2}\sum_{j=1}^{K_\beta}\frac{\partial}{\partial\boldsymbol{\theta}}\left[\boldsymbol{G}^{-1}\{\boldsymbol{\theta}(t)\}_{ij}|\boldsymbol{G}\{\boldsymbol{\theta}(t)\}|^{1/2}\right]dt + \left[\sqrt{\boldsymbol{G}\{\boldsymbol{\theta}(t)\}}d\boldsymbol{B}(t)\right] \tag{A.5}$$

The discrete form of the stochastic differential equation provides a proposal as follows:

$$\widetilde{\boldsymbol{\theta}}_i = \boldsymbol{\theta}_i^o + \frac{\varepsilon^2}{2}\{\boldsymbol{G}^{-1}(\boldsymbol{\theta}^o)\nabla_{\boldsymbol{\theta}}\,\mathcal{L}(\boldsymbol{\theta}^o)\}_i - \varepsilon^2\sum_{j=1}^{K_\theta}\left\{\boldsymbol{G}^{-1}(\boldsymbol{\theta}^o)\frac{\partial \boldsymbol{G}(\boldsymbol{\theta}^o)}{\partial\theta_i}\boldsymbol{G}^{-1}(\boldsymbol{\theta}^o)\right\}_{ij}$$
$$+\frac{\varepsilon^2}{2}\sum_{j=1}^{K_\theta}\{\boldsymbol{G}^{-1}(\boldsymbol{\theta}^o)\}_{ij}tr\left\{\boldsymbol{G}^{-1}(\boldsymbol{\theta}^o)\frac{\partial \boldsymbol{G}(\boldsymbol{\theta}^o)}{\partial\theta_i}\right\} + \left\{\varepsilon\sqrt{\boldsymbol{G}^{-1}(\boldsymbol{\theta}^o)}\xi^o\right\}_i \tag{A.6}$$
$$= \boldsymbol{\mu}(\boldsymbol{\theta}^o,\varepsilon)_i + \left\{\varepsilon\sqrt{\boldsymbol{G}^{-1}(\boldsymbol{\theta}^o)}\boldsymbol{\xi}^o\right\}_i$$

where $\boldsymbol{\beta}^o$ is the current draw.

The proposal density is:

$$q(\widetilde{\boldsymbol{\theta}}|\boldsymbol{\theta}^o) = \mathcal{N}_{K_\theta}\left(\widetilde{\boldsymbol{\theta}},\varepsilon^2\boldsymbol{G}^{-1}(\boldsymbol{\theta}^o)\right), \tag{A.7}$$

and convergence to the invariant distribution is ensured by using the standard form Metropolis-Hastings probability:

$$\min\left\{1,\frac{p(\widetilde{\boldsymbol{\theta}}|\cdot,\mathcal{Y})q(\boldsymbol{\theta}^o|\widetilde{\boldsymbol{\theta}})}{p(\boldsymbol{\theta}^o|\cdot,\mathcal{Y})q(\widetilde{\boldsymbol{\theta}}|\boldsymbol{\theta}^o)}\right\}. \tag{A.8}$$

The MCMC procedure is implemented using user-defined iterations (for instance, 150,000) and the first (1/3) iterations are omitted to mitigate start-up effects. For all parameters the prior is uniform over the interval $(-M,M)^{\dim(\boldsymbol{\theta})}$ where $M = 10^7$ and $\dim(\boldsymbol{\theta})$ is the dimensionality of the parameter vector.

Appendix A. Orientation Probabilities for Static DTDF

The implied probabilities of the four directional configurations are:

$$\begin{aligned}
&Pr(C_i = H_I \mid X_i) = Pr(Y_{D,i} = 0, Y_{Ln,i} = 1 \mid X_i) = \Phi_2(-X_i'\beta_D, X_i'\beta_{Ln}; -\rho)\\
&Pr(C_i = H_{II} \mid X_i) = Pr(Y_{D,i} = 1, Y_{Ln,i} = 0 \mid X_i) = \Phi_2(X_i'\beta_D, -X_i'\beta_{Ln}; -\rho)\\
&Pr(C_i = H_{III} \mid X_i) = Pr(Y_{D,i} = 1, Y_{Ln,i} = 1 \mid X_i) = \Phi_2(X_i'\beta_D, X_i'\beta_{Ln}; \rho)\\
&Pr(C_i = H_{IV} \mid X_i) = Pr(Y_{D,i} = 0, Y_{Ln,i} = 0 \mid X_i) = \Phi_2(-X_i'\beta_D, -X_i'\beta_{Ln}; \rho)
\end{aligned} \tag{A1}$$

where $\Phi_2(*,*;\rho)$ denotes the standard bivariate normal cumulative distribution function with correlation parameter ρ. This specification addresses whether the characteristics of banks occupying different positions in the balance-sheet and funding structure of the banking system systematically explain input-output orientations.

Appendix B. Posterior Uncertainty in Orientation Classification for Static DTDF

A hard classification based only on the posterior mean would be:

$$\hat{Y}_{D,i} = \mathbf{1}(\hat{g}_{D,i} > 0), \hat{Y}_{Ln,i} = \mathbf{1}(\hat{g}_{Ln,i} > 0) \tag{B1}$$

where $\hat{g}_{D,i}\ or\ \hat{g}_{Ln,i} = E[g_{D,i} \mid \mathcal{D}]\ or\ E[g_{Ln,i} \mid \mathcal{D}]$, respectively. This formulation is simple to report but discards posterior uncertainty, particularly for banks whose credible intervals include zero. Since the DTDF is estimated within a Bayesian MCMC framework, we propagate this uncertainty into the second-stage orientation analysis.

For posterior draw $s = 1, \dots, S$, we define:

$$Y_{D,i}^{(s)} = \mathbf{1}\left(g_{D,i}^{(s)} > 0\right), Y_{Ln,i}^{(s)} = \mathbf{1}\left(g_{Ln,i}^{(s)} > 0\right) \tag{B2}$$

Therefore, each draw induces an orientation classification of $C_i^{(s)} \in \{H_I, H_{II}, H_{III}, H_{IV}\}$.

The posterior probability that bank $i$belongs to orientation r is:

$$\pi_{ir} = Pr(C_i = r \mid \mathcal{D}) \approx \frac{1}{S}\sum_{s=1}^{S} \mathbf{1}\left(C_i^{(s)} = r\right) \tag{B3}$$

Rather than treating the posterior-mean classification as observed without error, we estimate the second-stage model by propagating the posterior draws from the DTDF stage. Let $\vartheta = (\beta_D, \beta_{Ln}, \rho)$ denote the parameters of the bivariate probit model. The modular posterior distribution is given:

$$p_{\text{cut}}(\vartheta \mid \mathcal{D}, X) = \int p(\vartheta \mid Y_D(g), Y_{Ln}(g), X) p(g \mid \mathcal{D}) dg \tag{B4}$$

which can be approximated using the MCMC draws from the DTDF model:

$$p_{\text{cut}}(\vartheta \mid \mathcal{D}, X) \approx \frac{1}{S}\sum_{s=1}^{S} p(\vartheta \mid Y_D^s, Y_{Ln}^s X) \tag{B5}$$

This procedure preserves the interpretation of the DTDF stage. The second-stage variables explain the estimated orientations, but they do not feed back into the estimation of the technology and direction vectors.

Appendix C. Marginal Effects

The quantities of economic interest are the average marginal effects of the explanatory variables on the probabilities of the orientation classes, reported as:

$$AME_{k,r} = \frac{1}{N}\sum_{i=1}^{N} \frac{\partial Pr(C_i = r \mid X_i)}{\partial X_{ik}}, r \in \{H_I, H_{II}, H_{III}, H_{IV}\} \tag{C1}$$

The intermediation hypothesis predicts that stronger deposit-based funding is associated with a higher probability of H_I:

$$\frac{\partial Pr(C_i = H_I \mid X_i)}{\partial DepositFundingShare_i} > 0 \tag{C2}$$

where $DepositFundingShare_{it} = \frac{D_{it}}{D_{it}+F_{it}} = 1 - WFR_{it}$. Since $DepositFundingShare_{it}$ and WFR_{it} are exact complements, they are never included simultaneously in a regression. Equivalently, the intermediation prediction may be expressed as:

$$\frac{\partial Pr(C_i = H_I \mid X_i)}{\partial \overline{WFR}_i} < 0 \qquad \text{C3}$$

The competing money-creation hypothesis predicts that greater balance-sheet centrality is associated with a higher probability of the reverse orientation:

$$\frac{\partial Pr(C_i = H_{II} \mid X_i)}{\partial \overline{Centrality}_i} > 0 \qquad \text{C4}$$

In the decomposed specification, supporting evidence would take the form of positive marginal effects of size, leverage, and wholesale-funding intensity on $Pr(H_{II})$:

$$\frac{\partial Pr(C_i = H_{II} \mid X_i)}{\partial \overline{Size}_i} > 0, \frac{\partial Pr(C_i = H_{II} \mid X_i)}{\partial \overline{Leverage}_i} > 0, \frac{\partial Pr(C_i = H_{II} \mid X_i)}{\partial \overline{WFR}_i} > 0 \qquad \text{C5}$$

These implications should be interpreted carefully. In particular, a positive association between wholesale-funding intensity and H_{II} is consistent with an FMC-like or wholesale-balance-sheet orientation when it occurs jointly with leverage and centrality. The wholesale funding ratio alone is not a direct measure of deposit creation, nor does it identify the structural mechanism through which lending creates liquidity claims.

Appendix D. Bayesian MCMC Appendix

The Bayesian estimation is implemented as a modular Bayesian two-stage procedure. In the first stage, the dynamic DTDF is estimated and posterior draws of the bank-year direction vectors g_{it} are retained. In the second stage, the annual orientation indicators $Y_{D,it}$ and $Y_{Ln,it}$ are constructed from the posterior output of the first stage and used in the correlated-random-effects panel bivariate probit. The second-stage model explains the estimated annual orientations but does not feed back into the estimation of the DTDF technology.

D.1 Bayesian MCMC for the Dynamic DTDF First Stage

The dynamic DTDF posterior is:

$$p\big(\theta, \Sigma, \{g_{it}\}, \{\mu_i\}, \mu_0, \Phi, Q_g, \Omega_\mu \mid \mathcal{D}\big) \qquad \text{(D1)}$$

A Gibbs-with-Metropolis sampler can be used. The sampler cycles through the following steps (blocks).

- *Step 1: Update the latent direction paths*

For each bank i, sample the full path:

$$g_i = \big(g_{i1}{}', \dots, g_{iT_i}{}'\big)' \qquad \text{(D2)}$$

from its conditional posterior:

$$p\left(g_i \mid \theta, \Sigma, \mu_i, \Phi, Q_g, \mathcal{D}_i\right) \propto \left[\prod_{t=1}^{T_i} p\left(z_{it} \mid \theta, g_{it}, \Sigma\right)\right] p\left(g_{i1} \mid \mu_i, \Phi, Q_g\right) \prod_{t=2}^{T_i} p\left(g_{it} \mid g_{i,t-1}, \mu_i, \Phi, Q_g\right) \tag{D3}$$

Because the DTDF observation equation is nonlinear in g_{it}, this block can be sampled using a block random-walk Metropolis-Hastings step, a manifold MALA proposal, or a local linearization combined with a simulation smoother.

- *Step 2: Update the bank-specific long-run direction vectors*

Conditional on the state paths, μ_i is updated from:

$$p\left(\mu_i \mid \{g_{it}\}, \mu_0, \Omega_\mu, \Phi, Q_g\right) \tag{D4}$$

Under Gaussian priors and the *AR(1)* state equation, this conditional distribution is Gaussian.

- *Step 3: Update the population mean direction vector*

Conditional on $\{\mu_i\}$ and Ω_μ, update:

$$\mu_0 \mid \{\mu_i\}, \Omega_\mu \sim N\left(\widetilde{m}_0, \tilde{V}_0\right) \tag{D5}$$

- *Step 4: Update cross-bank heterogeneity*

Conditional on $\{\mu_i\}$ and μ_0, update:

$$\Omega_\mu \mid \{\mu_i\}, \mu_0 \sim IW\left(\tilde{v}_\mu, \tilde{S}_\mu\right) \tag{D6}$$

- *Step 5: Update persistence and innovation variances*

For each direction component j, define:

$$\tilde{g}_{j,it} = g_{j,it} - \mu_{j,i} \tag{D7}$$

The state equation becomes:

$$\tilde{g}_{j,it} = \phi_j \tilde{g}_{j,i,t-1} + u_{j,it} \tag{D8}$$

The persistence coefficient ϕ_j is sampled from its truncated conditional posterior on $(-1,1)$, and the innovation variance is sampled from:

$$\sigma_{g,j}^2 \mid \{g_{j,it}\}, \phi_j, \mu_{j,i} \sim IG\left(\tilde{a}_{g,j}, \tilde{b}_{g,j}\right) \tag{D9}$$

- *Step 6: Update the technology parameters*

The technology parameter vector $\theta = (\alpha, \beta)$ is sampled from:

$$p(\theta \mid \{z_{it}\}, \{g_{it}\}, \Sigma) \propto \left[\prod_{i=1}^{N} \prod_{t=1}^{T_i} p\left(z_{it} \mid \theta, g_{it}, \Sigma\right)\right] p(\theta) \tag{D10}$$

subject to the translation restrictions. Because this conditional posterior is generally nonstandard, it can be updated using Metropolis-Hastings, MALA, or the Girolami-Calderhead manifold MALA step already used in the static DTDF estimation.

- *Step 7: Update the DTDF covariance matrix*

Let:

$$\hat{\varepsilon}_{it} = m_{it}(\theta, g_{it}) \tag{D11}$$

Then:

$$\Sigma \mid \{\hat{\varepsilon}_{it}\} \sim IW\left(\nu_\Sigma + n,\ S_\Sigma + \sum_{i=1}^{N}\sum_{t=1}^{T_i} \hat{\varepsilon}_{it}\,\hat{\varepsilon}_{it}'\right) \tag{D12}$$

where $n = \sum_i T_i$.

- *Step 8: Store posterior outputs*

For each retained MCMC draw $s = 1, \dots, S$, we store $\{g_{it}^{(s)}\}$, $Y_{D,it}^{(s)}$, $Y_{Ln,it}^{(s)}$, and $C_{it}^{(s)}$ for $i = 1, \dots, N$ and $t = 1, \dots, T_i$. These stored draws allow the uncertainty from the dynamic DTDF stage to be propagated into the second-stage panel bivariate probit. From these draws we compute the posterior mean deposit and loan directions $\bar{g}_{D,it} = \frac{1}{S}\sum_{s=1}^{S} g_{D,it}^{(s)}$ and $\bar{g}_{Ln,it} = \frac{1}{S}\sum_{s=1}^{S} g_{Ln,it}^{(s)}$. We also compute posterior orientation probabilities for deposits and loans, as $Pr(g_{D,it} > 0 \mid \mathcal{D}) \approx \frac{1}{S}\sum_{s=1}^{S} \mathbf{1}\left(g_{D,it}^{(s)} > 0\right)$ and $Pr(g_{Ln,it} > 0 \mid \mathcal{D}) \approx \frac{1}{S}\sum_{s=1}^{S} \mathbf{1}\left(g_{Ln,it}^{(s)} > 0\right)$. The hard annual classifications used in the baseline panel probit are $\hat{Y}_{D,it} = \mathbf{1}(\bar{g}_{D,it} > 0)$ and $\hat{Y}_{Ln,it} = \mathbf{1}(\bar{g}_{Ln,it} > 0)$. These variables are then combined into the annual orientation class:

$$\hat{C}_{it} = \begin{cases} H_I, & \hat{Y}_{D,it} = 0,\ \hat{Y}_{Ln,it} = 1, \\ H_{II}, & \hat{Y}_{D,it} = 1,\ \hat{Y}_{Ln,it} = 0, \\ H_{III}, & \hat{Y}_{D,it} = 1,\ \hat{Y}_{Ln,it} = 1, \\ H_{IV}, & \hat{Y}_{D,it} = 0,\ \hat{Y}_{Ln,it} = 0. \end{cases}$$

For uncertainty propagation, the posterior draw-specific classifications are retained as $Y_{D,it}^{(s)} = \mathbf{1}\left(g_{D,it}^{(s)} > 0\right)$ and $Y_{Ln,it}^{(s)} = \mathbf{1}\left(g_{Ln,it}^{(s)} > 0\right)$. These draw-specific outcomes allow the first-stage uncertainty in the dynamic DTDF directions to be carried into the second stage correlated-random-effects panel bivariate probit. In the baseline specification, the panel probit is estimated using the hard classifications based on posterior means. In the uncertainty-propagation specification, the panel probit can be re-estimated across posterior draws or across a selected subset of posterior draws.

D.2 Bayesian MCMC for the CRE Panel Bivariate Probit

Let the parameters of the panel bivariate probit be collected in:

$$\psi = (\beta_D, \beta_{Ln}, \pi_D, \pi_{Ln}, \delta_D, \delta_{Ln}, \{a_i\}, \Sigma_a, \rho_\varepsilon) \tag{D13}$$

The Bayesian sampler uses latent-variable augmentation. For each bank-year observation, define the latent vector:

$$Y_{it}^{\star} = \begin{pmatrix} Y_{D,it}^{\star} \\ Y_{Ln,it}^{\star} \end{pmatrix}.$$

Conditional on the observed binary outcomes, $Y_{it}^{\star}$ is sampled from a bivariate normal distribution truncated to the region implied by $(Y_{D,it}, Y_{Ln,it})$. Specifically:

$$Y_{D,it} = 1 \Rightarrow Y_{D,it}^{\star} > 0, \qquad Y_{D,it} = 0 \Rightarrow Y_{D,it}^{\star} \leq 0,$$

and

$$Y_{Ln,it} = 1 \Rightarrow Y_{Ln,it}^{\star} > 0, \qquad Y_{Ln,it} = 0 \Rightarrow Y_{Ln,it}^{\star} \leq 0.$$

The sampler cycles through the following blocks.

- *Step 1: Update latent utilities*

Sample:

$$Y_{it}^{\star} \mid Y_{it}, X_{it}, \bar{X}_i, \delta_t, a_i, \psi \tag{D14}$$

from the appropriate truncated bivariate normal distribution.

- *Step 2: Update regression coefficients*

Conditional on the latent utilities, random effects, year effects, and R_{ε}, the two-equation latent system is Gaussian. The coefficient blocks (β_D, π_D) and (β_{Ln}, π_{Ln}), or the stacked SUR coefficient vector, are updated from their Gaussian conditional posterior.

- *Step 3: Update year effects*

Conditional on the latent utilities and the remaining parameters, the year effects are updated from Gaussian conditional posteriors. To identify the model, one year is omitted or a zero-sum restriction is imposed, that is: $\sum_t \delta_{D,t} = 0$ and $\sum_t \delta_{Ln,t} = 0$

- *Step 4: Update bank random effects*

For each bank i, sample:

$$a_i = \begin{pmatrix} a_{D,i} \\ a_{Ln,i} \end{pmatrix}$$

from its bivariate normal conditional posterior:

$$p(a_i \mid Y_i^{\star}, X_i, \bar{X}_i, \beta, \pi, \delta, \Sigma_a, R_{\varepsilon}) \tag{D15}$$

- *Step 5: Update the random-effects covariance matrix*

Conditional on the bank random effects:

$$\Sigma_a \mid \{a_i\} \sim IW\left(\nu_a + N, S_a + \sum_{i=1}^{N} a_i\, a_i{}'\right) \tag{D16}$$

- *Step 6: Update the idiosyncratic correlation*

Because probit variances are normalized to one, the idiosyncratic covariance matrix is:

$$R_\varepsilon = \begin{pmatrix} 1 & \rho_\varepsilon \\ \rho_\varepsilon & 1 \end{pmatrix}.$$

The correlation parameter can be sampled using a Metropolis-Hastings step with transformation:

$$\rho_\varepsilon = \tanh(\eta_\varepsilon),$$

where $\eta_\varepsilon \in \mathbb{R}$.

The acceptance probability is based on the latent Gaussian likelihood of residuals:

$$e_{it} = Y_{it}^\star - \begin{pmatrix} X_{it}'\beta_D + \bar{X}_i'\pi_D + \delta_{D,t} + a_{D,i} \\ X_{it}'\beta_{Ln} + \bar{X}_i'\pi_{Ln} + \delta_{Ln,t} + a_{Ln,i} \end{pmatrix}.$$

- *Step 7: Compute predicted probabilities and AMEs*

For each retained draw, compute:

$$Pr(C_{it} = H_I \mid X_{it}), \quad Pr(C_{it} = H_{II} \mid X_{it}), \quad Pr(C_{it} = H_{III} \mid X_{it}), \quad Pr(C_{it} = H_{IV} \mid X_{it}).$$

Average marginal effects are computed as:

$$AME_{k,r} = \frac{1}{n}\sum_{i=1}^{N}\sum_{t=1}^{T_i}\frac{\partial Pr(C_{it} = r \mid X_{it})}{\partial X_{k,it}} \tag{D17}$$

for $r \in \{H_I, H_{II}, H_{III}, H_{IV}\}$.

D.3 Propagating First-Stage DTDF Uncertainty into the Panel Probit

The simplest implementation uses posterior-mean hard classifications from the dynamic DTDF stage. However, this treats estimated orientations as known and ignores first-stage uncertainty. To propagate uncertainty, we use the posterior draws of g_{it}.

For posterior draw s, we define:

$$Y_{D,it}^{(s)} = \mathbf{1}\left(g_{D,it}^{(s)} > 0\right),$$

and

$$Y_{Ln,it}^{(s)} = \mathbf{1}\left(g_{Ln,it}^{(s)} > 0\right).$$

Then, we estimate the panel bivariate probit conditional on each draw or on a selected subset of draws. The resulting cut posterior is:

$$p_{cut}(\psi \mid \mathcal{D}, X) = \int p(\psi \mid Y_D(g), Y_{Ln}(g), X) p(g \mid \mathcal{D})\, dg \tag{D18}$$

This is approximated by:

$$p_{cut}(\psi \mid \mathcal{D}, X) \approx \frac{1}{S}\sum_{s=1}^{S} p\left(\psi \mid Y_D^{(s)}, Y_{Ln}^{(s)}, X\right).$$

This procedure preserves the interpretation of the first-stage DTDF model. The second-stage panel probit explains the estimated annual orientations, but it does not feed back into the estimation of the technology and direction vectors.