Portfolio Construction Strategies in Emerging Markets: A Machine Learning Comparison Using BRICS Data

Abstract

Traditional portfolio optimization frameworks, such as the mean–variance and Sharpe-ratio models, often fail to capture the nonlinear and fat-tailed nature of emerging market returns. Our study evaluates whether machine learning approaches, specifically multifractal detrended fluctuation analysis (MF-DFA) and artificial neural networks (ANN), can improve portfolio performance under such conditions. Using daily data for 700 BRICS equities from January 2018 to December 2024 and twenty quarterly rebalancings, we compare eight portfolio configurations across eleven performance metrics. To the best of our knowledge, our study is the first to incorporate asymmetric tail modeling into neural network architectures in emerging stock markets. The results show that MF-DFA–Sharpe portfolios achieve annualized returns of 22.0% in Brazil and 27.3% in India, about 15 percentage points higher than traditional benchmarks. At the same time, asymmetric neural network models provide short-term advantages in Russia and South Africa. In contrast, the classical Markowitz minimum-variance model remains most effective in China for tail-risk control. Overall, our findings demonstrate that integrating asymmetry-aware and multifractal learning methods enhances portfolio resilience and performance in structurally inefficient markets, offering a practical pathway for adaptive investment strategies.

Keywords: Artificial Neural Networks; Investment Portfolio; Distribution with Long Tails; Portfolio Optimization.

1. Introduction

The optimization of investment portfolios has long been grounded in the mean–variance frameworks pioneered by Markowitz (1952) and the equilibrium pricing model of Sharpe (1964), both of which rest on assumptions of normally distributed returns, frictionless markets, and investor rationality (Lintner, 1965; Fama, 1970). Yet, empirical evidence reveals that asset returns exhibit fat tails, volatility clustering, and long-range dependencies; features at odds with these classical assumptions (Tiwari et al., 2019; Al-Yahyaee et al., 2020). In particular, emerging markets are characterized by structural inefficiencies such as lower liquidity, higher dividend yields, and greater exposure to political and regulatory shocks, conditions that amplify non-Gaussian behavior and generate exploitable anomalies (Boţoc & Pirtea, 2014; Alfaro & Mendoza, 2020; Salisu et al., 2021; Mohammed, 2025). From the emerging countries, BRICS markets have become economically relevant by their high share in global GDP (IMF, 2025). These countries are characteristic by available data, but high volatility, and market inefficiency at the same time. Such a combination makes them a proper sample for testing of added value of complex algorithms, which are able to account for such patterns (Oliveira et al., 2023). The Russia-Ukraine conflict and other recent geopolitical events further accentuate volatility in BRICS markets, underscoring the inadequacy of static models in these contexts (Peng et al., 2024; Gopal et al., 2025).

Despite these challenges, most advanced portfolio research remains focused on developed markets. In the S&P 500, for instance, multifractal detrended fluctuation analysis (MF-DFA) has been shown to detect persistence and multifractality in returns, improving risk-adjusted outcomes (Calvet & Fisher, 2002; Baruník & Křehlík, 2018). Likewise, artificial neural networks (ANNs) have demonstrated robust pattern-recognition capabilities, uncovering nonlinear relationships that elude quadratic optimisation (Chatterjee et al., 2000; Zhong & Enke, 2017; Ahmed et al., 2022). More recently, Oliveira et al. (2023) introduced an asymmetric Student's *t*-based ANN (ANNt) that explicitly models fat tails, producing improved Sharpe and Sortino ratios to both symmetric ANNs and traditional models. Moreover, hybrid approaches combining high-frequency data with deep learning architectures have further enhanced return predictability and responsiveness (Zhang et al., 2023; Ibikunle et al., 2024; Mohammed, 2024). Additionally, visual convex optimisation frameworks and fuzzy mean–variance models, integrating XGBoost, Elastic Net, and Random Forest, have demonstrated robustness under uncertainty and transaction constraints (Cajas, 2025; Zhang & Du, 2025).

However, the applicability of these advanced portfolio optimization techniques to emerging markets (particularly the BRICS economies) remains insufficiently explored. While recent studies have demonstrated the value of incorporating multifractality and asymmetry into machine learning models in developed markets, their transferability to more volatile and structurally distinct environments is not investigated. Emerging markets typically exhibit structural inefficiencies, including limited market liquidity, elevated dividend yields, and heightened vulnerability to political and regulatory disruptions. These characteristics tend to intensify deviations from normal return distributions and foster frequent regime shifts, which are characteristics under which machine learning models offer added value. As Gunjan and Bhattacharyya (2023) argue, such environments, marked by asymmetric information and institutional frictions, may be particularly conducive to the superior performance of neural networks and hybrid models. Yet, empirical applications of MF-DFA and ANNt in BRICS markets remain scarce, and no study to date has systematically examined the role of asymmetric tail modeling in machine learning (ML) frameworks within these contexts. This distinction is important because most related studies in emerging markets address only individual components of this problem rather than an integrated portfolio construction framework. Mei et al. (2017), for example, examine volatility forecasting and the role of realized skewness and kurtosis in emerging markets, but their study focuses on volatility rather than portfolio returns and does not employ machine learning portfolio models. Mensi et al. (2023) apply MF-DFA to examine return dynamics in Middle Eastern markets, but their framework does not integrate asset ranking using neural networks or asymmetric tail modelling. Similarly, Benbachir (2025) documents the relevance of MF-DFA for detecting multifractality in African stock markets, but the analysis does not incorporate neural networks or explicitly model asymmetric return behavior within portfolio construction. Therefore, while the existing literature provides evidence on volatility, multifractality, or return dynamics in emerging markets, it has not yet jointly examined portfolio construction strategies that combine multifractal signals, asymmetric tail modelling, and machine learning architectures in BRICS equity markets.

This gap motivates three key research questions: (1) To what extent do ML portfolio strategies, specifically MF-DFA and ANNt, offer added value over traditional approaches such as Markowitz minimum variance and Sharpe maximization in emerging markets? (2) How does the explicit modeling of asymmetry (via Student's *t* distribution in ANNt) contribute to performance improvements, particularly in terms of downside risk metrics such as Value at Risk (VaR) and Conditional VaR (CVaR)? (3) Which BRICS markets exhibit the most and least pronounced gains from ML strategies, and what structural characteristics account for these differences?

To address these questions, our study pursues three objectives. First, we compare the out-of-sample performance of traditional and ML portfolio strategies across BRICS equity markets over twenty quarterly rebalancing intervals from January 2018 to December 2024. Second, we assess the contribution of asymmetric tail modeling in ANNt to downside risk control. Third, we investigate cross-country heterogeneity in performance outcomes, identifying the local market features that may explain the differential effectiveness of ML optimization. By explicitly incorporating asymmetry and multifractality into portfolio construction and by applying these methods to structurally diverse emerging markets, our study brings novel empirical evidence to a literature that has thus far remained largely focused, to the best of our knowledge, on developed economies. In doing so, we aim to advance the understanding of how machine learning can be tailored to the unique challenges and opportunities presented by emerging financial systems.

Our methodological framework employs data from Bloomberg for 700 constituent stocks across the IBX 50 (Brazil), IMOEX (Russia), NIFTY 50 (India), SSE 50 (China), and FTSE/JSE Top 40 (South Africa). We train the portfolios on rolling biennial windows and rebalance them quarterly, yielding twenty out-of-sample evaluation periods. MF-DFA constructs portfolios by ranking assets according to the Market Deficiency Measure derived from the Hurst exponent (Hurst, 1951), which captures persistence and memory dependence. ANNt employs a single-hidden-layer neural network[1] whose output scores are transformed via the Student's *t* cumulative distribution to model fat tails (Student, 1908; Oliveira et al., 2023). While traditional portfolios follow quadratic optimisation rules (Markowitz, 1952; Sharpe, 1966), we evaluate performance across eleven metrics (including annualised return, Sharpe and Sortino ratios, Jensen's Alpha, VaR, CVaR, volatility, beta, and Treynor ratio) in local currencies and convert it to USD for comparability (Jensen, 1968; Sortino & Price, 1994).

[1] The Universal Approximation Theorem states that a feedforward neural network with one hidden layer and a suitable nonlinear activation function can approximate any continuous function on a compact domain, given a sufficiently large number of neurons (Cybenko, 1989; Hornik, 1991). It provides a theoretical rationale for employing a single hidden-layer ANN as a flexible nonlinear approximator.

Our out-of-sample results indicate that portfolios constructed via the MF-DFA–Sharpe approach deliver higher annualized returns in Brazil and India compared to classical mean–variance and Sharpe maximization strategies. By contrast, minimum-variance portfolios based on ANNt in these markets tend to exhibit slightly elevated volatility, although they achieve modest improvements in downside risk metrics. In China, the traditional Markowitz minimum variance model remains the most effective at controlling extreme losses, reflecting the market's tighter regulatory regime and state-driven trading dynamics. Russia and South Africa present a more nuanced picture: ANNt variants tend to outperform over shorter horizons but require ongoing risk management to sustain advantages over longer cycles, consistent with episodic regime shifts in those economies.

These findings carry significant theoretical and practical implications. Theoretically, we extend Modern Portfolio Theory by embedding multifractal and deep learning insights into emerging market contexts, thereby operationalising Simon's (1955) concept of bounded rationality: ML models serve as rational agents that mitigate cognitive and informational constraints in markets where prices deviate from fundamentals. Practically, our study equips portfolio managers and institutional investors with empirically validated toolkits driven by AI, demonstrating that ANNt is particularly effective in regimes with high volatility (Brazil, India), whereas MF-DFA excels where persistence dominates (South Africa). The framework provides a clear roadmap for both academic research and practical machine learning applications in emerging markets.

The study is organized as follows: in Section 2, we discuss the literature review. Section 3 describes the methods and the dataset. In Section 4, we provide the computations and comment on the results, while Section 5 concludes.

2. Literature review

2.1. Importance and Specificities of BRICS Financial Markets

The BRICS economies (Brazil, Russia, India, China, and South Africa) have increased in relevance within the global financial system, both as dynamic contributors to global growth and as sources of diversification for international investors. According to the IMF (2025), the BRICS countries collectively account for 24,87% of global GDP and more than 40% when measured in purchasing power parity. Their role in shaping global financial flows, commodity markets, and macroeconomic trends has been firmly established (Wilson, 2015), making their financial markets critical to understanding broader global dynamics. Despite their growing importance, the financial markets of BRICS nations display characteristics that distinguish them from those of developed economies. These include lower market liquidity, higher dividend yields, higher home biases, and greater exposure to external shocks such as geopolitical instability, exchange rate volatility, and abrupt capital flow reversals (Boţoc & Pirtea, 2014; Alfaro & Mendoza, 2020). Regulatory regimes across BRICS markets are also more fragmented and subject to sudden interventions, which often distort price discovery mechanisms and investor behavior (Salisu et al., 2021; Gopal et al., 2025).

Another common feature across these markets is the prevalence of informational inefficiencies (Anwar & Iwasaki, 2022). Due to limited analyst coverage, reduced transparency, and weaker investor protections, BRICS equity markets often reflect delayed responses to news and policy changes (Brody et al., 2012; Souza et al., 2018). These inefficiencies contribute to persistent deviations from the assumptions of classical financial theory, such as market efficiency and normal return distributions (Arshad et al., 2016). Behavioral biases and the dominance of retail investors further amplify return anomalies, making traditional portfolio optimization tools potentially less effective in these contexts. As emerging market assets continue to attract global attention, particularly in multi-asset portfolios seeking yield, understanding their structural distinctiveness becomes important. The case for developing more flexible, adaptive models to handle such complexity is supported by empirical findings demonstrating classical models' underperformance in environments with high volatility and asymmetry (Bağcı & Soylu, 2024; Georgantas et al., 2024).

2.2. Theoretical Foundations and Advances in Machine Learning for Portfolio Optimization

The inadequacy of traditional portfolio optimization frameworks in environments characterized by nonlinearity, non-stationarity, and structural breaks has led to increased interest in machine learning methods (López de Prado et al., 2025). These approaches have added value to classical parametric models in leveraging data-driven, non-assumptive architectures that adapt to complex patterns and allow for modeling asymmetric risks and evolving market conditions. At the core of this theoretical shift lies the notion of bounded rationality (Simon, 1955), which recognizes that investors operating in highly uncertain environments, such as those in emerging markets, face cognitive and informational limitations. ML models act as computational agents capable of learning from high-dimensional, noisy, and incomplete datasets, thus helping overcome such constraints. Their adaptability makes them particularly suitable for environments like BRICS, where market inefficiencies, episodic volatility, and structural changes are prevalent. Two methodological strands within ML have emerged as particularly relevant for portfolio construction in these contexts: memory-sensitive models and distribution-aware neural networks.

Memory-based models, particularly Multifractal Detrended Fluctuation Analysis (MF-DFA), capture dependencies over long range and the presence of autocorrelation in financial time series. The technique has been applied to evaluate deviations from market efficiency and to measure return predictability. Tiwari et al. (2019) demonstrated that MF-DFA is capable of distinguishing between random and persistent components in price dynamics. In the context of Brazil, Maciel (2021) operationalized this technique through the Market Deficiency Measure (MDM), which uses the Hurst exponent to quantify market inefficiency. The author's findings indicate that portfolios constructed by selecting assets with higher MDM values might improve performance over those built on more informationally efficient assets, suggesting that memory effects provide a valuable selection signal in emerging markets. In this context, MF-DFA aligns with the characteristics of BRICS markets. In markets where prices may not immediately reflect available information due to frictions or behavioral anomalies, long-memory models enable investors to exploit slow-moving patterns that are not captured by conventional approaches.

Another stream of machine learning approaches is artificial neural networks (ANNs). The approach, grounded in the universal approximation theorem (Hornik et al., 1989), has been recognized for the ability to capture nonlinear input-output relationships in financial data. Recent works (Kopeliovich & Pokojovy, 2024; Mohammed, 2024) further support the relevance of ANN models for financial prediction, demonstrating their effectiveness in capturing nonlinear patterns in stock price behavior through asymmetric learning frameworks. However, conventional ANN architectures often assume symmetric distributions and rely on loss functions that do not fully account for tail risk. It limits their effectiveness in highly volatile markets. To overcome these limitations, Oliveira et al. (2023) developed the ANNt framework, which replaces the normal output distribution with the Student's *t* distribution, thereby allowing the model to accommodate fat tails and skewness. Applied to the S&P 500, this model yielded improved downside risk control, particularly as measured by VaR and CVaR, and delivered stronger risk-adjusted performance through enhanced Sharpe and Sortino ratios. ANNt and similar adjustments bring support for use in environments where return distributions are characterized by asymmetry and fat tails. However, the application of these methods remains to be explored for the framework of emerging markets. Based on these findings, we define the first two hypotheses:

> *Hypothesis 1:* Machine learning portfolios do not improve risk and return indicators compared to standard portfolio strategies (Markowitz and Sharpe) in BRICS markets.

> *Hypothesis 2:* Incorporating return asymmetry through tail modeling adds no value in terms of risk and return compared to traditional portfolios and baseline machine learning strategies.

Beyond MF-DFA and ANNt, a growing body of research explores hybrid, ensemble, and uncertainty-aware models that integrate multiple learning techniques to improve portfolio construction. These include fuzzy logic systems, convex optimization with synthetic data, volatility clustering models, and advanced deep learning architectures. Cajas (2025) proposed a visual convex optimization model that integrates neural networks with synthetic data generation, demonstrating strong performance under data irregularity and limited observability. Furthermore, Zhang and Du (2025) developed a fuzzy mean-variance optimization framework using XGBoost, Elastic Net, and Random Forest, showing enhanced risk estimation under ambiguity. Yang and Liu (2023) combined multifractal volatility modeling with deep learning, enabling improved performance under volatility clustering. At the architectural level, Vuong et al. (2025) compared Transformer, BiLSTM, and CNN attention models, identifying Transformer-BiLSTM hybrids as effective in modeling sequential and nonlinear dynamics. In this context, their findings are relevant to markets, where structural breaks and regime switches are common. Complementary research by Kumaresh et al. (2025) and Selvam et al. (2025) emphasized the transferability of ANN systems developed for other domains (e.g., climate modeling, logistics) to high-uncertainty environments in finance. Moreover, the effectiveness of hybrid models is also supported by Zhong and Enke (2019), who demonstrated that combining ANN and SVM improves directional return forecasts. Meanwhile, Ibikunle et al. (2024) emphasized the role of ML in decoding high-frequency trading patterns.

Despite these frameworks bringing helpful insights into portfolio construction, their evaluation within emerging equity markets remains rare. While some case studies exist (e.g., Maciel (2021) on Brazil), comprehensive comparative analysis across BRICS countries using advanced ML remains underdeveloped. As Gunjan and Bhattacharyya (2023) note, recent evidence falls short of assessing generalizability to less stable markets. Thus, a gap remains: while ML techniques are theoretically suitable to bring added value for the portfolio construction within BRICS markets, their empirical implementation and evaluation in these markets lag behind. Furthermore, these markets have various context-specific constraints. Therefore, we define our third hypothesis:

> *Hypothesis 3:* There are no significant differences in the performance of machine learning-based portfolio strategies (MF-DFA and ANNt) across BRICS markets.

3. Data and methods

To ensure clarity and structure in the presentation of our data and methodology, we organize the relevant components across three tables. Table 1 presents the equations used to construct the compared portfolios and econometric models. Table 2 provides a detailed description of the dataset, including the characteristics of the portfolios under comparison, the number of assets included in each sample, and the sparse portfolios[2]. Table 3 outlines the validation parameters and performance metrics employed to evaluate the models. This organization facilitates a transparent and systematic understanding of the analytical framework applied in the study.

We apply asymmetric artificial neural network techniques (ANNt)[3] to individual assets within the stock market indices of the BRICS nations (Brazil, Russia, India, China,

[2] The restriction to 20 assets was imposed as a design choice to retain the advantages of a sparse portfolio, including reduced transaction and monitoring costs, increased robustness to noise, improved performance due to the optimizer's focus on the most informative assets, and greater ease of portfolio management and rebalancing (Brodie et al., 2009).

[3] We retained the shallow neural network architecture with one hidden layer, following de Oliveira et al. (2024). The choice was primarily motivated by consistency with the ANNt framework and by computational feasibility. Since the analysis required estimating more than 10,000 neural networks, with

and South Africa), as well as for the United States as a representative of a developed market. Specifically, we utilize constituent stocks from the IBX50 (Brazil), IMOEX (Russia), NIFTY 50 (India), SSE 50 (China), FTSE/JSE Top 40 (South Africa), and the S&P 500 (United States). All data are downloaded from Bloomberg. The analysis includes a total of 700 stocks, comprising 490 from the U.S. market and 210 from BRICS markets. Although the initial dataset consisted of 729 assets, we excluded a subset due to extended periods of missing data, reflecting typical challenges associated with market liquidity and data availability in emerging economies. We examine each BRICS portfolio independently to assess both commonalities and regional differences in performance dynamics. Our dataset spans from January 3, 2018, to December 31, 2024, on a daily frequency for each asset that makes up the benchmark index in each country analyzed, in relation to the return of the benchmark index itself.[4]

Stocks delisted during the sample period were not included in the dataset. The final dataset is restricted to assets with sufficient daily price information over the country-specific sample period. Moreover, all securities used in portfolio construction were required to exhibit daily trading activity throughout the analyzed period, reducing the influence of illiquid assets and improving the practical feasibility of the resulting portfolios. Returns were computed as discrete returns[5], as this measure provides a direct representation of realized cumulative portfolio performance across rebalancing horizons. While discrete returns may display stronger departures from normality than log returns, they are consistent with the observed price-based portfolio gains and losses and support the use of fat-tailed return distributions in the empirical framework.

A portfolio training is conducted on a rolling biennial basis, while we evaluate performance over a testing period that extends from January 3, 2020, to December 31, 2024 (incorporating quarterly rebalancing). The framework enables a comprehensive assessment of portfolio formation strategies across both long and short-term horizons,

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one ANN estimated for each asset in each rebalancing period, deeper architectures would substantially increase processing time. In our additional tests, the LSTM specification required more than one week of computation, compared with approximately two days for the shallow ANN, without providing a sufficiently consistent improvement across portfolio specifications to justify the additional computational burden. To document this choice, Table 17 (in Appendix) reports an architecture comparison based on the S&P 500 sample, contrasting the shallow ANN specification with a Long Short-Term Memory (LSTM) architecture.

[4] Using frequencies higher than daily, such as intraday data, would not be feasible, since not all assets traded on the stock exchange have high liquidity, which would exclude a significant number of stocks from the analysis. Lower frequencies, such as weekly data, would require a much longer training series to meet the restriction of observations exceeding the number of assets for the minimum variance solution matrix.

[5] $\left(p_{s(t)} - p_{s(t-1)}\right)/p_{s(t-1)}$, where, $p_{s(t)}$ is the price of the asset s in the period t and $p_{s(t-1)}$ is the price of the asset s in the period $t - 1$.

capturing market dynamics over the first half of the 2020s. In Figure 2, we illustrate the sequential structure of the training and testing periods, comprising a total of 20 quarterly rebalancing intervals. For each rebalancing, portfolios are trained using the preceding two years of data and subsequently tested until the end of 2024. Therefore, the training subsample has a fixed length of eight semesters (8Q), being rebalanced 20 times in the total sample (20 splits). The test subsample, on the other hand, has a variable length from 1 to 20 semesters (1Q to 20Q). For example, the first portfolio is trained from January 3, 2018, to December 31, 2019, and tested from January 2, 2020, to December 31, 2024. The second rebalancing employs a training window from April 1, 2018, to March 31, 2020, with testing from April 1, 2020, onward. This rolling approach continues through the 20th rebalancing, where training spans from September 1, 2022, to August 31, 2024, followed by a final testing period from September 1, 2024, to December 31, 2024.

In the empirical model, portfolio weights are assumed to remain constant within each testing period. Portfolios are recalculated quarterly, and the newly estimated weights are held fixed over the corresponding test period, independently of previous portfolio compositions. We construct portfolios using the Markowitz minimum variance method, the Sharpe ratio optimization, multifractal detrended fluctuation analysis (MF-DFA), and artificial neural networks with *t*-distribution classification (ANNt). For comparability, both the MF-DFA and ANNt portfolios are restricted to a maximum of 20 assets[6] per rebalancing cycle. All portfolios are constructed with the primary objective of maximizing the rate of return, subject to the corresponding portfolio constraints. Risk and return characteristics, including Jensen's Alpha, the Sharpe ratio, the Sortino ratio, and the other performance metrics reported in Table 3, are subsequently used for ex-post performance evaluation.

We process the data using the *R* software and employ the approach of Oliveira and Ceretta (2023)[7]. Table 1 outlines the model specifications used to construct the various portfolios. The technique, which incorporates asymmetric tail behavior through a Student's *t* distribution within artificial neural networks, has previously demonstrated superior performance for the S&P 500 (Oliveira et al., 2023). However, to the best of our knowledge, this approach has not yet been extended to emerging market equities. The minimum variation portfolio is subject to the weight restrictions $W_s \geq 0$ and $\sum_{s=1}^{N} W_s =$

[6] The choice of 20 assets is justified by the literature as the number of assets needed to eliminate the specific (diversifiable) risk of a portfolio (Evans & Archer, 1968).
[7] The code is available at: https://github.com/aleoli05/ANNt.

1, where s represents assets from 1 for N; $\sigma_p^2 = W^T \Phi W$ is the portfolio variance, T is the total period analyzed (test sample period), and Φ is the covariance matrix of order NxN. By limiting the portfolio to covered positions, we obtain sparse portfolios, which have advantages in reduced costs associated with buying, monitoring, and selling many positions. The maximum Sharpe portfolio has restrictions given by $\sum_{s=1}^{N} W_s = 1$. $R_p = (r_1, r_2, \dots, r_N)^T$ is the average vector of asset returns. The Sharpe portfolio emphasizes the maximized value of return relative to risk. In the Multifractal Trendless Fluctuation Analysis (MF-DFA) or simply (MF) portfolio, $y_v(t)$ is the first difference between the observed value of the Return (r_s) and the Average Return $(\bar{r_s})$ of the asset (s) for each subinterval $[v = 1, \dots, 2T_s]$, where T is the number of observations. $F^2(s, v)$ is the estimated variance for $v = 1, \dots, T_s$ and $v = T_s + 1, \dots, 2T_s$. $F_q(s)$ is the fluctuation function of order q. We use the Hurst Exponent $(h(q))$ to calculate the Market Deficiency Measure (MDM). The lower the MDM value, the more efficient the market is considered. The MDM is used to classify the assets that make up the MF-DFA portfolio (Hurst, 1951).

In artificial neural network portfolios described in Table 1, a mathematical model of a neuron (Haykin, 1994; Tyagi & Abraham, 2022) presents the activation potential[8] represented by (v_k) that is mathematically the sum of the linear combination (u_k), and the bias term (b_0). u_k is a linear combination of the synaptic weights w_{kj}[9] of k neurons with each of the inputs. It means that u_k is a weighted linear combination of each input variable or input signal (x_j). To amplify the activation signal, we add a bias constant b, which increases the observed signal (if positive) or decreases the observed signal (if negative), to the resulting linear combination u_k. It is an external parameter to the neuron. In short, this is called the activation potential v_k, which is then maintained by the activation function $\varphi(.)$. $\hat{y}_k$ is the output signal or final result (probability of return superior to the benchmark performance). $e_j(n)$ is the error signal of neuron j; n is the training iteration; $d_j(n)$ is the desired response for neuron j, and $y_j(n)$ is the output signal from neuron j. The cumulative probability function $(F(t))$ of a given value $P(X \leq$

[8] It denotes the weighted aggregation of a neuron's inputs combined with the bias term, forming the activation potential that serves as the argument to the activation function. If this potential is high enough to reach a certain threshold, the neuron will be "activated," producing an output value, similar to the cell body in an organic neuron (Figure 1a and 1b).

[9] We initialized the network using Gaussian random weights, which provides moderate numerical stability, and tested alternative activation functions, retaining the specification with the lowest output variance as an indicator of reduced sensitivity to initialization.

x) in the Student's t distribution is then applied to each asset, ordered from largest to smallest, to form portfolios, which can gain values between 0 and 1[10] .

We initiate our analysis by modeling the excess returns of individual assets using artificial neural networks (ANNs). To enhance the probabilistic interpretation of the model outputs, we incorporate the Student's t distribution, which allows us to estimate the likelihood that a given asset will outperform the benchmark index. The approach, referred to as ANNt (de Oliveira et al., 2024), combines machine learning techniques with econometric rigor. The formal specification of the model is given by the following equation:

$$r_{exc\_p\_s(t)} = \varphi\left(\sum_{kjj=1}^{N} w_{k^{(l)}j} x_{Hk^{(l)}j} + b\right), \tag{1}$$

or

$$r_{exc\_p\_s(t)} = \varphi\left\{\sum_{k^{(l)}j=1}^{N} w_{k^{(l)}j}\left[\varphi^{(l)}\left(w_{1j}R_{SP500\,(t-1)} + \sum_{i=1}^{5} w_{ij} r_{exc_{s(t-i)}} + b^{(l)}\right)\right] + b\right\}. \tag{2}$$

The dependent variable is the daily excess return of each asset ($r_{exc\_p\_s(t)}$) relative to the return of the benchmark of each country (e.g., benchmark for the U.S. is the S&P500 portfolio (R_{SP500})). In this case, the independent variables include a lag of one day of the S&P500 returns ($R_{SP500\,(t-1)}$) and the lagged excess return of the asset itself, $r_{exc\_s_{(t-dn)}} = r_{s(t-dn)} - R_{SP500\,(t-dn)}$, with the lag ranging from 1 to 5 days, $dn = (-5,..,-1)$. φ[11] is the output layer sigmoid activation function; N is the number of days for the training sample; w_{kj} is the synaptic weight of information j in neuron k; b[12] is the bias of the output layer; $x_{Hk^{(l)}j}$ is the hidden layer with N hidden neurons $k^{(l)}$. $\varphi^{(l)}$ is the

[10] CDF denotes the cumulative distribution function, which converts a given value into a probability between 0 and 1. We considered both the standard Student's t CDF, with location zero and scale one, and a non-standard version with asset-specific location and scale parameters. The final specification uses the standard Student's t CDF, while asymmetry is incorporated through an adjustment based on the direction of skewness and the corresponding standard error. The specification was retained because it produced more stable and superior portfolio results in our robustness checks.

[11] Notation adapted from Haykin (1994), Goodfellow et al. (2016), Jagtap et al. (2020), Dubey et al. (2022), and de Oliveira et al. (2024).

[12] Notation adapted from Haykin (1994), Goodfellow et al. (2016), Tyagi & Abraham (2022), and de Oliveira et al. (2024).

hidden layer hyperbolic tangent activation function, and $b^{(l)}$ is the bias of the hidden layer.

To account for distributional asymmetries, we compute the skewness and kurtosis of each excess return series and adjust the average probability curve using the standard error to better reflect these higher-order moments. The extension enhances the conventional ANN framework by explicitly incorporating asymmetric return profiles, thereby improving downside risk management (Rachev et al., 2010). To capture the sense of asymmetry, whether positive or negative, it is necessary to obtain Pearson's asymmetry coefficient (A_s). If $A_s < 0$, the distribution is negative skew; if $A_s = 0$, the distribution is symmetric; and if $A_s > 0$ the distribution is positive skew. To measure the kurtosis, we use the moment coefficient α_4. If $\alpha_4 < 3$, the distribution is platykurtic; if $\alpha_4 = 3$, the distribution is mesokurtic; and if $\alpha_4 > 3$, leptokurtic.

To mitigate overfitting, we follow the previous research by de Oliveira et al. (2024). Therefore, we train ANNt models using a single hidden layer with several neurons equal to the number of assets in the respective market proxy and iterate over 2,000 training steps, with batch gradient descent training (8 semesters, approximately 500 days, depending on the rebalancing quarter), and a step size of 1/500. A large batch size is used because it is a hybrid model, in which the calculation of Sharpe and Markowitz portfolios requires more observations in the sample than the number of analyzed assets. In the model by de Oliveira et al. (2024), the learning rate is adaptively tested between 0.005 and 0.3. The model's predicted excess returns ($r_{exc\_p\_s(t)}$) are compared with the actual excess returns ($r_{exc_{s(t-1)}}$), using the simple difference or error (ε) between the values ($\varepsilon = r_{exc\_p\_s(t)} - r_{exc_{s(t)}}$), as a loss function. The objective function $J(\varphi)$ is the cost function of Mean Squared Error (MSE), without regularization: $J(\varphi) = \frac{1}{T}\sum_{t=1}^{T}\left(r_{exc\_p\_s(t)} - r_{exc_{s(t)}}\right)^2$.

The neural network is stable, not exhibiting explosive movements with increasing probability amplitude, since the iterative prediction of the neural network occurs over the established total epochs (2000 iterations) to generate the weights during the training phase. In the testing phase, the ANN weights remain constant, but excess returns are fed back with each new market closing during the testing period, preventing the model from being explosive when calculating the Student's *t* probability. The enhancement is practiced by portfolio managers, who monitor price movements and feed their models with the most recent information.

Once the predicted excess return is obtained, it is treated according to the Student's t distribution, being individually corrected for the standard error if the distribution presents skewness and kurtosis in the direction of the $Median$ ($Correction\ of\ Assimetry = r_{exc\_p\_s(t)} \pm {}^{\sigma_s}/_{\sqrt{T}}$), minus if negative skewness, and plus if positive skewness. We use the t-distribution instead of the normal distribution because it better fits the series of financial returns, which are overwhelmingly leptokurtic, with heavy tails and skewness (Campbell & Akhtar, 2000; Chen et al., 2001; Laurent, 2002; Rachev et al., 2005; Ibragimov et al., 2015). The objective is to capture the assets that have the highest probability of exceeding the market proxy return, with adequate skewness. Then, the probability of the asset exceeding the market proxy return is calculated, and all assets are ranked in descending order of probability of exceeding the return. The cutoff point for the ANNt portfolio is the 20 assets with the highest probability. With these 20 assets, we generate ANNt portfolios with equal weights (ANNt_EQ); the ANNt portfolio with a sample of 20 assets with minimum variance weights (ANNt_Mkw), and the ANNt portfolio with a sample of 20 assets with maximum Sharpe weights (ANNt_Sharpe). Therefore, by employing the ANNt technique, we filter out the excess returns in each market.

In summary, we perform in five steps to obtain the investment portfolio. (i) A neural network predicts excess returns relative to a market proxy. (ii) We calculate the probability of returns exceeding the market proxy return by using the t-distribution (considering skewness and kurtosis)[13] . (iii) Then, we rank the assets in descending order of probability of exceeding market returns. (iv) Followingly, we establish the number of assets in order based on ranking (the 20 assets with the highest probability of overperforming the market). (v) From the 20 assets, we assemble portfolios with equal weights, minimum variance, and maximum Sharpe.

In Table 2, we present an overview of the portfolio configurations derived from the models described in Table 1. These include the benchmark portfolio, traditional Markowitz and Sharpe optimized portfolios, two portfolios based on multifractal detrended fluctuation analysis (MF-DFA), and three ANNt portfolios. The Markowitz and Sharpe portfolios are optimized over the full sample of 70 selected assets. Table 2

[13] The original Student's t-distribution introduced by William Sealy Gosset under the pseudonym "Student" (Student, 1908) is symmetric and therefore does not capture skewness. Asymmetric extensions, such as skew-t specifications, were developed later. Azzalini and Capitanio (2003) are often cited as a theoretical framework for skew-t distributions, while Hansen (1994) is frequently regarded as an early econometric framework for modelling conditional densities with skewness and fat tails.

also reports portfolio sparsity, measured as the number of assets[14] with non-zero portfolio weights after optimization (the average for balanced portfolios)[15]. Although the MF-DFA and ANNt procedures initially select up to 20 candidate assets, the final optimized portfolios may contain fewer active positions. The MF_MKW portfolio contains 13, 15, 11, 9, 17, and 17 non-zero weights in the Brazilian, Russian, Indian, Chinese, South African, and U.S. markets, respectively. The MF_SHARPE portfolio is substantially more concentrated, with only 1, 1, 1, 1, 2, and 1 non-zero weights across the same markets, indicating that this specification provides limited diversification and does not fully eliminate idiosyncratic risk[16]. The ANNt_MKW portfolio contains 10, 15, 12, 5, 17, and 12 non-zero weights, respectively, while the ANNt_SHARPE portfolio contains 5, 7, 3, 3, 5, and 6 non-zero weights. These results clarify that the 20-asset selection rule defines the candidate set, whereas actual portfolio sparsity is determined by the number of non-zero weights produced by the subsequent weighting procedure. The MF_SHARPE portfolio is the sparsest specification, with only one or two assets receiving non-zero weights across most markets. While this extreme concentration is associated with the highest returns in markets such as Brazil and India, it is also accompanied by substantially higher volatility, VaR, and CVaR, indicating a trade-off between return maximization and diversification. By contrast, the ANNt portfolios are generally less sparse, containing a larger number of assets with non-zero weights. Although less concentrated portfolios do not systematically generate higher returns, they tend to exhibit more balanced risk-return characteristics and greater robustness from a portfolio management perspective. Overall, our results suggest that sparse portfolios may outperform in terms of return under

[14] The number of assets refers to the average number of constituent stocks with non-zero portfolio weights across all rebalancing periods. Therefore, it represents the effective number of companies included in the final portfolio after the optimization procedure, rather than the total number of assets available in the initial sample.

[15] The reported values are obtained by averaging the number of stocks with non-zero portfolio weights across the 20 quarterly rebalancing periods. Since portfolio holdings are discrete entities, each rebalancing produces an integer number of invested companies. For simplicity and readability, the mean values are reported as the nearest whole number. Consequently, these figures represent the typical number of companies included in the portfolio after optimization and rebalancing over the sample period.

[16] The extreme sparsity of the MF_SHARPE portfolios (typically one or two assets with non-zero weights) raises an important practical consideration. Although such concentration may arise from the optimizer exploiting persistent multifractal signals in inefficient markets, it may also reflect an increased sensitivity to sample-specific return patterns and, consequently, a higher risk of overfitting. Therefore, the superior return performance observed for MF_SHARPE portfolios in Brazil and India should not be interpreted as evidence that highly concentrated portfolios represent a universally optimal investment strategy. Rather, the results suggest a trade-off between return maximization and diversification. Consistent with diversification theory (Evans & Archer, 1968), the lack of breadth is associated with substantially higher volatility and tail-risk measures (VaR and CVaR) reported in our analysis. By comparison, ANNt portfolios maintain a broader asset allocation and achieve more balanced risk-adjusted outcomes, indicating greater robustness from a portfolio management perspective.

specific market conditions, whereas less sparse portfolios provide superior diversification and more stable risk-adjusted performance.

To validate the results of our study, we apply traditional return and risk backtesting (more details are provided in Table 3). For returns, we observe the following parameters: average return, annualized return, and accumulated return. To measure risk, we utilize Jensen's Alpha, the Sharpe Ratio, the Sortino Ratio, the Treynor Ratio, Beta (CAPM), annualized volatility, Value at Risk (VaR), and Conditional Value at Risk (CVaR), following the methodology of Oliveira et al. (2023). In the validation parameter equations, $r_{p,t}$ is the portfolio return (p) in the period (t) with the series length (T) being the standard year with trading 252 days. $\gamma = 5\%$ is the significance level of the normal distribution at the 95% confidence level. $p(x)$ is the probability density of the return value (x). c is the cutoff point of the analyzed distribution. We use the risk-free rate (r_f) with zero value. $\sigma_{p,d}^{A}$ is the annualized standard deviation of negative returns; $cov(r_{p,t}, r_{SP500,t})$ is the covariance of the returns of the portfolio p with the market index in a period t, and $\beta_{p,d}^{A}$ is the annualized beta of negative returns.

We estimate the parameters in the local currencies of each of the U.S. and BRICS markets: Brazilian Real (BRL), Russian Ruble (RUB), Indian Rupee (INR), Chinese Yuan Renminbi (CNY), South African Rand (ZAR), and United States Dollar (USD). Then, we convert these results to USD for performance comparison purposes (Table 7). Tables 8 and 9 are included to test Hypothesis 1 using paired mean-difference tests, thereby strengthening the robustness of the empirical results. In the Appendix, we further extend the analysis to the 2025 period to provide additional confirmatory evidence. For Hypothesis 2, Table 10 reports tests confirming the presence of skewness, kurtosis, and non-normality based on the Kolmogorov-Smirnov (KS) test. These tests are performed for all assets included in the analyzed BRICS and U.S. market indices across the 24 rebalancing periods of the sample.

4. Results

4.1 The dynamic risk-return trade-off among traditional and machine learning portfolio approaches in emerging countries

The integration of machine learning techniques into portfolio optimization has attracted attention, particularly in response to the limitations of traditional models in environments characterized by structural inefficiencies, regime shifts, and heavy-tailed return distributions. While early advances, such as those proposed by Chatterjee et al. (2000) and Zhong and Enke (2019), demonstrated the potential of artificial neural networks for capturing nonlinear dependencies in developed markets, recent research has expanded the methodological frontier by incorporating multifractal metrics (Benbachir, 2025; Maciel, 2025) and architectures accounting for asymmetries like ANNt (Oliveira et al., 2023). These approaches challenge the assumptions of normality and stationarity embedded in classical models, such as those proposed by Markowitz (1952) and Sharpe (1964), instead offering adaptive, data-driven alternatives that can model real-world market behavior more accurately (Abir et al., 2025; Acikgoz, 2025).

Moreover, relevant are the advances in multifractal detrended fluctuation analysis (MF-DFA), which captures long memory and persistence in price dynamics, and in asymmetric neural networks (ANNt), which utilize fat-tailed Student's *t* distributions to reflect events causing extreme volatility. The benefits of such models have been evidenced in high-frequency trading contexts (Ibikunle et al., 2024), regime switching environments (Yang & Liu, 2023), and volatility clustering scenarios (Zhang et al., 2023). Furthermore, ensemble approaches and hybrid ML models (such as the fuzzy mean-variance optimization frameworks) have demonstrated robustness under informational noise and ambiguous market signals (Zhang & Du, 2025). However, these empirical applications remain centered on developed markets, with relatively little attention given to how such models perform under the distinct structural conditions of emerging economies.

In light of these gaps, this section empirically assesses the comparative performance of traditional and machine learning portfolio strategies across BRICS equity markets. We evaluate eight portfolio configurations (including Markowitz, Sharpe optimized, MF-DFA, and ANNt variants) using eleven distinct performance metrics. We cover return, volatility, tail risk, and risk-reward ratios. The multidimensional assessment allows us to move beyond single metric conclusions and robustly test whether ML frameworks offer meaningful advantages in the volatile, fragmented information context of emerging markets. In doing so, we aim to extend the generalizability of recent findings (e.g., Gunjan and Bhattacharyya, 2023; Oliveira and Ceretta, 2023) while providing new evidence on the contextual validity of ML portfolio construction techniques in markets characterized by lower liquidity, weaker investor protections, and greater regulatory discretion.

Brazil

The Brazilian equity market is characterized by pronounced informational inefficiencies, episodic volatility, and strong multifractality characteristics previously documented by Tiwari et al. (2019) and Maciel (2021). In Figures 3a to 13a, we observe the extent to which machine learning approaches could bring added value to portfolios consisting of Brazilian stocks.

As shown in Figure 3a, the MF_SHARPE portfolio achieves the highest average return in 19 out of 20 rebalancing periods, with only one negative quarter. It aligns with the findings of Maciel (2021), who demonstrated that Brazilian equities ranked by the Market Deficiency Measure (MDM) exhibit persistent return predictability. In Figure 4a (annualized returns), the MF_SHARPE portfolio again leads, often doubling the performance of the traditional Markowitz or Sharpe optimized portfolios. These outcomes reinforce the claim that long memory models like MF-DFA can extract hidden structure in inefficient markets (Benbachir, 2025). However, such results must be viewed cautiously: high persistence may also reflect exposure to autocorrelated risk rather than true alpha. Despite high returns, the MF_SHARPE portfolio exhibits substantial tail risk. In Figure 12a, the portfolio achieves the highest Value at Risk (VaR) in 17 of 20 quarters, while Figure 13a confirms that its Conditional VaR (CVaR) at the beginning of the period exceeded 8%. It is consistent with Oliveira et al. (2023), who noted that multifractal portfolios are particularly sensitive to shocks on the left tail in fat-tailed distributions. The elevated downside risk suggests a trade-off familiar from fractal volatility models (Yang and Liu 2023), where long-term return efficiency may come at the cost of extreme event exposure.

The Markowitz portfolio offers a markedly different profile. It consistently shows lower volatility (Figure 11a), lower VaR and CVaR (Figures 12a and 13a), and a stable Sharpe ratio (Figure 7a). While its return is moderate (Figure 3a), it remains above the benchmark IBRX 50 index. These results align with Zhang and Du (2025), who emphasized the robustness of variance-minimization frameworks under regulatory constraints and limited data visibility, both of which characterize the Brazilian equity environment. For long-horizon or risk-averse investors, this supports the view of Pan et al. (2025), who argue that traditional frameworks can still be effective if risk control is prioritized over seeking alpha. As for the ANNt, the performance varies. The ANNt_EQ strategy underperforms across most metrics, delivering negative returns in multiple quarters (Figure 3a) and high downside risk (Figure 13a). It supports earlier findings

(Chatterjee et al., 2000; Mohammed, 2024) that ANNs without risk weighting or tail modeling may fail in volatile contexts. However, the ANNt_MKW variant offers more balanced results. It shows competitive returns (Figure 4a), reduced volatility (Figure 11a), and reasonable Jensen's Alpha (Figure 6a). These outcomes are in line with the portfolio design of Oliveira et al. (2023), where incorporating a Student's *t* distribution into neural networks helped improve the trade-off between performance and risk. It also echoes the argument from Ibikunle et al. (2024), who observed that fat-tail aware models are better suited for real-world market regimes.

Sharpe and Sortino ratios (Figures 7a and 8a) again favor MF_SHARPE, but the advantage is accompanied by significantly higher volatility and beta (Figures 11a and 10a). While MF_SHARPE achieves the highest Treynor ratio (Figure 9a), this may simply reflect leveraged market exposure rather than systematic skill. As Zhang et al. (2023) note, strategies that overfit past high-volatility episodes can appear deceptively strong in ex-post metrics. Therefore, while these results highlight the potential of adaptive ML methods in enhancing returns, they also raise caution around out-of-sample robustness.

The evidence from Brazil underscores the core tension in portfolio optimization within emerging markets: models that exploit long memory (MF-DFA) or nonlinear scoring (ANNt_MKW) can achieve higher returns, but only with careful management of extreme risks. Consistent with the theoretical framework of bounded rationality (Simon 1955), ML models like ANNt and MF-DFA can help overcome noisy signals in Brazil's fragmented markets. However, their success depends heavily on risk controlling mechanisms, adaptive rebalancing, and realistic scenario testing, elements considered in recent literature (Cajas, 2025; Vuong et al., 2025). Overall, the Brazilian case suggests that while ML portfolios offer upside potential, the Markowitz strategy remains a rational benchmark for conservative allocations. For tactical investors, hybrid approaches that combine ANNt scoring with variance constraints (as in ANNt_MKW) may provide an effective middle ground between return maximization and capital preservation.

Russia

The Russian equity market is also characterized by episodic geopolitical shocks, regulatory opacity, and capital flow volatility. However, it provides a different environment from Brazil. Prior literature (Alfaro & Mendoza, 2020; Gopal et al., 2025) documents how policy risk and information asymmetries in Russia challenge stable return generation. Our analysis of portfolio strategies in this market (Figures 3b to 13b) examines these dynamics and reveals that model performance is sensitive to horizon length and volatility regimes.

In Figure 3b, the MF_MKW portfolio delivers the highest average return in 8 of 20 quarters, with ANNt_MKW close behind. These two strategies, multifractal with minimum variance and ANNs accounting for asymmetries, both outperform the benchmark IMOEX index and most classical configurations. In annualized terms (Figure 4b), ANNt_MKW leads in 10 rebalancings, suggesting that it adapts well to short-lived opportunity windows. The observation is consistent with the hybrid modeling literature (e.g., Yang and Liu 2023), which emphasizes the role of fractal signals and regime-aware weighting in highly stochastic markets. Cumulative return analysis (Figure 5b) supports the robustness of ANNt_MKW, which accumulates gains steadily, especially during short-term rallies and post-sanction rebounds. The MF_MKW variant tracks similarly but exhibits more pronounced drawdowns, hinting at reduced adaptability to regime breaks. It underscores the conclusions of Gunjan and Bhattacharyya (2023), who observed that model flexibility (rather than static optimization) is critical in navigating price distortions with a political background.

On the other hand, Markowitz once again provides the most consistent protection against losses, with the lowest volatility (Figure 11b), VaR (Figure 12b), and CVaR (Figure 13b). It confirms its utility as a baseline strategy, particularly during phases with elevated uncertainty, such as the 2022–2023 post-invasion period. Yet, ANNt_MKW narrows the gap in downside protection more effectively than other ML portfolios. Its VaR curve in Figure 12b is notably smoother, and its CVaR (Figure 13b) remains moderate relative to return gains. In Figure 6b, we study these methods from the perspective of Jensen’s Alpha. The metric is the highest for ANNt_MKW in 9 quarters, indicating strong risk-adjusted performance. Moreover, Sharpe and Sortino ratios (Figures 7b and 8b) also rank ANNt_MKW among the top in most periods, though values fluctuate, reflecting inherent return volatility. These results are in line with findings by Mohammed (2024), who demonstrated that ANN architectures yield improved ratios in high-risk financial domains (the author highlighted short to medium term horizons as suitable for these approaches).

As for exposure and systematic risk, Figure 10b shows that ANNt_MKW maintains a relatively stable beta, mostly below one. It is consistent with the notion that

ML portfolios can select stocks with less systemic exposure, potentially avoiding sectors prone to sanctions, resource dependency, or capital controls. Such adaptive behavior echoes the mechanisms described in the synthetic convex optimization approaches (Cajas, 2025), where portfolios dynamically shift toward "resilient" assets under data irregularity. Despite its improved risk metrics, the Sharpe optimized traditional portfolio fails to consistently deliver competitive returns (Figure 3b) and underperforms in cumulative terms (Figure 5b), suggesting that linear risk-reward maximization may not be suited to Russia's regime-switching market[17]. It is in line with the critique by Al-Yahyaee et al. (2020) that static optimizers overfit to historical noise in non-stationary environments.

Overall, the Russian case illustrates that tail-sensitive, adaptive ML portfolios (particularly ANNt_MKW) can exploit transient inefficiencies more effectively than both multifractal and traditional approaches, though only with proper risk controls and horizon awareness. These observations support the principle of bounded rationality (Simon, 1955), where data-driven models enable investors to navigate in incomplete and ambiguous market signals (such as under institutional and macroeconomic uncertainty). However, long-term investors may still find the stability of Markowitz's variance minimization preferable, particularly in light of exogenous shocks beyond model anticipation. Therefore, our findings suggest that hybrid ML portfolios, such as ANNt_MKW, may serve as overlays, while traditional frameworks offer a conservative core, echoing Zhang and Du (2025), who advocate for integration of AI tools in portfolio design dependent on context.

[17] In a war-economy environment characterized by sanctions, state intervention, abrupt regime shifts, and elevated uncertainty, the ANNt framework can be adapted to improve robustness in three ways. First, the degrees-of-freedom parameter (v) of the Student's t distribution may be treated as time-varying; during periods of severe market stress, calibrating v toward lower values (e.g., 3-4) allows the model to accommodate extreme observations and heavier tails without disproportionately inflating overall portfolio risk estimates. Second, the conventional mean squared error (MSE) objective may be replaced by a loss function based on the Student's t log-likelihood. This approach reduces the influence of extreme and potentially transient shocks, thereby limiting the impact of short-lived market dislocations on parameter estimation. Third, the optimization window can be shortened through a rolling-window framework, allowing the model to place greater weight on recent information and to adapt more rapidly when the return-generating process changes due to geopolitical events or policy interventions. These modifications are particularly relevant for the Russian market, where wartime conditions may weaken the predictive value of parameters estimated under normal economic regimes and require more frequent recalibration of model assumptions (Briegel & Tresp, 1999; Fernandez, 1999).

India

The Indian equity market presents a combination of strong domestic investor participation, moderate regulatory efficiency, and recurring behavioral anomalies (Boțoc and Pirtea, 2014; Arshad et al., 2016). Figures 3c and 4c highlight the improved results of the MF_SHARPE portfolio, which records the highest average and annualized returns in multiple rebalancing periods. In cumulative returns (Figure 5c), it consistently outperforms other portfolios. These results echo prior work by Tiwari et al. (2019), who found strong multifractality and long-memory features in Indian stock returns, suggesting that MF-DFA methods are well-positioned to exploit these inefficiencies. This performance further confirms Maciel's (2021) findings from the Brazilian market and extends them to India, where return persistence appears similarly actionable. The result also lends empirical support to the notion advanced by Yang and Liu (2023) that multifractal models embedded in portfolio selection may outperform standard parametric frameworks in growing, volatile markets.

On the other hand, MF_SHARPE's risk profile is less favorable. Figures 12c and 13c show that this portfolio consistently carries the highest VaR and CVaR values, with peaks above 6% and 9%, respectively. Similarly, volatility in Figure 11c often exceeds 60%, which underscores the downside exposure associated with asset selection driven by momentum. These patterns align with the caution noted by Oliveira et al. (2023) that MF-DFA portfolios, though statistically appealing, can be vulnerable to severe drawdowns in turbulent periods without volatility controls. ANNt_SHARPE emerges as the strongest among the neural-network portfolios, delivering positive returns in 19 out of 20 quarters (Figure 3c) and consistently high Sharpe and Sortino ratios (Figures 7c and 8c). It rejects our second hypothesis about the lack of added value of accounting for asymmetries in the emerging markets. Furthermore, Figure 6c confirms this portfolio's efficiency through Jensen's Alpha, with ANNt_SHARPE ranking second only to MF_SHARPE. However, its volatility and CVaR remain substantial, suggesting that ML return enhancement in India comes with considerable exposure to downside tails, reiterating the findings of Rachev et al. (2010) regarding the trade-off between predictive accuracy and risk concentration in non-Gaussian settings.

Markowitz portfolios again provide a risk-controlled benchmark. As seen in Figures 11c and 12c, they consistently deliver the lowest volatility and VaR levels, confirming their usefulness for long-term institutional investors. While their average return lags behind the ML portfolios, their Sharpe ratio (Figure 7c) is competitive, especially during periods of macroeconomic stability. These results are congruent with the risk-constrained optimization literature (Emmanuel et al., 2025), which advocates for low-variance allocations in politically or economically uncertain contexts, despite the seemingly attractive return potential elsewhere. Treynor ratios in Figure 9c suggest that MF_SHARPE leverages systemic risk more aggressively, often reflecting market betas above 1.5 (Figure 10c). ANNt_SHARPE and ANNt_MKW maintain more moderate betas, which suggests better risk-adjusted performance under CAPM assumptions.

The analysis suggests that the Indian market offers perhaps the strongest validation of ML techniques in our sample. Both MF_SHARPE and ANNt_SHARPE outperform traditional strategies across most metrics, rejecting Hypothesis 1 of this study. However, their higher volatility and tail risk caution against unrestricted deployment, especially in periods of regime shift or macro shock. The observation is echoed by Gunjan and Bhattacharyya (2023), who emphasize the need for filtered ML integration, where hybrid strategies selectively combine alpha extraction with robust volatility control based on signal. In this regard, the ANNt_MKW portfolio performs notably well as a middle ground: it substitutes returns for improved Sortino and Treynor ratios (Figures 8c and 9c), while maintaining controlled downside exposure. In sum, while India's semi-efficient market structure rewards ML strategies, these must be deployed with strong attention to risk asymmetries. MF-DFA methods (especially MF_SHARPE) are optimal for return maximization but require volatility protection. ANNt_SHARPE and ANNt_MKW offer more balanced profiles, capable of delivering both alpha and risk control. Additionally, classical models remain relevant for conservative allocations but appear less responsive to return-generating anomalies.

China

Among the BRICS economies, China is characterized by a financial system managed by the state, heavy regulatory oversight, and episodic market interventions (Pan & Qian, 2025; Yin & Zhang, 2025). As for returns, Figure 3d demonstrates that the Markowitz portfolio generates the highest average returns in 13 of 20 rebalancing periods. It outpaces both multifractal and neural network alternatives. Further, the annualized return patterns in Figure 4d reinforce this finding, especially over multi-quarter horizons. These results resonate with prior studies by Peng et al. (2024), who argue that China's financial market microstructure, marked by capital controls and investor behavior controlled by the state, limits the emergence of persistent anomalies that ML methods typically exploit.

Interestingly, cumulative return curves (Figure 5d) validate the Markowitz portfolio as the leader over the longer run. Its consistency may reflect that price dynamics in the SSE50 are more driven by policy than sentiment or memory, reducing the relative advantage of models like MF-DFA, which assume return persistence (Yang and Liu 2023). In Figures 11d through 13d, we examine the risk metrics. These figures confirm the defensive profile of the Markowitz strategy: it exhibits the lowest volatility in 16 out of 20 quarters and consistently outperforms other portfolios on VaR and CVaR metrics. It is in line with the findings of Pekanov Starcevic et al. (2019) and Hirano et al. (2020), who emphasize that allocations minimizing variance remain optimal when markets are subject to regulation and low transparency. In contrast, MF_SHARPE, though effective in markets like India and Brazil, underperforms here, frequently yielding the worst CVaR and VaR figures and producing multiple negative return intervals. It suggests that multifractal asset selection based on signals may lack predictive power in environments where price distortions are driven by exogenous state intervention rather than endogenous memory patterns. Among the neural-network configurations, ANNt_MKW demonstrates balanced performance. It ranks second in average return (Figure 3d), while maintaining relatively low CVaR and volatility. Figures 6d, 8d, and 9d also highlight solid performance in Jensen's Alpha, Sortino, and Treynor ratios, respectively. These results take a step further from Oliveira et al. (2023), who found that ML models accounting for asymmetries can contribute to downside protection. However, the authors accounted only for developed markets. Nevertheless, as seen in Figures 7d and 10d, ANNt portfolios show inconsistent Sharpe ratios and fluctuating beta, revealing limited robustness when exposed to China's regime-driven risk factors.

Altogether, our findings suggest that China's market conditions constrain the comparative advantage of complex ML approaches. Classical mean–variance optimization, grounded in stable covariance structures, appears to align more closely with the underlying mechanics of the SSE50. It contradicts results from Brazil and India, where ML portfolios thrived due to the abundance of long-memory signals and behavioral anomalies (as previously confirmed by Maciel (2021)). However, ANNt_MKW provides a promising tactical option for short to medium-term horizons. It captures upward momentum during less restrictive quarters (e.g., early 2020, late 2023), yet avoids the worst drawdowns seen in MF models. This flexible risk-responsiveness matches the framework proposed by Jain (2025), where synthetic convex optimization and adaptive ML are embedded into regulatory-aware portfolio construction. In sum, China's capital markets validate the relevance of classical finance: the Markowitz model demonstrates durability across both return and risk metrics. ML approaches, especially those relying on long-range dependencies or volatility clustering, face headwinds in this policy-influenced environment. Still, the selective application of models like ANNt_MKW can improve responsiveness when volatility regimes shift.

South Africa

The South African equity market (represented in our analysis by the FTSE/JSE Top 40 index) exhibits a hybrid profile. It combines traits of both developed and emerging economies: relatively deep financial markets and strong institutional frameworks on one hand, but also periodic shocks from commodity price cycles, currency volatility, and domestic policy risk on the other (Chen et al., 2025).

Figures 3e and 4e bring us the results for return analysis. These figures reveal that the MF_SHARPE and ANNt_MKW portfolios alternate leadership in terms of average and annualized returns. The MF_SHARPE configuration achieves the highest return in the sample but leads in only six quarters. Meanwhile, ANNt_MKW outperforms in a comparable number of intervals but exhibits greater short-term stability. The pattern resembles the Indian market, where machine-learning models also outperformed on multiple fronts, though with higher volatility. In contrast to China, where the Markowitz model led across most return metrics, South Africa offers more room for tactical ML deployment. The cumulative return patterns in Figure 5e further support this interpretation. While MF_SHARPE delivers sharp upside potential, ANNt_MKW achieves more consistent accumulation of gains, particularly across 2022 to 2024, a period marked by domestic political and energy sector instability. These results echo the conclusions of Selvam et al. (2025), who highlighted the suitability of neural network adaptive models in semi-efficient markets linked by resources such as South Africa.

On the risk side, Figures 11e (volatility), 12e (Value at Risk), and 13e (Conditional Value at Risk) show that the Markowitz portfolio remains the least volatile, and consistently exhibits the lowest Value at Risk (VaR) and Conditional Value at Risk (CVaR). Its performance provides a strong argument for maintaining a risk-averse core allocation, as was also observed in the Chinese and Russian markets. However, ANNt_MKW manages to reduce volatility relative to other ML variants and also posts lower tail-risk exposures in multiple quarters. It contradicts our hypothesis about no added value of accounting for tails in emerging markets (Hypothesis 2). Figures from 6e to 9e show that ANNt_MKW often outpaces other strategies in Jensen's Alpha, Sortino, and Treynor ratios, confirming its superior risk-reward efficiency in turbulent windows. Its Sharpe ratio performance (Figure 7e) is also strong, especially when viewed alongside the underlying volatility dynamics. These results are consistent with Naik et al. (2025), who emphasize that the integration of probabilistic scoring functions into ML architectures enhances their robustness in real-world applications, provided that rebalancing frequencies and selection criteria are calibrated to market rhythms.

By contrast, MF portfolios perform inconsistently. MF_EQ and MF_MKW exhibit relatively poor return stability and high tail risk, while MF_SHARPE, despite its standout quarters, also incurs large drawdowns. The pattern is reminiscent of the critique by Zhang et al. (2023), who note that multifractal models, though theoretically elegant, can be overly sensitive to noise in periods of low autocorrelation or regime instability. It is in contrast to their stronger performance in Brazil and India, where market memory and inefficiency were more pronounced. The beta dynamics (Figure 10e) show that ANNt_MKW consistently avoids overexposure to systemic shocks, maintaining unlevered profiles in every quarter. The characteristic is especially relevant in South Africa, where market turbulence often originates from exogenous sources, such as global commodity shocks or rand volatility, rather than firm-level signals. It contrasts with Brazil and India, where endogenous volatility and autocorrelated returns created more favorable environments for high-beta strategies like MF_SHARPE. Additionally, it confirms prior findings from Ibikunle et al. (2024) that ML strategies can exhibit adaptive hedging by shifting weight away from highly exposed sectors during turbulent times.

Overall, the South African case favors a diversified implementation of portfolio strategies. Classical models like Markowitz retain their appeal for defensive mandates, yet machine learning approaches, particularly ANNt_MKW, offer meaningful incremental gains when applied selectively. These findings resonate with the bounded rationality argument posed by Simon (1955), wherein algorithmic agents augment investor decisions under limited foresight and data fragmentation. Additionally, the hybrid nature of the South African market supports the broader proposition by Cajas (2025) that core strategies based on blending rules with adaptive ML overlays may be the most prudent path in transitional economies. To conclude, machine learning models in

South Africa deliver credible performance, but their benefits hinge on precise tailoring and risk oversight. Among the tested strategies, ANNt_MKW stands out as the most practically viable ML option, offering a favorable compromise between return-seeking and risk control. Meanwhile, the Markowitz model remains a valuable anchor, especially for institutional investors prioritizing downside protection over tactical alpha. When viewed in context with the other BRICS markets, South Africa illustrates that success with ML portfolios is conditional, not just on model design, but on the regime characteristics and structural idiosyncrasies of each market.

Moreover, Table 11 shows that incorporating transaction costs does not materially reduce the attractiveness of the proposed strategies, nor does it change the relative ranking of portfolio performance.

Benchmarking against the U.S. Market: Lessons from Developed Markets

To evaluate the comparison of our findings to developed markets, we replicate our portfolio experiments in the U.S. equity market (S&P 500 index). Figures 3f to 13f indicate that the S&P 500 portfolio delivers the highest cumulative and annualized returns across most rebalancing intervals (Figures 4f, 5f), outperforming both machine learning and traditional optimization strategies. The result contrasts with BRICS markets, where passive benchmarks (e.g., IBRX50, IMOEX) were consistently outperformed by traditional (Markowitz or Sharpe) or machine learning approaches. The outperformance of the S&P 500 portfolio reflects the hallmark features of developed markets, such as deep liquidity, strong market efficiency, and transparent price discovery mechanisms (Ali et al., 2018).

Moreover, the Markowitz portfolio in the U.S. maintains the lowest levels of volatility, Value at Risk (VaR), and Conditional Value at Risk (CVaR) (Figures 11f–13f). These findings echo similar outcomes in the Chinese market, where centralized monetary authority and regulatory control create quasi-developed conditions (Peng et al., 2024). In both environments, the stability of variance-covariance structures makes the mean–variance optimization approach more effective. The result is consistent with literature emphasizing the robustness of Markowitz allocation under conditions of low structural asymmetry and stationarity (Christoffersen, 1998; Zhang and Du, 2025). In contrast, the same Markowitz strategy was suboptimal in volatile BRICS contexts such as Brazil and India, where persistent inefficiencies and regime shifts impair its assumptions. In these markets, machine learning based approaches such as MF-DFA and ANNt provided superior risk-adjusted returns, aligning with prior research highlighting their effectiveness in structurally unstable environments (Tiwari et al., 2019; Acikgoz, 2025; Maciel, 2025). For instance, the MF_SHARPE portfolio, while highly effective in Brazil and India, delivered only discontinuous returns in the U.S., accompanied by elevated volatility and poor tail-risk metrics (Figures 11f–13f). It suggests that the multifractal memory effects leveraged in emerging markets are arbitraged in efficient developed markets.

Among the machine learning strategies, ANNt_MKW offered the most stable results in the U.S., with moderate Jensen's Alpha (Figure 6f) and competitive Sortino and Treynor ratios (Figures 8f and 9f). However, its relative advantage was smaller than in Russia or South Africa, where the same portfolio led in cumulative and risk-adjusted returns. It reinforces the conclusion from recent literature that neural networks with asymmetric tail modeling deliver their strongest performance in non-Gaussian, fat-tailed regimes (Oliveira et al., 2023; Mohammed, 2024). Indeed, in stable markets such as the U.S., models relying on long memory (MF-DFA) or tail adaptive scoring (ANNt) offer diminishing returns as price signals become more efficient and dominated by noise (Zhong and Enke, 2019; Ahmed et al., 2022).

The empirical divergence between U.S. and BRICS results contradicts our third hypothesis: there are no significant differences in the performance of machine learning-based portfolio strategies across markets. In efficient markets, traditional models perform sufficiently well, offering robust returns with minimal risk. However, in emerging markets (marked by regulatory unpredictability, liquidity constraints, and behavioral inefficiencies), ML strategies provide additional gains by adapting to evolving return distributions and capturing transient pricing anomalies. Rather than assuming full information and rational processing, these results illustrate how adaptive machine learning frameworks can navigate uncertainty and noise more effectively, consistent with Simon's (1955) view that real-world decision making requires heuristics tailored to environmental complexity. Consequently, investors and asset allocators may benefit from a dual approach. In developed markets, passive indexing or mean-variance optimization remains appropriate. In contrast, for BRICS markets and similar semi-efficient environments, deploying ML strategies (especially those incorporating asymmetry and memory) can significantly enhance portfolio performance. It echoes broader asset allocation frameworks that advocate tailoring optimization techniques to the informational structure and volatility regime of each market (Cajas, 2025; Vuong et al., 2025).

4.2 Static risk-return assessment of machine learning approaches

The static risk-return evaluation summarized in Tables 4 and 5 provides a broader lens through which to assess the long-term performance of traditional and machine learning based portfolio strategies across structurally heterogeneous markets. While dynamic assessments capture regime shifts and rebalancing effects, the static perspective helps clarify whether certain methods exhibit stable advantages independent of horizon or timing. It also enables us to examine the three hypotheses of this study with a more cross-sectional lens.

In Brazil, the MF_SHARPE strategy dominates metrics accounting for returns, achieving the highest average daily return (0.10%) and annualized performance (22.00%), ahead of both classical and other ML strategies. The result is broadly consistent with earlier studies suggesting strong memory effects and low market efficiency in Brazil (Maciel, 2021). However, it is accompanied by significant volatility (35.60%) and downside risk (CVaR above 4.50%), which brings into question the robustness of the strategy in real-world applications with capital constraints. While the gains may stem from meaningful signal exploitation, the possibility of return autocorrelation being mistaken for persistent alpha cannot be ruled out. It aligns with concerns voiced by Christoffersen (1998) and Rachev et al. (2010), who caution against interpreting statistical persistence as a stable investment opportunity. In the same market, the Markowitz minimum-variance portfolio demonstrates a conservative risk profile, with the lowest CVaR (1.79%) and volatility (14.00%), though with a lower return (6.80%). It is consistent with the long-standing view that mean–variance optimization is appropriate for risk-managed mandates in volatile markets, particularly when transaction costs or rebalancing constraints are present (Treynor, 1965; Sarfarazurrehman et al., 2025). Meanwhile, ANNt_MKW performs moderately well on both return and risk, offering a compromise profile. However, the variation across ANNt variants is substantial, and the weakest (ANNt_EQ) underperforms not just on return, but also on risk, reinforcing the argument that model architecture and calibration strongly influence ML outcomes (in agreement with Mohammed (2024)).

In Russia, where political risk and policy opacity frequently alter asset pricing dynamics (Alfaro and Mendoza, 2020), ANNt_MKW shows stable outperformance in static terms (an annualized return of 10.60%), a relatively low beta, and the highest Jensen's Alpha among all tested portfolios. These results contradict Hypotheses 1 and 2 about no added value of asymmetries and ML approaches, particularly in the context of the difficulty of constructing consistent alpha in geopolitically exposed markets. Still, the Markowitz strategy remains competitive in terms of volatility and tail-risk control. The underperformance of MF_SHARPE (−8.05% annualized return) suggests that multifractal methods may be less effective in environments where shocks are exogenous and discontinuous (as argued in Gopal et al. (2025)).

Furthermore, India emerges as the most favorable environment for ML strategies. MF_SHARPE achieves the highest returns overall (27.34% p.a.), while ANNt_SHARPE ranks close behind in both return and alpha. These results reflect earlier empirical observations of structural inefficiency and long-memory effects (Campbell & Thompson, 2008), but also suggest that fat-tail modeling offers incremental value, particularly in controlling downside risk. However, both strategies remain highly volatile (especially MF_SHARPE), with volatility exceeding 46%, which limits their appeal for institutional investors with fixed risk budgets. The Markowitz strategy again demonstrates risk efficiency, recording the lowest volatility (11.80%) among all BRICS portfolios. Thus, while ML approaches confirm their potential, their benefits appear conditional on market conditions and risk appetite, as previously discussed by Burki et al. (2025).

In China, the traditional models outperform machine learning approaches. The Markowitz portfolio records the best results across nearly all metrics, including return (29.60% p.a.), alpha (0.09), and both VaR and CVaR (see Tables 4 and 5). These outcomes are in line with previous research identifying the Chinese market structure as heavily influenced by highly regulatory policy measures and limited price discovery mechanisms (Peng et al., 2024). ML strategies relying on data-driven inference may be constrained in such contexts, particularly when persistent patterns are interrupted by regulatory intervention. ANNt_MKW still achieves second-best performance in return and maintains moderate risk, suggesting limited but non-trivial value for tactical deployment. On the contrary, MF_SHARPE performs poorly (−13.80% annualized return), indicating that multifractal signals are largely ineffective under strong regulatory regimes.

In this case, South Africa presents a transitional market combining elements of both developed and emerging structures. As the results from Tables 4 and 5 indicate, the market yields more balanced results. ANNt_MKW emerges as the outperformer with the highest alpha (0.02) and Sortino ratio (0.49). Its volatility remains relatively contained (14.90%), confirming that machine learning approaches may work in semi-efficient environments when carefully specified. In contrast, MF_SHARPE exhibits both high variance and weak consistency, echoing critiques that multifractal models may overfit to transient patterns in moderately noisy environments (Zhang et al., 2023). Markowitz's portfolio again confirms robustness in risk control, though at the cost of lower returns. It suggests that a hybrid allocation framework may be most appropriate in such contexts, as proposed by Selvam et al. (2025), where ML overlays are used on top of a core that minimizes variance.

Additionally, the U.S. market presents a useful benchmark to contrast these findings (as a representative of developed markets with a deeper and more efficient market structure). The S&P 500 index outperforms all constructed strategies in return terms (15.20% p.a.), while the Markowitz portfolio achieves the lowest volatility (11.50%). ML strategies, including MF_SHARPE and ANNt variants, perform inconsistently and, in some cases, negatively (ANNt_SHARPE: −7.70%). It supports prior literature indicating that in environments characterized by high liquidity and efficient pricing, machine learning strategies are less effective (Ali et al., 2018; Ahmed et al., 2022). Notably, ANNt_MKW shows moderate risk-adjusted performance but offers little incremental benefit over traditional methods.

Taken together, the static risk to return comparison confirms and evaluates findings from subsection 4.1 that machine learning–based portfolio strategies yield superior results primarily in markets with structural inefficiencies, strong return autocorrelation, or behavioral biases (conditions most visible in Brazil, India, and to a lesser extent, South Africa). In contrast, in markets with stronger institutions or centralized control, such as China, traditional frameworks remain dominant. It rejects the general logic of Hypothesis 3 about no context dependency and emphasizes the importance of tailoring portfolio strategies to local market structure. Moreover, while both MF-DFA and ANNt models show potential, their effectiveness is neither universal nor unconditional, and remains sensitive to calibration, time horizon, and market regime.

4.3 Robustness analysis from the investment horizon and domestic currency perspectives

To evaluate whether the observed performance patterns are robust to real-world portfolio implementation, we assess how the ratios and cumulative returns of each strategy vary under different rebalancing frequencies (Table 4). It serves as a robustness check on the hypotheses, particularly whether there exist benefits of machine learning models (Hypothesis 1) and incorporating asymmetries (Hypothesis 2) across horizons, and whether their effectiveness is dependent on structural features of each market (Hypothesis 3). Statistically, Tables 8 and 9 show that the main conclusions reported throughout the analysis are supported by paired mean-difference tests based on cumulative returns and annualized volatility. In most markets, the ANNt and MF portfolios exhibit statistically significant differences relative to traditional portfolio strategies over the first half of the 2020s. Table 10 further confirms the presence of skewness, kurtosis, and non-normality in abnormal return series, supporting the use of tail-sensitive modelling. The Appendix extends the analysis to 2025 and confirms the statistical significance of the ANNt_SHARPE portfolio relative to the traditional SHARPE portfolio in terms of cumulative returns. To address market frictions and elevated transaction costs, we additionally penalize cumulative portfolio returns at each rebalancing date, providing a robustness assessment of whether the economic gains persist after implementation costs are considered (Tables 11 and 12).

In Brazil, ML strategies, especially MF_SHARPE, consistently outperform under short rebalancing intervals, but their returns decline markedly with lower frequency (Table 4). Such frequency sensitivity reflects the short-lived predictability and high return autocorrelation reported in Brazilian equity markets (see Section 4.1), which makes frequent adjustment critical. Similar volatility clustering has been observed in emerging markets with speculative dynamics and weak institutional anchoring (Bekaert and Harvey, 1995). The ANNt_MKW portfolio performs more steadily across frequencies, which suggests better adaptability to structural noise, a feature theoretically supported by models accounting for asymmetric return distributions (Rachev et al., 2010). In Russia, ANNt_MKW again emerges as the most robust ML strategy, with stable Sharpe ratios across all frequencies and a cumulative return exceeding 26%. It reflects its ability to capture extreme movements without overreacting to transient noise. The weakness of MF_SHARPE in Russia supports findings from Albrecht and Kočenda (2024), who emphasize the dominance of exogenous geopolitical shocks over internal shock persistence in emerging markets. In such environments, model robustness is less about granularity and more about managing discontinuities, favoring approaches based on fat-tailed error structures. Additionally, India stands out as the most transparent case of ML robustness. MF_SHARPE and ANNt_SHARPE sustain high Sharpe ratios across all rebalancing frequencies and produce the highest cumulative returns in the sample (see Table 4). These results are consistent with past evidence of long memory and herding behavior in Indian markets (Saldanha et al., 2025), which creates stable inefficiencies exploitable across multiple horizons. The stability of the ML strategies suggests that the local process generating returns is persistent and nonlinear, a premise aligned with the bounded rationality framework (Simon, 1955).

China, in contrast, confirms the dominance of traditional models, as previously outlined in Sections 4.1 and 4.2. The Markowitz portfolio delivers the highest Sharpe ratios among rebalancing frequencies, and ML strategies (particularly MF_SHARPE) fail to outperform. These results are consistent with previous research (Cheng et al., 2022), emphasizing the centralized role of state-owned enterprises and policy signals in price formation. The lack of exploitable patterns challenges the core assumption of data-driven learning: that historical signals reflect future dynamics. When market movements are driven more by administrative interventions than by investor behavior, machine learning methods tend to overfit noise or lag behind sudden shifts. South Africa presents a hybrid case. ANNt_MKW performs robustly across frequencies, with a relatively stable Sharpe ratio and moderate cumulative gains. It suggests partial market inefficiencies that are more persistent than in China or the U.S., but less exploitable than in India or Brazil. The volatility of MF_SHARPE under longer horizons points to instability in signal structure, consistent with transitional economies where capital flow regimes, investor base, and regulation fluctuate over time (Wang et al., 2020). Traditional strategies remain relevant but often underperform in terms of returns.

Last but not least, in the U.S. market, the results reinforce the consensus that machine learning strategies offer little incremental value in more efficient systems. Both MF_SHARPE and ANNt_SHARPE display poorer performance across rebalancing frequencies, often underperforming even passive benchmarks. The S&P 500 and Markowitz portfolios deliver consistently high Sharpe ratios and cumulative returns. These results are in line with studies showing that in efficient markets with strong arbitrage mechanisms, persistent return patterns are rare and easily arbitraged away (Fama, 1970; Ali et al., 2018). Moreover, higher model complexity may lead to increased overfitting and transaction costs without clear performance gains (Ahmed et al., 2022). Taken together, the rebalancing analysis confirms that the benefits of machine learning in portfolio construction are not uniform. In markets like India and Brazil, ML strategies can maintain their edge across frequencies, suggesting structural inefficiencies and stable nonlinear dynamics. In Russia and South Africa, their robustness depends on model design and market regime. In China and the U.S., classical strategies remain dominant, with machine learning methods offering little value under realistic rebalancing constraints. These findings contribute to a more nuanced view of ML portfolio optimization, which shows that robustness is highly context dependent and that no single strategy is optimal across markets or frequencies.

4.4 Implications for investors and policymakers

The empirical findings from our analyses in Sections 4.1 to 4.3 offer clear guidance for both portfolio managers and regulators in BRICS markets. From an investment management standpoint, the superior long-term performance of the MF-DFA–Sharpe strategy in Brazil and India directly reflects the multifractal inefficiency ranking demonstrated by Tiwari et al. (2019) in emerging market contexts. By incorporating the Hurst exponent Market Deficiency Measure into their security selection, practitioners can isolate the 15–20 most promising equities for an "alpha sleeve," while still preserving a core allocation to the classical minimum-variance portfolio. This hybrid allocation balances enhanced return potential with the downside protection that institutional investors expect.

Our results also underscore the tactical value of ANNt–minimum-variance configurations (see Table 6). Oliveira et al. (2023) have shown that embedding a Student's *t*–based probability ranking within neural-network outputs better captures extreme-return behavior. In line with Gunjan and Bhattacharyya's (2023) findings for different markets, the ANNt–MKW variant in Russia and South Africa excelled at exploiting short-lived opportunities for specific regimes. However, it suggested that investors apply stringent risk filters, such as volume and volatility thresholds (as recommended by Zhang et al. (2023)), and maintain disciplined stop-loss rules to guard against sudden drawdowns. Conversely, in China's tightly regulated equity market (characterized by significant state participation), the classical Markowitz minimum variance model remains the most reliable tool for tail-risk control. Peng et al. (2024) attribute this dominance to lower informational asymmetries and policy-driven trading constraints that diminish the marginal benefits of aggressive ML allocations. Accordingly, asset managers should dynamically adjust their ML exposure, favoring higher weights in minimum-variance mandates when VaR and CVaR targets become binding.

At the policy level, our evidence points to the urgent need for BRICS regulators and central banks to modernize market infrastructure. Authorities should grant qualified participants access to high-frequency trade and order book data, thereby enabling robust backtests of both MF-DFA and ANNt algorithms under realistic conditions. In addition, establishing minimum standards for risk management (such as the stress scenario validation protocols proposed by Vuong et al. (2025)) would help mitigate systemic risks associated with algorithmic herding. Finally, mandating transparent disclosure of ML portfolio performance (including tail-risk contributions and drawdown statistics) would strengthen market confidence and foster a level playing field for all investors.

Taken together, these investor-focused tactics and policy reforms can improve both portfolio returns and market resilience across BRICS equity markets. By adopting the multifractal and asymmetric ANN approaches validated here and by ensuring a supportive regulatory environment, stakeholders can better navigate the complexities and volatilities inherent to emerging-market investing.

Conclusion

Our study provides novel empirical evidence on the effectiveness of machine learning techniques in portfolio optimization across emerging markets, with a particular focus on the BRICS countries. By comparing traditional models (Markowitz and Sharpe) with machine learning approaches such as multifractal detrended fluctuation analysis (MF-DFA) and artificial neural networks with asymmetric probability ranking (ANNt), we demonstrate the superior performance of machine learning strategies under conditions of market inefficiency and heightened volatility.

Our findings reveal that MF_SHARPE and ANNt_MKW portfolios consistently outperform traditional benchmarks in several BRICS markets in both dynamic and static frameworks. These machine learning portfolios offer not only higher returns but also enhanced risk-adjusted metrics, particularly in regions characterized by structural inefficiencies and regulatory asymmetries. While the Markowitz portfolio remains robust in less volatile environments (most notably in China and the U.S.), its effectiveness diminishes in more turbulent markets, where adaptive strategies prove more valuable. Notably, this study is the first to apply asymmetric neural network architectures to a comprehensive sample of emerging equity markets, empirically demonstrating their effectiveness beyond the institutional and informational structures of developed financial systems. By integrating cumulative sum control charts and currency-adjusted performance evaluations, we provide a practical supporting framework for investors managing diversified portfolios across heterogeneous market environments. Together, these innovations extend the methodological frontier of portfolio optimization by linking asymmetric tail learning and multifractal diagnostics to real-world investment applications in the BRICS context.

Overall, the study highlights the relevance of incorporating machine learning into global asset allocation frameworks. For investors, adopting machine learning enhanced strategies (such as MF-DFA–Sharpe in volatile environments and ANNt frameworks with built-in controls for downside risk) can materially boost risk-adjusted returns. Embedding these machine learning models into existing allocation processes, supported by real-time data analytics and rigorous stress-testing, enables portfolio managers to uncover hidden opportunities, dynamically rebalance exposures, and navigate shifting market regimes more effectively. At the policy level, our evidence underscores the need for regulators and market infrastructure bodies in BRICS to facilitate wider adoption of machine learning portfolio tools. Specifically, enhanced data transparency requirements (e.g., more granular trade and order-book disclosures) and sandboxes for fintech innovation would lower barriers for neural network and multifractal engines to access high-quality inputs. Moreover, central banks and securities commissions should consider guidelines on model risk management for machine learning based strategies, ensuring appropriate backtesting standards and stress scenario validation in highly volatile regimes.

Notwithstanding these insights, our analysis has several limitations. First, we have abstracted from liquidity constraints and market impact; incorporating realistic trading frictions could temper some of the out-of-sample gains. Second, the quarterly rebalancing cadence, while common in institutional practice, may understate the potential of higher-frequency updates in volatile periods. Third, our focus on 700 liquid stocks within major BRICS indices may not capture the full breadth of small-cap and frontier segments, where inefficiencies can be even more pronounced. Future studies should therefore extend this framework by embedding dynamic rebalancing rules, potentially leveraging reinforcement learning agents for real-time adjustment. Integrating macro indicators (e.g., interest rate differentials, political risk indices) as inputs could also improve robustness during regime shifts.

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List of abbreviations

ANNs	Artificial neural networks
ANNt	Artificial neural networks with *t* Student's Distribution
ANNt_EQ	ANN portfolio with the highest Student's *t* probability and equal weights
ANNt_Mkw	ANN portfolio with the highest Student's *t* probability and minimum variance weights
ANNt_SHARPE	ANN portfolio with the highest Student's *t* probability and maximum Sharpe weights
BRICS	Brazil, Russia, India, China, and South Africa
BRL	Brazilian Real
CAPM	Capital Asset Price Model
CNY	Chinese Yuan Renminbi
Cvar	Conditional Value at Risk
FTSE/JSE40	South African market proxy of the Johannesburg Stock Exchange
IBRX50	Brazilian market proxy of the B[3] Stock Exchange
INR	Indian Rupee
IMF	International Monetary Fund
IMOEX	Russian market proxy of the Moscow Stock Exchange
MARKOWITZ	Minimum variance portfolio (Markowitz) applied to all assets in the sample
MDM	Market Deficiency Measure
MF	Multifractal trend fluctuation analysis abbreviation
MF-DFA	Multifractal trend fluctuation analysis
MF_EQ	Multifractal Portfolio with the lowest MDM and equal weights
MF_MKW	Multifractal Portfolio with the lowest MDM and minimum variance weights
MF_SHARPE	Multifractal Portfolio with the lowest MDM and maximum Sharpe weights
NIFTY_50	Indian market proxy of the National Stock Exchange
PPP	Purchasing power parity
RUB	Russian Ruble
SHARPE	The maximum Sharpe portfolio is applied to all assets in the
S&P500	sample
SSE50	US market proxy of the New York Stock Exchange (SP500)
Var	Chinese market proxy of the Shanghai Stock Exchange
USD	Value at Risk
ZAR	United States Dollar
	South African Rand

References

Abir, S. I., Sarwer, M. H., Hasan, M., Sultana, N., Dolon, M. S. A., Arefeen, S. M. S., Shimanto, A. H., Zakaria, R. M., Shiam, S. A. A., Shoha, S., & Saha, T. R. (2025). Deep Learning for Financial Markets: A Case-Based Analysis of BRICS Nations in the Era of Intelligent Forecasting. *Journal of Economics, Finance and Accounting Studies*, *7*(1), 01–15. https://doi.org/10.32996/jefas.2025.7.1.1

Acikgoz, T. (2025). The Multifractal Nature of Cross-Correlations between Emerging Market Equities and Financial Assets: An Econophysics Perspective. *Computational Economics*. https://doi.org/10.1007/s10614-025-10991-4

Ahmed, S., Alshater, M. M., Ammari, A. E., & Hammami, H. (2022). Artificial intelligence and machine learning in finance: A bibliometric review. *Research in International Business and Finance*, *61*, 101646. https://doi.org/10.1016/j.ribaf.2022.101646

Alfaro, L., & Mendoza, E. (2020). Current Policy Challenges Facing Emerging Markets. *IMF Economic Review*, *68*(3), 453–455. https://doi.org/10.1057/s41308-020-00119-z

Ali, S., Shahzad, S. J. H., Raza, N., & Al-Yahyaee, K. H. (2018). Stock market efficiency: A comparative analysis of Islamic and conventional stock markets. *Physica A: Statistical Mechanics and Its Applications*, *503*, 139–153. https://doi.org/10.1016/j.physa.2018.02.169

Al-Yahyaee, K. H., Mensi, W., Ko, H.-U., Yoon, S.-M., & Kang, S. H. (2020). Why cryptocurrency markets are inefficient: The impact of liquidity and volatility. *The North American Journal of Economics and Finance*, *52*, 101168. https://doi.org/10.1016/j.najef.2020.101168

Anwar, A., & Iwasaki, I. (2022). Institutions and FDI from BRICS countries: A meta-analytic review. *Empirical Economics, 63*, (63), 417–468. https://doi.org/https://doi.org/10.1007/s00181-021-02145-w

Arshad, S., Rizvi, S. A. R., Ghani, G. M., & Duasa, J. (2016). Investigating stock market efficiency: A look at OIC member countries. *Research in International Business and Finance*, *36*, 402–413. https://doi.org/10.1016/j.ribaf.2015.09.026

Bachelier, L. (1900). Théorie de la spéculation. *Annales Scientifiques de l'École Normale Supérieure*, *17*, 21–86. https://doi.org/10.24033/asens.476

Bağcı, M., & Soylu, P. K. (2024). Optimal portfolio selection with volatility information for a high frequency rebalancing algorithm | Financial Innovation | Full Text. *Financial Innovation*, *10*(1). https://doi.org/https://doi.org/10.1186/s40854-023-00590-3

Baruník, J., & Křehlík, T. (2018). Measuring the Frequency Dynamics of Financial Connectedness and Systemic Risk. *The Journal of Financial Econometrics, 16*(2), 271–296.

Benbachir, S. (2025). Exploring multifractality in African stock markets: A multifractal detrended fluctuation analysis approach. *Investment Management & Financial Innovations*, *22*(1), 35–51. https://doi.org/10.21511/imfi.22(1).2025.04

Boţoc, C., & Pirtea, M. (2014). Dividend Payout-Policy Drivers: Evidence from Emerging Countries. *Emerging Markets Finance and Trade, 50*(S4), 95–112.

Briegel, T., & Tresp, V. (1999). Robust Neural Network Regression for Offline and Online Learning. *Advances in Neural Information Processing Systems*, *12*. https://papers.nips.cc/paper_files/paper/1999/hash/db29450c3f5e97f97846693611f98c15-Abstract.html

Brody, D. C., Meister, B. K., & Parry, M. F. (2012). Informational inefficiency in financial markets. *Mathematics and Financial Economics*, *6*(3), 249–259. https://doi.org/10.1007/s11579-012-0078-1

Cajas, D. (2025). *Advanced Portfolio Optimization: A Cutting-edge Quantitative Approach*. Springer Nature Switzerland. https://doi.org/10.1007/978-3-031-84304-4

Calvet, L., & Fisher, A. (2002). Multifractality in Asset Returns: Theory and Evidence. *The Review of Economics and Statistics*, *84*(3), 381–406. https://doi.org/10.1162/003465302320259420

Campbell, J. Y., & Thompson, S. B. (2008). Predicting Excess Stock Returns out of Sample: Can Anything Beat the Historical Average? *The Review of Financial Studies*, *21*(4), 1509–1531.

Campbell, R. H., & Akhtar, R. S. (2000). Conditional Skewness in Asset Pricing Tests. *The Journal of Finance*, *55*(3), 1263–1295. https://doi.org/https://doi.org/10.1111/0022-1082.00247

Chatterjee, A., Ayadi, O. F., & Boone, B. E. (2000). Artificial Neural Network and the Financial Markets: A Survey. *Managerial Finance*, *26*(12), 32–45. https://doi.org/10.1108/03074350010767034

Chen, B., Jiang, Y., & Zhou, D. (2025). Risk contagion network and characteristic measurement among international financial markets. *Pacific Basin Finance Journal*, *92*. https://doi.org/10.1016/j.pacfin.2025.102766

Chen, J., Hong, H., & Stein, J. C. (2001). Forecasting crashes: Trading volume, past returns, and conditional skewness in stock prices. *Journal of Financial Economics*, *61*(3), 345–381. https://doi.org/10.1016/S0304-405X(01)00066-6

Christoffersen, P. F. (1998). Evaluating Interval Forecasts. *International Economic Review*, *39*(4), 841–862. https://doi.org/10.2307/2527341

de Oliveira, A. S., Ceretta, P. S., & Pastorek, D. (2024). An experiment with ANNs and Long-Tail Probability Ranking to Obtain Portfolios with Superior Returns. *Computational Economics*, *65*, 1819–1853. https://doi.org/10.1007/s10614-024-10605-5

Dubey, S. R., Singh, S. K., & Chaudhuri, B. B. (2022). Activation functions in deep learning: A comprehensive survey and benchmark. *Neurocomputing*, *503*, 92–108. https://doi.org/10.1016/j.neucom.2022.06.111

Embrechts, P., Klüppelberg, C., & Mikosch, T. (1997). *Modelling Extremal Events*. Springer. https://doi.org/10.1007/978-3-642-33483-2

Emmanuel, S., Sathasivam, S., Velavan, M., & Govinda, V. (2025). Adaptive Portfolio Strategies: Comparing Pre-, during and Post-COVID-19 Dynamics using Mean-Variance Optimization. *Semarak International Journal in Modern Accounting and Finance*, *4*(1), 1–17. https://doi.org/10.37934/sijmaf.4.1.117

Evans, J. L., & Archer, S. H. (1968). Diversification and the Reduction of Dispersion: An Empirical Analysis. *The Journal of Finance*, *23*(5), 761–767. https://doi.org/10.2307/2325905

Fama, E. F. (1970). Efficient Capital Markets: A Review of Theory and Empirical Work. *The Journal of Finance*, *25*(2), 383–417. https://doi.org/10.2307/2325486

Fernandez, C. (1999). Multivariate Student-t regression models: Pitfalls and inference. *Biometrika*, *86*(1), 153–167. https://doi.org/10.1093/biomet/86.1.153

Gauss, C. F. (1809). *Theoria motus corporum coelestium in sectionibus conicis solem ambientium*. Sumtibus F. Perthes et I.H. Besser.

Georgantas, A., Doumpos, M., & Zopounidis, C. (2024). Robust optimization approaches for portfolio selection: A comparative analysis. *Ann Oper Res.*, (339), 1205–1221. https://doi.org/https://doi.org/10.1007/s10479-021-04177-y

Goodfellow, I., Bengio, Y., & Courville, A. (2016). *Deep Learning*. MIT Press. https://mitpress.mit.edu/9780262035613/deep-learning/

Gopal, S., Thangaraj, V., Kumara, R., & Rupa, R. (2025). Geopolitical shockwaves: The Russia-Ukraine war's impact on BRICS financial markets. *COGENT ECONOMICS & FINANCE*, *3*(1). https://doi.org/10.1080/23322039.2025.2476096

Gunjan, A., & Bhattacharyya, S. (2022). A brief review of portfolio optimization techniques. *Artif. Intell. Rev.*, *56*(5), 3847–3886. https://doi.org/10.1007/s10462-022-10273-7

Haykin, S. (1994). *Neural networks. A comprehensive foundation.* Pearson Prentice Hall.

Hebb, D. O. (1949). The Organization of Behavior. *Brain Research Bulletin*, *50*(5), 437. https://doi.org/10.1016/S0361-9230(99)00182-3

Hirano, M., Izumi, K., Shimada, T., Matsushima, H., & Sakaji, H. (2020). Impact Analysis of Financial Regulation on Multi-Asset Markets Using Artificial Market Simulations. *Journal of Risk and Financial Management*, *13*(4). https://doi.org/10.3390/jrfm13040075

Hornik, K., Stinchcombe, M., & White, H. (1989). Multilayer feedforward networks are universal approximators. *Neural Networks*, *2*(5), 359–366. https://doi.org/10.1016/0893-6080(89)90020-8

Hurst, H. E. (1951). Long-Term Storage Capacity of Reservoirs. *Transactions of the American Society of Civil Engineers*, *116*(1), 770–799. https://doi.org/10.1061/TACEAT.0006518

Ibikunle, G., Moews, B., & Rzayev, K. (2024). *Can Machine Learning Unlock New Insights into High-Frequency Trading?* (SSRN Scholarly Paper N. 4826698). Social Science Research Network. https://doi.org/10.2139/ssrn.4826698

Ibragimov, M., Ibragimov, R., & Walden, J. (2015). Introduction. Em M. Ibragimov, R. Ibragimov, & J. Walden (Org.), *Heavy-Tailed Distributions and Robustness in Economics and Finance* (p. 1–9). Springer International Publishing. https://doi.org/10.1007/978-3-319-16877-7_1

IMF. (2025). *World Economic Outlook (October 2024)—GDP, current prices*. https://www.imf.org/external/datamapper/NGDPD@WEO

Jagtap, A. D., Kharazmi, E., & Karniadakis, G. E. (2020). Conservative physics-informed neural networks on discrete domains for conservation laws: Applications to forward and inverse problems. *Computer Methods in Applied Mechanics and Engineering*, *365*, 113028. https://doi.org/10.1016/j.cma.2020.113028

Jain, R. (2025). *Cutting-Edge Developments in Science, Engineering and Technology: A Multidisciplinary Review* (SSRN Scholarly Paper N. 5197905). Social Science Research Network. https://doi.org/10.2139/ssrn.5197905

Jensen, M. C. (1968). The Performance of Mutual Funds in the Period 1945–1964. *The Journal of Finance*, *23*(2), 389–416. https://doi.org/10.1111/j.1540-6261.1968.tb00815.x

Kopeliovich, Y., & Pokojovy, M. (2024). Portfolio optimization with feedback strategies based on artificial neural networks. *Finance Research Letters*, *69*, 106185. https://doi.org/10.1016/j.frl.2024.106185

Kumaresh, S. S., Devarapalli, R., García Márquez, F. P., & Acaroğlu, H. (2025). A comprehensive review of optimization, market strategies, and AI applications in energy storage systems. *Evolutionary Intelligence*, *18*(4). https://doi.org/10.1007/s12065-025-01066-2

Laurent, S. (2002). *Asymmetry and fat-tails in financial time series*. Universiteit Maastricht. https://doi.org/10.26481/dis.20020620sl

Lintner, J. (1965). The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets. *The Review of Economics and Statistics*, *47*(1), 13. https://doi.org/10.2307/1924119

López de Prado, M., Simonian, J., Fabozzi, F. A., & Fabozzi, F. J. (2025). Enhancing Markowitz's portfolio selection paradigm with machine learning. *Annals of Operations Research*, *346*(1), 319–340.

Maciel, L. (2021). A new approach to portfolio management in the Brazilian equity market: Does assets efficiency level improve performance? *The Quarterly Review of Economics and Finance*, *81*, 38–56. https://doi.org/10.1016/j.qref.2021.04.017

Maciel, L. S. (2025). Multifractality and the economic value of equity risk premium forecasts. *Applied Economics Letters*, *0*(0), 1–8. https://doi.org/10.1080/13504851.2025.2498060

Markowitz, H. (1952). Portfolio Selection. *The Journal of Finance*, *7*(1), 77–91. https://doi.org/10.2307/2975974

McCulloch, W. S., & Pitts, W. (1943). A logical calculus of the ideas immanent in nervous activity. *The Bulletin of Mathematical Biophysics*, *5*(4), 115–133. https://doi.org/10.1007/BF02478259

Mohammed, S. A. S. A. (2024). Artificial Neural Networks (ANN) for Stock Price Prediction: A Financial Machine Learning Analysis. Em B. Verma, A. Mittal, M. Raman, & B. Sindhav (Org.), *Augmenting Retail Reality, Part B: Blockchain, AR, VR, and AI* (p. 0). Emerald Publishing Limited. https://doi.org/10.1108/978-1-83608-708-320241019

Mohammed, S. A. S. A. (2025). Support Vector Machines in Stock Price Prediction. Em *Financial Innovation for Global Sustainability* (p. 561–578). John Wiley & Sons, Ltd. https://doi.org/10.1002/9781394311682.ch22

Naik, A. S., Yeniaras, E., Hellstern, G., Prasad, G., & Vishwakarma, S. K. L. P. (2025). From portfolio optimization to quantum blockchain and security: A systematic review of quantum computing in finance. *Financial Innovation*, *11*(1), 88. https://doi.org/10.1186/s40854-025-00751-6

Oliveira, A. S. de, & Ceretta, P. S. (2023). *ANNt Package for R*. Pacote ANNt. https://github.com/aleoli05/ANNt

Oliveira, A. S. de, Ceretta, P. S., & Albrecht, P. (2023). Performance comparison of multifractal techniques and artificial neural networks in the construction of investment portfolios. *Finance Research Letters*, *55*, 103814. https://doi.org/10.1016/j.frl.2023.103814

Pan, Q., Lin, Y., Wu, J., & Zhong, J. (2025). Intertemporal constrained portfolio optimization problem with tracking-error. *Applied Mathematics in Science and Engineering*, *33*(1), 2463456. https://doi.org/10.1080/27690911.2025.2463456

Pan, X., & Qian, M. (2025). *6: Corporate governance in China: new perspectives and future research* (p. 138–155). EE Elgaronline. https://www.elgaronline.com/edcollchap/book/9781035312610/chapter6.xml

Pekanov Starcevic, D., Zrnic, A., & Jaksic, T. (2019). Possibilities of Applying Markowitz Portfolio Theory on the Croatian Capital Market. Em D. Barkovic, B. Crnkovic, M. Z. Susac, K. H. Dernoscheg, N. Pap, B. Runzheimer, & D. Wentzel (Org.), *INTERDISCIPLINARY MANAGEMENT RESEARCH XV (IMR)* (V. 15, p. 520–538). Josip Juraj Strossmayer Univ Osijek.

Peng, Z., Xiong, K., & Yang, Y. (2024). Microstructure of the Chinese stock market: A historical review. *Pacific-Basin Finance Journal*, *88*(December), 34. https://doi.org/https://doi.org/10.1016/j.pacfin.2024.102551

Rachev, S. T., Menn, C., & Fabozzi, F. J. (2005). *Fat-Tailed and Skewed Asset Return Distributions: Implications for Risk Management, Portfolio Selection, and Option Pricing*. John Wiley & Sons.

Rachev, Z., Racheva-lotova, S., & Stoyanov, S. (2010, abril 30). *Capturing fat tails*. Risk.Net. https://www.risk.net/node/1603830

S, A. B., Hn, D., S, B., Jangid, D., & Km, H. (2025). Investify—No Sharks Required. *International Journal of Latest Technology in Engineering, Management & Applied Science*, *14*(6), 70–74.

Saldanha, D., Mallikarjunappa, T., & Oware, K. M. (2025). Determinants of Adaptive Behaviour in Stock Market: A Review. *Vision*, *29*(1), 112–124. https://doi.org/10.1177/09722629221148809

Salisu, A. A., Cuñado, J., Isah, K., & Gupta, R. (2021). Stock markets and exchange rate behavior of the BRICS. *Journal of Forecasting*, *40*(8), 1581–1595. https://doi.org/10.1002/for.2795

Sarfarazurrehman, S., Mane, V., & Doshi, A. (2025). AI and Machine Learning Models in Cross-Asset Class Investment Risk Analysis: A Case Study of Real Estate and Equities Markets. *2025 3rd International Conference on Self Sustainable*

Artificial Intelligence Systems (ICSSAS), 1170–1177. https://doi.org/10.1109/ICSSAS66150.2025.11081061

Selvam, K., Ramasamy, V., Ganesan, V., Dey, N., Hota, S. B., & Chowdhury, S. (2025). Leveraging Machine Learning Models in Data Science for Share Market Analysis—A Comprehensive Review and Future Directions. Em N. H. Long, J. M. R. S. Tavares, S. Pal, L. H. Son, & B. Thai Pham (Org.), *Proceedings of Third International Conference on Intelligent System* (p. 367–378). Springer Nature. https://doi.org/10.1007/978-981-96-4093-5_32

Sharpe, W. F. (1964). Capital Asset Prices: A Theory of Market Equilibrium Under Conditions of Risk*. *The Journal of Finance*, *19*(3), 425–442. https://doi.org/10.1111/j.1540-6261.1964.tb02865.x

Sharpe, W. F. (1966). Mutual Fund Performance. *The Journal of Business*, *39*(1), 119–138.

Simon, H. A. (1955). A Behavioral Model of Rational Choice. *The Quarterly Journal of Economics*, *69*(1), 99–118. https://doi.org/10.2307/1884852

Sortino, F. A., & Price, L. N. (1994). Performance Measurement in a Downside Risk Framework. *The Journal of Investing*, *3*(3), 59–64. https://doi.org/10.3905/joi.3.3.59

Souza, M. J. S. de, Ramos, D. G. F., Pena, M. G., Sobreiro, V. A., & Kimura, H. (2018). Examination of the profitability of technical analysis based on moving average strategies in BRICS. *Financial Innovation*, *4*(1), 1–18.

Student. (1908). The Probable Error of a Mean. *Biometrika*, *6*(1), 1–25. https://doi.org/10.2307/2331554

Tiwari, A. K., Aye, G. C., & Gupta, R. (2019). Stock market efficiency analysis using long spans of Data: A multifractal detrended fluctuation approach. *Finance Research Letters*, *28*, 398–411. https://doi.org/10.1016/j.frl.2018.06.012

Treynor, J. L. (1962). *Jack Treynor's "Toward a Theory of Market Value of Risky Assets"* (SSRN Scholarly Paper N. 628187). https://doi.org/10.2139/ssrn.628187

Treynor, J. L. (1965). How to rate management of investment funds. *Harvard Business Review*, *Harvard Business Review*, *43*(1), 63–75.

Tyagi, A., & Abraham, A. (2022). *Recurrent Neural Networks. Concepts and Applications.* (1º ed.). CRC Press Taylor & Francis Group. https://www.taylorfrancis.com/books/edit/10.1201/9781003307822/recurrent-neural-networks-amit-kumar-tyagi-ajith-abraham

Vuong, P. H., Phu, L. H., Van Nguyen, T. H., Duy, L. N., Bao, P. T., & Trinh, T. D. (2025). A comparative study of deep learning approaches for stock price prediction. *Digital Finance*. https://doi.org/10.1007/s42521-025-00149-0

Wang, F., Yan, X. (Sterling), & Zheng, L. (2020). *Should Mutual Fund Investors Time Volatility? 77*(1). https://doi.org/10.1080/0015198X.2020.1822705

Wilson, J. D. (2015). Resource powers? Minerals, energy and the rise of the BRICS. *Third World Quarterly*, *36*(2), 223–239.

Yang, Y., & Liu, H. (2023). Multifractal volatility modeling with deep learning: Evidence from emerging markets. *Physica A: Statistical Mechanics and its*

Applications, *621*(128912). https://doi.org/https://doi.org/10.1016/j.physa.2023.128912

Yin, H., & Zhang, H. (2025). *11: Financial analysts in China* (p. 226–242). EE Elgaronline. https://www.elgaronline.com/edcollchap/book/9781035312610/chapter11.xml

Zhang, P., & Du, B. (2025). Fuzzy Mean-Variance Portfolio Selection Based on Machine Learning. *Computational Economics*. https://doi.org/10.1007/s10614-025-11017-9

Zhang, X., Huang, Y., Xu, K., & Xing, L. (2023). Novel modelling strategies for high-frequency stock trading data. *Financial Innovation*, *9*(1), 39. https://doi.org/10.1186/s40854-022-00431-9

Zhong, X., & Enke, D. (2017). A comprehensive cluster and classification mining procedure for daily stock market return forecasting. *Neurocomputing*, *267*, 152–168. https://doi.org/10.1016/j.neucom.2017.06.010

Zhong, X., & Enke, D. (2019). Predicting the daily return direction of the stock market using hybrid machine learning algorithms. *Financial Innovation*, *5*(1), 24. https://doi.org/10.1186/s40854-019-0138-0

Tables

Table 1: Equations for generating compared portfolios and econometric models

Portfolio	Equation	Reference
Markowitz	$min_W\left[\sigma_p^2\right] = min_W\left[W^T \Phi W\right]$	(Markowitz, 1952)
Sharpe	$max_W\left[SR_p\right] = max_W \frac{W^T R_p - r_f}{\sqrt{W^T \Phi W}}$	(Sharpe, 1966)
Fluctuation Analysis with Multifractal Trend (MF-DFA)	$F^2(s,v) = \frac{1}{s}\sum_{t=1}^{s}\{y[(v-1)s+t] - y_v(t)\}^2$ $F^2(s,v) = \frac{1}{s}\sum_{t=1}^{s}\{y[T-(v-T_s)s+t] - y_v(t)\}^2$ $F_q(s) = \left\{\frac{1}{2T_s}\sum_{v=1}^{2T_s}[F^2(s,v)]^{q/2}\right\}^{1/q}$ $MDM = \frac{1}{2}(\lvert h(q_{min}) - 0{,}5\rvert + \lvert h(q_{max}) - 0{,}5\rvert)$	(Ali et al., 2018) (Tiwari et al., 2019) (L. Maciel, 2021)
Artificial Neural Networks with classification by Student's t probability (ANNt)	$u_k = \sum_{j=1}^{k} w_{kj} x_j$ $v_k = u_k + b$ $\hat{y}_k = \varphi(v_k)$ $e_j(n) = d_j(n) - y_j(n)$ $P(X \leq x) = \int_{-\infty}^{x} f(t)dt$ $r_{exc\_p\_s(t)} = \varphi\left(\sum_{k'j=1}^{N} w_{k'j} x_{Hk'j} + b,\right)$	(Student, 1908) (McCulloch & Pitts, 1943) (Hebb, 1949) (de Oliveira et al., 2024)

Note: The minimum variation portfolio is weight-restricted by $W_s > 0$ and $\sum_{s=1}^{N} W_s = 1$; s represents assets from 1 to N; $\sigma_p^2 = W^T \Phi W$ is the portfolio variance; T is the total period analyzed (test sample period) and Φ is the covariance matrix of size NxN. The maximum Sharpe portfolio is restricted by $\sum_{s=1}^{N} W_i = 1$. $R_p = (r_1, r_2, \ldots, r_N)^T$ is the average vector of asset returns. $y_v(t)$ is the first difference between the observed value of the Return (r_s) and the Average Return ($\bar{r}_s$) of the asset (s). For each subinterval $[v = 1, \ldots, 2T_s]$, T is the number of observations. $F^2(s,v)$ is the estimated variance for $v = 1, \ldots, T_s$ and $v = T_s + 1, \ldots, 2T_s$. $F_q(s)$ is the fluctuation function of order q. $h(q)$) is the Hurst Exponent, MDM is the Market Deficiency Measure. u_k is a linear combination of the synaptic weights w_{kj} of k neurons with each of the inputs. b is the bias; v_c is the activation potential. $\varphi(.)$ is the activation function. $\hat{y}_k$ is the output signal or final. $e_j(n)$ is the error signal of neuron j; n is the training iteration; $d_j(n)$ is the desired response for neuron j, and $y_j(n)$ is the output signal from neuron j. $F(t)$ is the cumulative probability function of a given value $P(X \leq x)$ in the Student's t distribution. $r_{exc\_p\_s(t)}$ is the daily excess return of each asset, w_{kj} is the synaptic weight of information j in neuron k, $x_{Hk'j}$ is the hidden layer with N hidden neurons k', and $b,$ is the bias of the hidden layer.

Table 2: Data description, compared sparse portfolios, and the number of assets considered in the samples

Portfolio	Raw Sample	Brazil**	Russian**	India**	China**	South Africa**	USA**
IBRX50	50	50					
IMOEX	36		26				
NIFTY_50	50			50			
SSE50	50				41		
FTSE/JSE_40	40					40	
S&P500	503						490
MARKOWITZ	Proxy	15	18	24	9	21	44
SHARPE	Proxy	10	10	13	9	8	23
MF_EQ	20 from Proxy *	20	20	20	20	20	20
MF_MKW	20 from Proxy *	13	15	11	9	17	17
MF_SHARPE	20 from Proxy *	1	1	1	1	2	1
ANNT_EQ	20 from Proxy *	20	20	20	20	20	20
ANNT_MKW	20 from Proxy *	10	15	12	5	17	12
ANNT_SHARPE	20 from Proxy *	5	7	3	3	5	4

Portfolio	Description
IBRX50	Brazilian market proxy of the B[3] Stock Exchange
IMOEX	Russian market proxy of the Moscow Stock Exchange
NIFTY_50	Indian market proxy of the National Stock Exchange
SSE50	Chinese market proxy of the Shanghai Stock Exchange
FTSE/JSE_40	South African market proxy of the Johannesburg Stock Exchange
S&P500	US market proxy of the New York Stock Exchange
MARKOWITZ	Minimum variance portfolio (Markowitz) applied to all assets in the sample
SHARPE	Maximum Sharpe portfolio applied to all assets in the sample
MF_EQ	Multifractal Portfolio with the lowest MDM and equal weights
MF_MKW	Multifractal Portfolio with the lowest MDM and minimum variance weights
MF_SHARPE	Multifractal Portfolio with lowest MDM and maximum Sharpe weights
ANNt_EQ	ANN portfolio with highest Student's *t* probability and equal weights
ANNt_MKW	ANN portfolio with highest Student's *t* probability and minimum variance weights
ANNt_SHARPE	ANN portfolio with highest Student's *t* probability and maximum Sharpe weights

Note: Dataset spans from January 3, 2018, to December 31, 2024. The portfolio training is conducted on a rolling biennial basis, while performance is evaluated over a testing period that extends from January 3, 2020, to December 31, 2024, based on quarterly rebalancing. Markowitz and Sharpe portfolios use samples of 36, 40, 50, or 503 assets, respectively, for each market proxy.

* Denotes the assets that make up each of the BRICS market proxies are used, but portfolios are forced to choose 20 market proxy assets that best meet the specific portfolio selection criteria. The selection of 20 assets is made because the size of the best-performing ANNt and MF-DFA portfolios is between 15 and 20 assets (de Oliveira et al., 2024). The number of non-zero asset weights in each portfolio is the average number of assets in each rebalanced portfolio.

** Market samples without N/A.

Table 3: Validation parameters or metric performance

Validation Parameter	Equation	Reference
Average Return ($\bar{r}_p$)	$\bar{r}_p = \sum_{t=1}^{T} \frac{r_{p,t}}{T}$	(Bachelier, 1900)
Annualized Return (r_p^A)	$r_p^A = \left[\prod_{t=1}^{T} (1 + r_{p,t})\right]^{252/T} - 1$	(Bachelier, 1900)
Accumulated Return (r_p^C)	$r_p^C = \left[\prod_{t=1}^{T} (1 + r_{p,t})\right] - 1$	(Bachelier, 1900)
Jensen's Alpha (α)	$\alpha = r_{p,t} - [r_f + (r_{m,t} - r_f)\, \beta]$	(Jensen, 1968)
Sharpe Ratio (Sp)	$S = \frac{r_p^A - r_f}{\sigma_p^A}$	(Sharpe, 1966)
Sortino Ratio ($Sort$)	$Sort = \frac{r_p^A - r_f}{\sigma_{p,d}^A}$	(Sortino & Price, 1994)
Treynor Ratio (Try)	$Tr = \frac{r_p^A - r_f}{\beta_{p,d}^A}$	(Treynor, 1965)
Beta (β)	$\beta = \frac{cov(r_{p,t}, r_{m,t})}{\sigma_m^2}$	(Treynor, 1962),(Sharpe, 1964), (Lintner, 1965)
Annualized Volatility (σ_p^A)	$\sigma_p^A = \sqrt{252}\sigma_p = \sqrt{252}\sqrt{\frac{1}{T}\sum_{t=1}^{T}(r_{p,t} - \bar{r}_p)^2}$	(Gauss, 1809)
Value at risk ($VaR_{1-\gamma}$)	$VaR_{1-\gamma} = inf\{x \in \Re : Prob(r_p < x) \leq \alpha\}$	(Christoffersen, 1998)
Conditional Value at Risk ($CVaR_{1-\gamma}$)	$CVaR_{1-\gamma} = \frac{1}{1-c}\int_{-1}^{VaR_{1-\gamma}} xp(x)dx$	(Embrechts et al., 1997)

Note: $r_{p,t}$ is the portfolio return, p is the period, t denotes the series length (T), σ_m^2 is the market variance, σ_p is the portfolio´s standard deviation, and $\gamma = 5\%$ is the significance level of the normal distribution. $p(x)$ is the probability density of the return value (x). c is the cutoff point of the analyzed distribution. r_f is the risk-free rate, $\sigma_{p,d}^A$ is the annualized standard deviation of negative returns, $cov(r_{p,t}, r_{SP500,t})$ is the covariance of the returns of the portfolio, and $\beta_{p,d}^A$ is the annualized beta of negative returns.

Table 4: Average of parameters in BRICS markets (best performances in bold) between 2020 and 2024

Brazilian Market									
Ratio	IBRX50	MARKOWITZ	SHARPE	MF_EQ	MF_MKW	MF_SHARPE	ANNt_EQ	ANNt_MKW	ANNt_SHARPE
$\bar{r}_p$	0.004	0.029	0.032	-0.014	0.018	**0.104**	-0.032	0.004	0.010
r_p^A	-0.139	6.798	6.916	-4.394	3.609	**21.984**	-8.042	1.023	2.018
r_p^C	6.259	19.758	6.081	3.838	16.754	**89.836**	-2.677	9.034	-0.703
α	0.000	0.022	0.026	-0.016	0.011	**0.098**	-0.030	0.003	0.002
S	-0.219	0.281	0.348	-0.405	0.016	**0.544**	-0.520	-0.098	0.041
$Sort$	0.004	0.052	0.053	-0.015	0.028	**0.069**	-0.027	0.014	0.024
Tr	-0.010	0.032	0.029	-0.021	0.011	**0.086**	-0.030	-0.002	0.017
β	1.000	**0.590**	0.838	1.077	0.640	1.044	1.142	0.791	0.896
σ_p^A	16.952	**14.028**	20.404	19.703	14.874	35.603	21.502	17.405	24.971
$VaR_{1-\gamma}$	1.753	**1.425**	2.081	2.057	1.524	3.585	2.260	1.799	2.576
$CVaR_{1-\gamma}$	2.199	**1.793**	2.619	2.574	1.916	4.522	2.824	2.258	3.235
Russian Market									
Ratio	IMOEX	MARKOWITZ	SHARPE	MF_EQ	MF_MKW	MF_SHARPE	ANNt_EQ	ANNt_MKW	ANNt_SHARPE
$\bar{r}_p$	0.010	0.047	-0.007	0.016	0.028	-0.012	0.012	**0.050**	0.002
r_p^A	-0.930	9.948	-4.764	1.656	4.226	-8.048	0.302	**10.595**	-2.816
r_p^C	0.491	23.831	-4.765	9.690	24.285	-12.769	5.714	**26.937**	-3.415
α	0.000	0.038	-0.015	0.007	0.015	-0.025	0.003	**0.040**	-0.007
S	-0.107	0.280	-0.312	-0.013	0.070	-0.296	-0.068	**0.300**	-0.206
$Sort$	0.012	0.046	-0.007	0.018	0.026	-0.006	0.014	**0.048**	0.002
Tr	-0.002	0.049	-0.023	0.006	0.025	-0.033	0.000	**0.053**	-0.010
β	1.000	**0.755**	0.716	0.877	0.806	0.806	0.886	0.799	0.779
σ_p^A	26.006	**24.718**	26.120	25.571	25.678	35.084	25.936	25.940	27.738
$VaR_{1-\gamma}$	2.687	**2.514**	2.714	2.632	2.635	3.648	2.676	2.638	2.871
$CVaR_{1-\gamma}$	3.369	**3.164**	3.401	3.305	3.311	4.571	3.358	3.321	3.603
Indian Market									
Ratio	NIFTY_50	MARKOWITZ	SHARPE	MF_EQ	MF_MKW	MF_SHARPE	ANNt_EQ	ANNt_MKW	ANNt_SHARPE
$\bar{r}_p$	0.042	0.049	0.048	0.064	0.063	**0.114**	0.056	0.058	0.084
r_p^A	10.748	13.630	12.199	17.242	17.061	**27.344**	15.315	15.601	21.651
r_p^C	45.127	57.996	43.862	80.004	73.817	**96.692**	78.832	72.570	75.770
α	0.000	0.017	0.008	0.022	0.028	**0.054**	0.012	0.023	0.042
S	0.541	0.891	0.519	0.923	**1.068**	0.722	0.796	0.895	0.897
$Sort$	0.068	0.103	0.062	0.094	**0.117**	0.078	0.082	0.098	0.094
Tr	0.029	0.0580	0.038	0.052	0.070	**0.090**	0.044	0.057	0.074
β	1.000	**0.685**	1.016	1.000	0.745	1.550	1.062	0.864	1.079
σ_p^A	13.913	**11.801**	20.45	15.304	12.833	46.486	16.189	14.331	24.733
$VaR_{1-\gamma}$	1.401	**1.172**	2.071	1.523	1.266	4.702	1.622	1.427	2.479
$CVaR_{1-\gamma}$	1.765	**1.484**	2.609	1.926	1.605	5.925	2.048	1.804	3.130
Chinese Market									
Ratio	SSE50	MARKOWITZ	SHARPE	MF_EQ	MF_MKW	MF_SHARPE	ANNt_EQ	ANNt_MKW	ANNt_SHARPE
$\bar{r}_p$	0.020	**0.106**	0.009	0.003	0.012	-0.048	0.045	0.090	0.033
r_p^A	3.655	**29.575**	2.041	-1.166	1.572	-13.809	10.504	25.044	7.940
r_p^C	2.719	**64.277**	-8.875	-9.994	-1.500	-34.062	11.74	43.376	-0.863

α	0.000	**0.088**	-0.003	-0.017	-0.006	-0.066	0.026	0.072	0.021
S	-0.006	**1.570**	0.077	-0.210	-0.076	-0.427	0.324	1.162	0.392
$Sort$	0.024	**0.154**	0.023	0.003	0.016	-0.023	0.052	0.121	0.049
Tr	0.008	**0.184**	0.040	-0.008	0.001	-0.040	0.033	0.106	0.104
β	1.000	**0.490**	0.898	1.133	0.986	1.244	1.020	0.738	0.786
σ_p^A	17.727	**16.048**	22.753	22.656	21.490	35.911	20.062	18.021	25.427
$VaR_{1-\gamma}$	1.817	**1.556**	2.348	2.345	2.213	3.768	2.034	1.778	2.602
$CVaR_{1-\gamma}$	2.284	**1.980**	2.948	2.941	2.779	4.713	2.562	2.251	3.271
South-African Market									
Ratio	FTSE/JSE40	MARKOWITZ	SHARPE	MF_EQ	MF_MKW	MF_SHARPE	ANNt_EQ	ANNt_MKW	ANNt_SHARPE
$\bar{r}_p$	0.027	0.036	0.024	0.028	0.018	-0.009	0.033	**0.041**	0.021
r_p^A	5.640	8.968	4.700	6.300	4.139	-6.667	7.729	**10.014**	2.738
r_p^C	19.132	25.034	12.333	25.668	14.053	-14.212	**29.855**	28.748	4.089
α	0.000	0.017	0.000	0.004	-0.004	-0.038	0.009	**0.020**	-0.005
S	0.132	0.466	0.078	0.169	0.052	-0.164	0.262	**0.492**	0.043
$Sort$	0.037	0.073	0.032	0.04	0.032	0.005	0.049	**0.074**	0.028
Tr	0.014	0.046	0.016	0.022	0.010	-0.012	0.026	**0.047**	0.016
β	1.000	**0.544**	0.791	0.817	0.621	1.024	0.863	0.651	0.860
σ_p^A	16.273	**12.918**	19.260	15.476	14.252	37.523	16.113	14.921	22.027
$VaR_{1-\gamma}$	1.661	**1.302**	1.970	1.576	1.459	3.897	1.637	1.504	2.262
$CVaR_{1-\gamma}$	2.088	**1.642**	2.478	1.984	1.833	4.884	2.059	1.897	2.842

Note: $\bar{r}_p$ is the Average Return; r_p^A is the Annualized Return; r_p^C is the Accumulated Return; σ_p^A is the Annualized Volatility; $VaR_{1-\gamma}$ is the Value at Risk; $CVaR_{1-\gamma}$ is the Conditional Value at Risk; Sp is the Sharpe Ratio; $Sort$ is the Sortino Index; β is Beta; and α is Jensen's Alpha. The highest $\bar{r}_p$, r_p^A, and r_p^C is the MF_SHARPE portfolio in the Indian Market, with 0.114% p.d., 27.344% p.a., and 96.692% p.p. The best α are those of the MF_SHARPE with 0.098 in the Brazilian Market. The highest S is from the MARKOWITZ portfolio, with 1.57% in the Chinese Market. The top Sort is also the ANNt_MKW portfolio in the South African Market, with 0.492%. The best Tr is that of the MARKOWITZ portfolio with 0.184% in the Chinese Market. The lowest β is that of the MARKOWITZ portfolio, with a value of 0.490 in the Chinese Market. The portfolio with the lowest σ_p^A is the MARKOWITZ with a value of 11.801% in the Indian Market. The smallest $VaR_{1-\gamma}$ and $CVaR_{1-\gamma}$ is the MARKOWITZ Portfolio, with 1.172 and 1.484%, respectively, also in the Indian Market.

Table 5: Average of parameters in the U.S. market (best performances in bold) between 2020 and 2024

U.S. Market									
Ratio	S&P500	MARKOWITZ	SHARPE	MF_EQ	MF_MKW	MF_SHARPE	ANNt_EQ	ANNt_MKW	ANNt_SHARPE
$\bar{r}_p$	**0.062**	0.022	0.025	0.029	0.034	0.012	0.032	0.028	-0.007
r_p^A	**15.221**	5.180	4.654	6.772	8.052	-2.687	6.074	5.564	-7.694
r_p^C	**41.632**	12.226	10.392	26.108	24.886	13.274	33.624	29.789	-0.962
α	**0.000**	-0.008	-0.027	-0.019	**0.000**	-0.057	-0.043	-0.030	-0.094
S	**0.852**	0.220	0.125	0.212	0.373	-0.059	0.116	0.084	-0.206
$Sort$	**0.096**	0.048	0.032	0.044	0.060	0.016	0.032	0.028	0.004
Tr	0.048	0.049	0.020	0.020	**0.057**	0.006	0.018	0.014	-0.010
β	1.000	**0.395**	0.861	0.777	0.474	1.109	1.260	0.966	1.490
σ_p^A	15.357	**11.491**	19.788	14.891	13.738	39.455	23.285	21.158	40.119
$VaR_{1-\gamma}$	1.532	**1.170**	2.026	1.514	1.389	4.078	2.380	2.165	4.165
$CVaR_{1-\gamma}$	1.936	**1.470**	2.546	1.906	1.752	5.117	2.993	2.721	5.220

Note: $\bar{r}_p$ is the Average Return; r_p^A is the Annualized Return; r_p^C is the Accumulated Return; σ_p^A is the Annualized Volatility; $VaR_{1-\gamma}$ is the Value at Risk; $CVaR_{1-\gamma}$ is the Conditional Value at Risk; Sp is the Sharpe Ratio; $Sort$ is the Sortino Index; β is Beta; and α is Jensen's Alpha. The highest $\bar{r}_p$, r_p^A, and r_p^C is the S&P500, with 0.062% p.d., 15.221% p.a., and 41.632% p.p. The best α are those of the S&P500 and MF_MKW with 0.000. The highest S is from the Markowitz portfolio, with 11.491% in the USA Market. The top Sort is also the S&P500 portfolio, with 0.096%. The best Tr is that of the MF_MKW portfolio with 0.057% in the USA Market. The lowest β is that of the Markowitz portfolio, with a value of 0.395 in the USA Market. The portfolio with the lowest σ_p^A is the Markowitz with a value of 11.801% in the USA Market. The smallest $VaR_{1-\gamma}$ and $CVaR_{1-\gamma}$ is the Markowitz Portfolio, with 1.170 and 1.470%, respectively, also in the USA Market.

Table 6: Summary of strategies with superior results in the Long Term (LT) and Short Term (ST) in the first 5 years of the 20s of the 21st century

Market	**Higher Return Strategy**	**Lower Risk Strategy**
Brazilian	LT – MF_SHARPE ST - ANNt_SHARPE	LT - MARKOWITZ ST - MARKOWITZ
Russian	LT - ANNt_MKW ST - ANNt_MKW	LT - MARKOWITZ ST - MARKOWITZ
Indian	LT – MF_SHARPE ST _ MF_SHARPE	LT - MARKOWITZ ST - MARKOWITZ
Chinese	LT – MARKOWITZ ST – ANNt_MKW	LT - MARKOWITZ ST - ANNt_SHARPE
South African	LT – ANNt_MKW ST – MF_SHARPE	LT - MARKOWITZ ST - MARKOWITZ
U.S.	LT – S&P500 ST - ANNt_SHARPE	LT - MARKOWITZ ST - MARKOWITZ

Table 7: Annualized return and annualized volatility of the BRICS and the U.S. markets in % p.a., calculated in local currency and in U.S. dollars between 2020 and 2024

Annualized Return of BRICS and U.S. Stock Markets (% p.a.)					
Brazil	Russian	India	China	South Africa	USA
BRL	RUB	INR	CNY	ZAR	USD
21.984	10.595	27.344	**29.575**	10.014	15.221
USD	USD	USD	USD	USD	USD
22.024	10.810	**27.181**	25.071	10.088	15.221
Annualized volatility of BRICS and USA (% p.a.)					
Brazil	Russian	India	China	South Africa	USA
BRL	RUB	INR	CNY	ZAR	USD
14.028	24.718	11.801	16.048	12.918	**11.491**
USD	USD	USD	USD	USD	USD
13.910	24.347	11.706	16.049	12.851	**11.491**

Note: The numbers in bold represent the highest annualized returns and lowest market volatility. In local currency, China has the highest annualized return (CNY 29.575), while in US dollars, India has the highest annualized return (USD 27.181). The market with the lowest volatility is the US market (USD 11.491).

Table 8: t-test pair portfolios's p-value considering the trade-off between Cumulative Return ($RCum$) and Annualized Volatility for the BRICS Markets between 2020 and 2024

Brazil Market									
t_test	IBRX50	MARKOWITZ	SHARPE	MF_EQ	MF_MKW	MF_SHARPE	ANNt_EQ	ANNt_MKW	ANNt_SHARPE
SP500	1.0000	**0.0030**	**0.0400**	**0.0050**	**0.0270**	**0.0000**	**0.0000**	0.6200	**0.0010**
MARKOWITZ	**0.0090**	1.0000	**0.0000**	**0.0000**	0.2660	**0.0000**	**0.0000**	**0.0000**	**0.0000**
SHARPE	0.9800	0.0660	1.0000	0.6500	**0.0010**	**0.0000**	0.4810	0.0600	0.0810
MF_EQ	0.7220	**0.0230**	0.7940	1.0000	**0.0000**	**0.0000**	**0.0280**	**0.0040**	**0.0240**
MF_MKW	0.0530	0.5630	0.1590	0.0710	1.0000	**0.0000**	**0.0000**	**0.0010**	**0.0000**
MF_SHARPE	**0.0080**	**0.0230**	**0.0080**	**0.0070**	**0.0180**	1.0000	**0.0000**	**0.0000**	**0.0020**
ANNt_EQ	0.1160	0.0000	0.2580	0.3680	**0.0020**	**0.0040**	1.0000	**0.0000**	0.1250
ANNt_MKW	0.6330	0.0670	0.7060	0.4820	0.2040	**0.0100**	0.0680	1.0000	**0.0020**
ANNt_SHARPE	0.4970	0.0530	0.5550	0.6840	0.1000	**0.0060**	0.8500	0.3620	1.0000
Russian Market									
t_test	IMOEX	MARKOWITZ	SHARPE	MF_EQ	MF_MKW	MF_SHARPE	ANNt_EQ	ANNt_MKW	ANNt_SHARPE
IMOEX	1.0000	0.3490	0.9430	0.7700	0.8110	**0.0000**	0.9630	0.9620	0.2740
MARKOWITZ	**0.0030**	1.0000	0.2030	0.3450	0.1580	**0.0000**	0.1960	0.1050	**0.0060**
SHARPE	0.3770	**0.0000**	1.0000	0.6580	0.6850	**0.0000**	0.8840	0.8730	0.2310
MF_EQ	0.1910	0.0780	0.0280	1.0000	0.9050	**0.0000**	0.7410	0.7000	0.0760
MF_MKW	**0.0190**	0.9650	0.0040	0.1480	1.0000	**0.0000**	0.7830	0.7270	0.0550
MF_SHARPE	0.1060	**0.0000**	0.2930	**0.0100**	**0.0010**	1.0000	**0.0000**	**0.0000**	**0.0000**
ANNt_EQ	0.4400	**0.0230**	0.0940	0.5780	0.0650	**0.0300**	1.0000	0.9970	0.1450
ANNt_MKW	**0.0040**	0.7380	**0.0000**	0.0570	0.8110	0.0000	**0.0190**	1.0000	0.1040
ANNt_SHARPE	0.5630	**0.0010**	0.8250	0.0720	**0.0080**	0.2580	0.1920	**0.0010**	1.0000
Indian Market									
t_test	NIFTY_50	MARKOWITZ	SHARPE	MF_EQ	MF_MKW	MF_SHARPE	ANNt_EQ	ANNt_MKW	ANNt_SHARPE
NIFTY_50	1.0000	**0.0000**	**0.0000**	**0.0070**	**0.0260**	**0.0000**	**0.0010**	0.4730	**0.0010**
MARKOWITZ	0.3750	1.0000	**0.0000**	**0.0000**	**0.0170**	**0.0000**	**0.0000**	**0.0000**	**0.0000**
SHARPE	0.9240	0.3280	1.0000	**0.0010**	**0.0000**	**0.0000**	**0.0050**	**0.0000**	0.1650
MF_EQ	0.0750	0.2720	0.0650	1.0000	**0.0000**	**0.0000**	0.1350	0.0780	**0.0030**
MF_MKW	0.0960	0.3750	0.0810	0.7750	1.0000	**0.0000**	**0.0000**	**0.0070**	**0.0000**
MF_SHARPE	0.2400	0.3780	0.2280	0.7120	0.6070	1.0000	**0.0000**	**0.0000**	**0.0000**
ANNt_EQ	0.1060	0.3270	0.0930	0.9620	0.8260	0.6960	1.0000	**0.0070**	**0.0060**
ANNt_MKW	0.1330	0.4400	0.1160	0.7420	0.9520	0.5910	0.7910	1.0000	**0.0010**
ANNt_SHARPE	0.1140	0.3710	0.0990	0.8560	0.9280	0.6440	0.9000	0.8870	1.0000
China Market									
t_test	SSE50	MARKOWITZ	SHARPE	MF_EQ	MF_MKW	MF_SHARPE	ANNt_EQ	ANNt_MKW	ANNt_SHARPE
SSE50	1.0000	**0.0250**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0040**	0.7100	**0.0000**
MARKOWITZ	**0.0000**	1.0000	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0080**	**0.0000**
SHARPE	0.2600	**0.0000**	1.0000	0.9230	0.1900	**0.0000**	**0.0080**	**0.0000**	0.1750
MF_EQ	**0.0050**	**0.0000**	0.9160	1.0000	0.1290	**0.0000**	**0.0020**	**0.0000**	0.1440
MF_MKW	0.3510	**0.0000**	0.4950	0.1340	1.0000	**0.0000**	**0.0450**	**0.0000**	**0.0400**
MF_SHARPE	**0.0000**	**0.0000**	0.0590	0.0150	0.0020	1.0000	**0.0000**	**0.0000**	**0.0000**
ANNt_EQ	**0.0070**	**0.0000**	0.0550	0.0000	0.0100	0.0000	1.0000	**0.0090**	**0.0070**
ANNt_MKW	**0.0000**	**0.0020**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	1.0000	**0.0000**

ANNt_SHARPE	0.7470	**0.0000**	0.5870	0.4330	0.9560	**0.0210**	0.2690	**0.0010**	1.0000

South Africa Market									
t_test	FTSE/JSE40	MARKOWITZ	SHARPE	MF_EQ	MF_MKW	MF_SHARPE	ANNt_EQ	ANNt_MKW	ANNt_SHARPE
FTSE/JSE40	1.0000	**0.0000**	0.0880	0.2640	**0.0110**	**0.0000**	0.8370	0.1920	**0.0150**
MARKOWITZ	0.2550	1.0000	**0.0010**	**0.0020**	0.0990	**0.0000**	**0.0000**	0.0630	**0.0000**
SHARPE	0.2500	**0.0230**	1.0000	**0.0300**	**0.0060**	**0.0000**	0.0710	**0.0220**	0.3020
MF_EQ	0.3490	0.9230	0.0680	1.0000	0.0650	**0.0000**	0.3470	0.5590	**0.0060**
MF_MKW	0.3020	**0.0140**	0.7390	0.0760	1.0000	**0.0000**	**0.0140**	0.4990	**0.0020**
MF_SHARPE	**0.0100**	**0.0030**	**0.0370**	**0.0040**	**0.0240**	1.0000	**0.0000**	**0.0000**	**0.0000**
ANNt_EQ	0.1640	0.5070	0.0290	0.6260	0.0320	**0.0020**	1.0000	0.2390	**0.0130**
ANNt_MKW	0.1300	0.5250	**0.0150**	0.6800	**0.0130**	**0.0020**	0.8910	1.0000	**0.0040**
ANNt_SHARPE	**0.0180**	**0.0010**	0.1970	**0.0050**	0.0740	0.1440	**0.0020**	**0.0010**	1.0000

Note: The lower left triangle in the table shows the pairwise t-test for differences in means of the portfolios considering the cumulative return series of the rebalanced portfolios with each of the strategies. The upper right triangle shows the pairwise t-test for differences in means of the Annualized Volatility series of the rebalanced portfolios. In bold, p-values indicate a significance level of 5%.

In the Brazilian market, regarding the traditional MF portfolios, only the MF_EQ and MF_SHARPE portfolio show a significant difference in $RCum$. The ANNt portfolios do not show a statistically significant difference, strictly speaking, placing the ANNt_SHARPE technically at 5%. Annualized Volatility, however, shows a statistically significant difference in practically all portfolio pairs.

In Russian marketing, regarding $RCum$, ANNt´s portfolios stand out compared to traditional portfolios, showing a statistically significant difference at the 5% level. However, the volatility of most portfolios does not show a significant difference.

In the Indian market, no portfolio is significantly different in terms of $RCum$, while Annualized Volatility is statistically different across virtually all portfolios.

In the Chinese market, the $RCum$ series of ANNt wallets shows a significant difference compared to traditional wallets. Similarly, most wallets have statistically significant differences from one another.

In the South African market, ANNt and MF portfolios differ from traditional portfolios in the $RCum$ series. Similarly, most portfolios exhibit a significant difference in annualized volatility.

Table 9: t-test pair portfolios' p-value considering the trade-off between Cumulative Return ($RCum$) and Annualized Volatility for the USA Market between 2020 and 2024

USA Market									
t_test	S&P500	MARKOWITZ	SHARPE	MF_EQ	MF_MKW	MF_SHARPE	ANNt_EQ	ANNt_MKW	ANNt_SHARPE
SP500	1.0000	**0.0000**	**0.0010**	0.5850	**0.0300**	**0.0000**	**0.0000**	**0.0000**	**0.0000**
MARKOWITZ	**0.0000**	1.0000	**0.0000**	**0.0000**	**0.0060**	**0.0000**	**0.0000**	**0.0000**	**0.0000**
SHARPE	**0.0010**	0.8010	1.0000	0.0010	**0.0000**	**0.0000**	**0.0080**	0.2990	**0.0000**
MF_EQ	0.1100	0.0850	0.1290	1.0000	0.1520	**0.0000**	**0.0000**	**0.0000**	**0.0000**
MF_MKW	**0.0340**	**0.0230**	0.0940	0.8910	1.0000	**0.0000**	**0.0000**	**0.0000**	**0.0000**
MF_SHARPE	0.1180	0.9500	0.8730	0.4830	0.5050	1.0000	**0.0010**	**0.0000**	0.9150
ANNt_EQ	0.5430	0.0840	0.0950	0.5890	0.4930	0.3190	1.0000	**0.0180**	**0.0020**
ANNt_MKW	0.4510	0.2380	0.2320	0.8210	0.7480	0.4530	0.8360	1.0000	**0.0010**
ANNt_SHARPE	**0.0010**	0.1940	0.3440	**0.0320**	**0.0230**	0.4600	**0.0270**	0.0830	1.0000

Note: The lower left triangle in the table shows the pairwise t-test for differences in means of the portfolios considering the cumulative return series of the rebalanced portfolios with each of the strategies. The upper right triangle shows the pairwise t-test for differences in means of the Annualized Volatility series of the rebalanced portfolios. In bold, p-values indicate a significance level of 5%. Regarding the traditional MF portfolios, only the MF_MKW portfolio shows a significant difference in $RCum$. The ANNt portfolios do not show a statistically significant difference. Annualized Volatility, however, shows a statistically significant difference in practically all portfolio pairs.

Table 10 Classification parameters of the *t* distribution of the first 20 assets with major probability outperforming the SP500 portfolio between 2020 and 2024

USA Market													
Ticker	Probab.	Mean	Median	St. Dev.	Kurtosis	Skew.	Min.	Max.	xi	omega	Alpha	nu	KS
NFLX	0.775	-0.105	-1.000	0.995	-1.957	0.212	-1.000	1.000	-1.000	0.000	116604793.416	0.140	0.000
WBD	0.761	-0.045	-1.000	0.995	-1.989	0.089	-1.000	1.000	-1.000	0.000	578755197.323	0.134	0.000
OXY	0.756	-0.014	-1.000	1.000	-2.001	0.028	-1.000	1.000	-1.000	0.000	2780487509.792	0.132	0.000
COR	0.598	0.052	0.068	0.268	-0.109	-0.254	-0.780	0.792	0.291	0.360	-1.511	7528284.235	0.000
AZO	0.594	0.024	0.037	0.222	0.448	-0.261	-0.814	0.752	0.322	0.378	-4.183	18.504	0.000
L	0.594	0.024	0.029	0.138	1.829	-0.408	-0.790	0.494	0.101	0.139	-0.821	9.056	0.000
ETN	0.591	0.026	0.039	0.175	0.880	-0.421	-0.752	0.525	0.141	0.184	-1.010	8.685	0.000
MLM	0.591	0.019	0.020	0.150	0.516	-0.069	-0.646	0.528	0.224	0.265	-4.129	18.503	0.000
EME	0.591	0.051	0.059	0.265	0.843	-0.164	-0.967	0.856	0.225	0.300	-1.005	18.509	0.000
IBM	0.591	0.031	0.035	0.182	1.125	-0.292	-0.791	0.613	0.113	0.172	-0.639	7.531	0.000
JBL	0.589	0.028	0.038	0.203	0.521	-0.210	-0.657	0.625	0.293	0.348	-3.898	18.504	0.000
RSG	0.587	0.024	0.033	0.172	0.878	-0.401	-0.666	0.609	0.142	0.186	-1.069	9.576	0.000
APA	0.584	0.021	0.032	0.233	0.788	-0.226	-0.944	0.915	0.327	0.393	-4.080	18.498	0.000
ARE	0.583	0.032	0.039	0.275	0.338	-0.047	-0.893	0.921	0.391	0.471	-3.809	18.501	0.000
TSN	0.582	0.015	0.032	0.219	0.824	-0.443	-0.937	0.625	0.301	0.364	-4.025	18.506	0.000
EXPD	0.581	0.016	0.025	0.175	1.335	-0.335	-0.873	0.569	0.241	0.292	-3.903	18.500	0.000
CBRE	0.579	0.015	0.024	0.177	0.885	-0.304	-0.831	0.592	0.248	0.302	-4.035	18.504	0.000
LLY	0.578	0.043	0.048	0.276	0.404	-0.184	-0.882	0.906	0.192	0.291	-0.761	14.421	0.000
CHD	0.578	0.009	0.016	0.133	1.106	-0.419	-0.628	0.441	0.185	0.226	-4.308	18.501	0.000
ED	0.577	0.012	0.020	0.155	0.900	-0.274	-0.617	0.525	0.215	0.263	-4.097	18.499	0.000

Note: Parameters Probability of outperformance the proxy market (Prob.), mean, median, standard-deviation (S.D.), kurtosis, skewness (Skew), minimum (Min.) and maximum (Max.) observation, the parameter for the location of the distribution peak (xi), the scale that defines the width of the distribution (omega), the skewness parameter (alpha), the number of degrees of freedom, which define the shape of the tails (nu), and the Kolmogorov-Smirnov (KS) normality test.

Table 11: Transaction Cost considering turnover's portfolio and BRICS market frictions between 2020 and 2024

Brazil Market									
Variable	IBRX50	MARKOWITZ	SHARPE	MF_EQ	MF_MKW	MF_SHARPE	ANNt_EQ	ANNt_MKW	ANNt_SHARPE
$RCum$	6.259	19.76	6.08	3.84	16.75	89.84	-2.68	9.03	-0.70
$Turnover$	0.00	0.16	0.43	0.14	0.44	0.28	0.41	0.59	0.77
$RCum\ Cost$	6.259	19.40	4.97	3.53	16.08	87.65	-3.32	8.18	-2.54
Russian Market									
Variable	IMOEX	MARKOWITZ	SHARPE	MF_EQ	MF_MKW	MF_SHARPE	ANNt_EQ	ANNt_MKW	ANNt_SHARPE
$RCum$	0.491	23.83	-4.76	9.69	24.28	-12.77	5.71	26.94	-3.42
$Turnover$	0.00	0.18	0.44	0.04	0.36	0.53	0.18	0.39	0.58
$RCum\ Cost$	0.491	23.22	-5.32	9.60	23.35	-14.11	5.39	25.80	-4.31
Indian Market									
Variable	NIFTY_50	MARKOWITZ	SHARPE	MF_EQ	MF_MKW	MF_SHARPE	ANNt_EQ	ANNt_MKW	ANNt_SHARPE
$RCum$	45.127	58.00	43.86	80.00	73.82	96.69	78.83	72.57	75.77
$Turnover$	0.00	0.20	0.41	0.13	0.30	0.32	0.30	0.52	0.56
$RCum\ Cost$	45.127	56.99	42.42	78.86	71.31	93.76	76.79	69.31	72.34
China Market									
Variable	SSE50	MARKOWITZ	SHARPE	MF_EQ	MF_MKW	MF_SHARPE	ANNt_EQ	ANNt_MKW	ANNt_SHARPE
$RCum$	2.7185	64.28	-8.88	-9.99	-1.50	-34.06	11.74	43.38	-0.86
$Turnover$	0.00	0.26	0.50	0.01	0.28	0.68	0.27	0.44	0.43
$RCum\ Cost$	2.7185	62.89	-10.81	-10.02	-1.91	-37.06	11.4	41.47	-2.71
South African Market									
Variable	FTSE/JSE40	MARKOWITZ	SHARPE	MF_EQ	MF_MKW	MF_SHARPE	ANNt_EQ	ANNt_MKW	ANNt_SHARPE
$RCum$	19.132	25.03	12.33	25.67	14.05	-14.21	29.86	28.75	4.09
$Turnover$	0.00	0.21	0.43	0.09	0.27	0.74	0.26	0.46	0.59
$RCum\ Cost$	19.132	24.59	11.73	25.32	13.65	-17.24	29.04	27.51	3.23

Note: $RCum$ is the Return Cumulative without Cost Transactions, $Turnover$ is the mean of turnover of the portfolio rebalancing. $RCum\ Cost$ is the Return Cumulative discounted of the transaction cost. It is obtained with the $RCum$ discounted with the difference of the specific turnover of each portfolio rebalancing multiplied by the cost of transaction, considering the market frictions. The values considered was 5% to buy and 5% to sell (10% total) due to the friction market and the spread bid-ask.

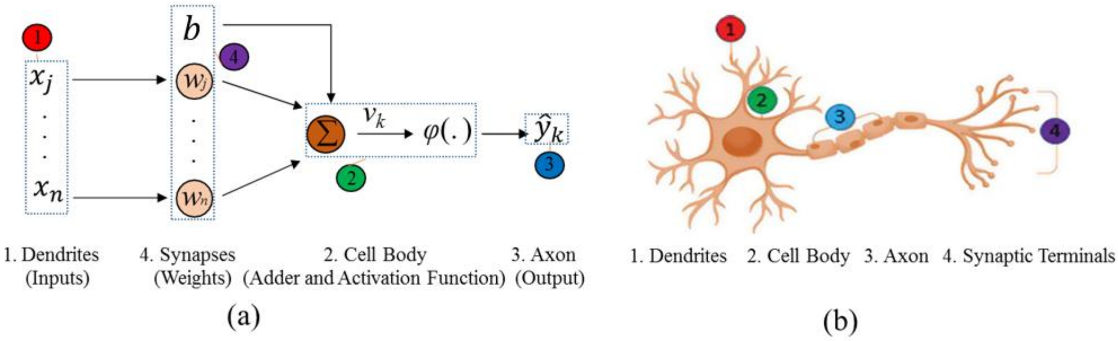
Table 12: Transaction Cost considering turnover's portfolio and U.S.A. market frictions between 2020 and 2024

U.S.A. Market									
Variable	SP500	MARKOWITZ	SHARPE	MF_EQ	MF_MKW	MF_SHARPE	ANNt_EQ	ANNt_MKW	ANNt_SHARPE
$RCum$	41.6325	12.23	10.39	26.11	24.89	13.27	33.62	29.79	-0.96
$Turnover$	0.00	0.31	0.50	0.30	0.38	0.84	0.66	0.87	0.82
$RCum\ Cost$	41.6325	12.09	9.99	25.78	24.53	11.48	32.97	28.43	-2.03

Note: $RCum$ is the Return Cumulative without Cost Transactions, $Turnover$ is the mean of turnover of the portfolio rebalancing. $RCum\ Cost$ is the Return Cumulative discounted of the transaction cost. It is obtained with the $RCum$ discounted with the difference of the specific turnover of each portfolio rebalancing multiplied by the cost of transaction, considering the market frictions. The values considered was 2% to buy and 2% to sell (4% total) due to bid-ask spread.

Figures

Figure 1: Artificial Neuron x Organic Neuron



Note: 1(a) displays the analogous components of the organic neuron; 1(b) in an artificial neuron, adapted from (de Oliveira et al., 2024). The inputs x_j to x_n are the dendrites, the bias b and weights w_j to w_n are the synaptic terminals, the adder (Σ), the activation function ($\varphi(.)$), the cell body, and the output (the axon ($\hat{y}_k$)).

Figure 2: Demonstration of the training and testing periods of the 20 rebalancings, carried out quarterly during the first 5 years of the 20s of the 21st century

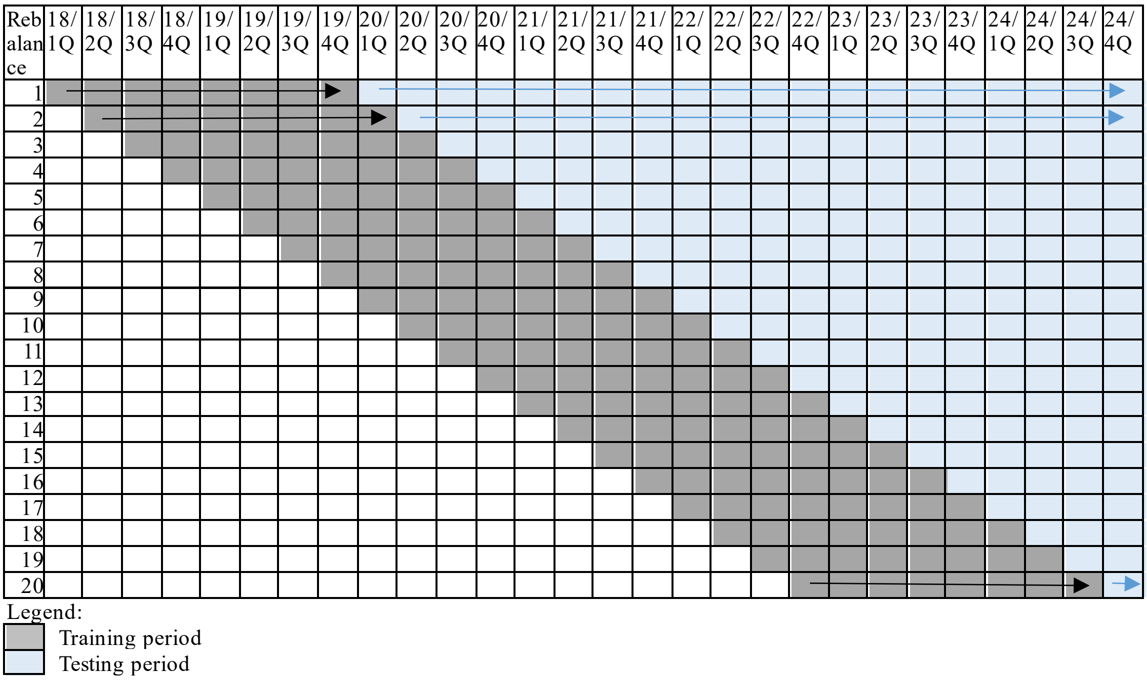
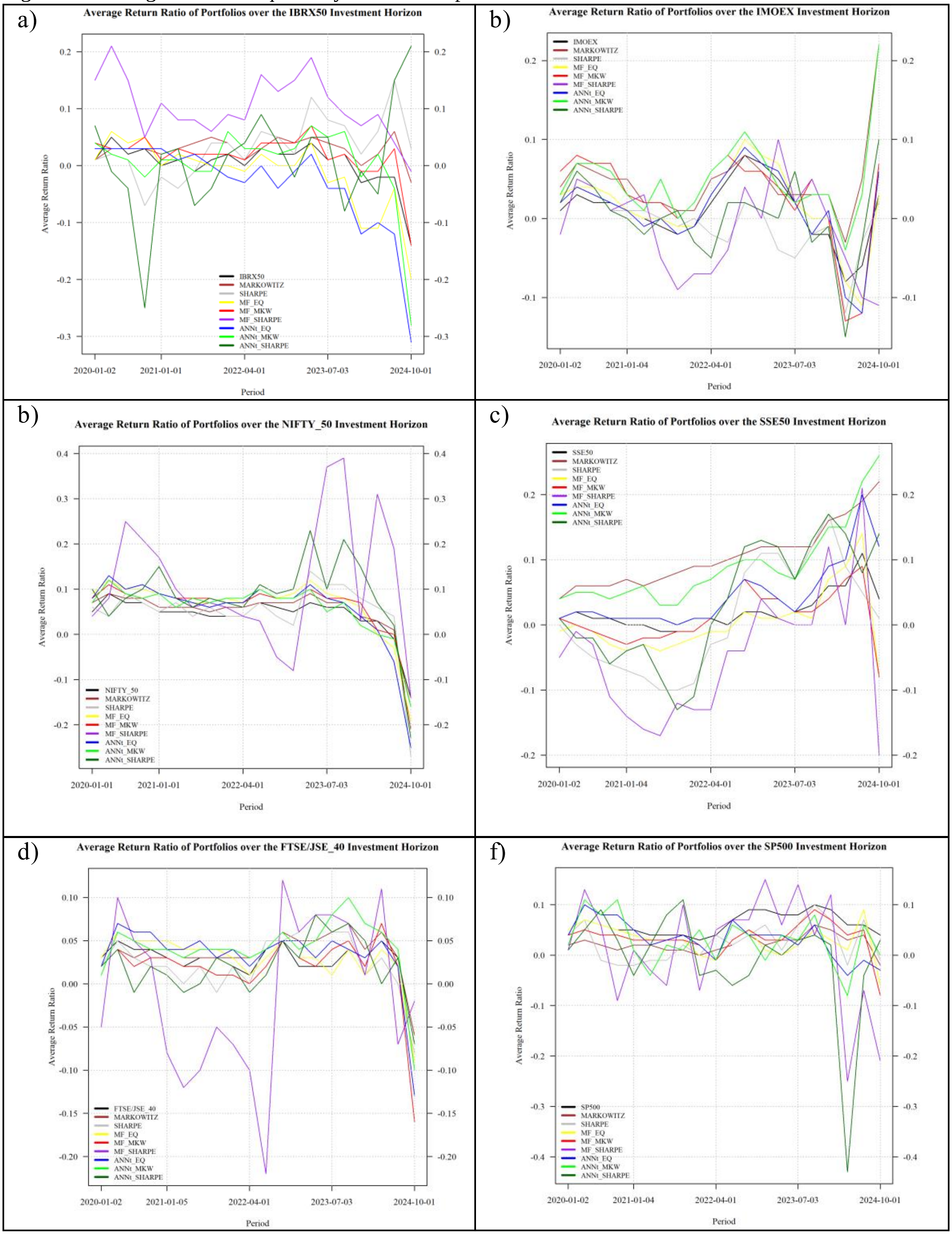
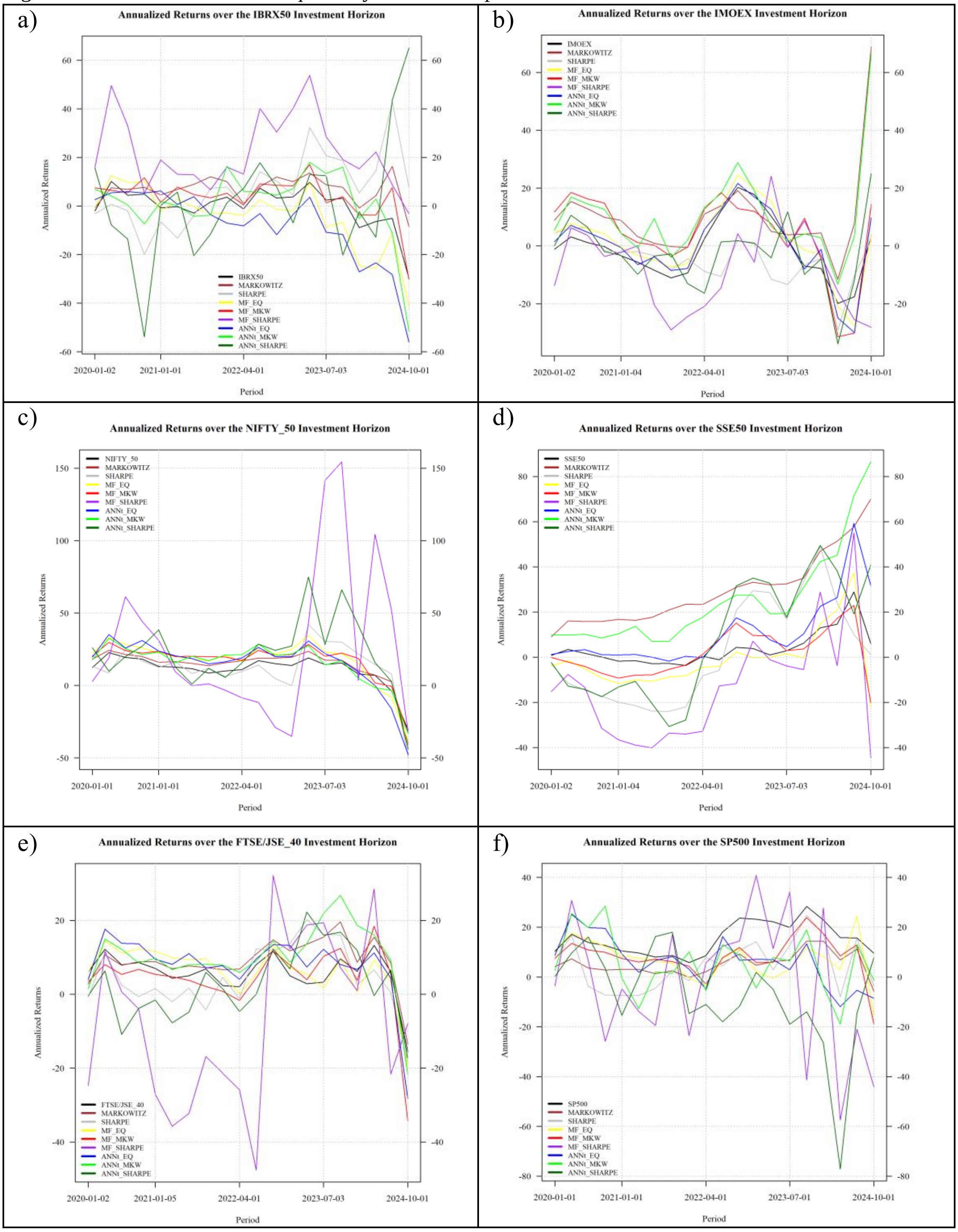


Figure 3: Average returns of quarterly rebalanced portfolios between 2020 and 2024



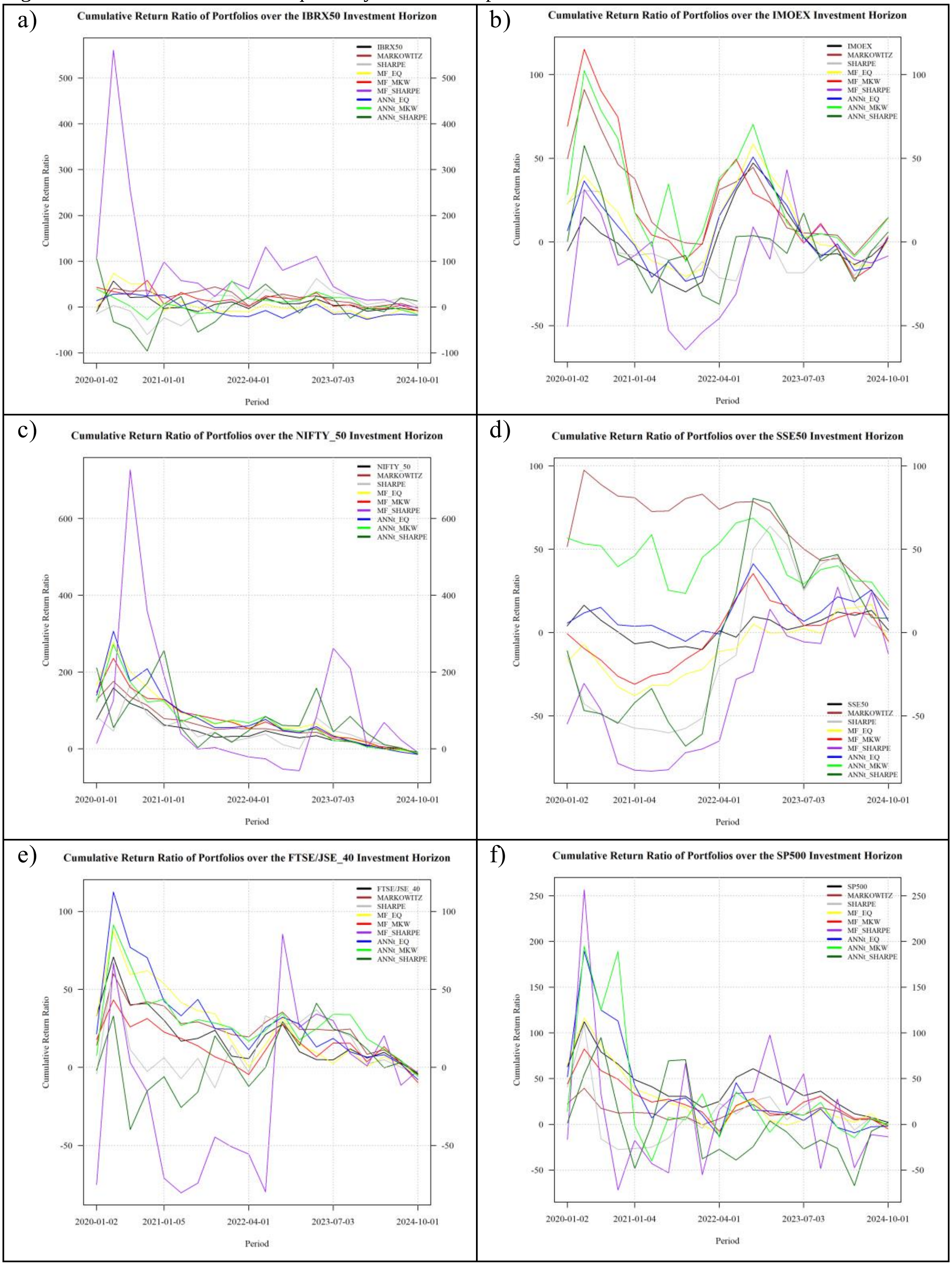
Note: Figure 3a analyzes the Brazilian market, where the MF_SHARPE portfolio reports the highest average returns in 19 of the 20 periods analyzed, presenting a negative return only in the short term of one quarter. Figure 3b analyzes the Russian market, where the ANNt_MKW portfolio has the best average return in 9 of the 20 quarters analyzed and also the highest average return, which occurs in the short term. Figure 3c analyzes the Indian market, where the MF_SHARPE portfolio oscillates with 17 positive and 3 negative average returns. Figure 3d analyzes the Chinese market, where the MARKOWITZ portfolio performs best in 13 periods, especially in the long term. Figure 3e analyzes the South African market, where the ANNt_MKW portfolio has the highest average return over 6 quarters, being the one that describes the highest frequency of best results, with good stability. Figure 3f analyzes the U.S. market. The SP500 portfolio presents a higher average return in 8 of the 20 rebalancings.

Figure 4: Annualized returns of quarterly rebalanced portfolios between 2020 and 2024



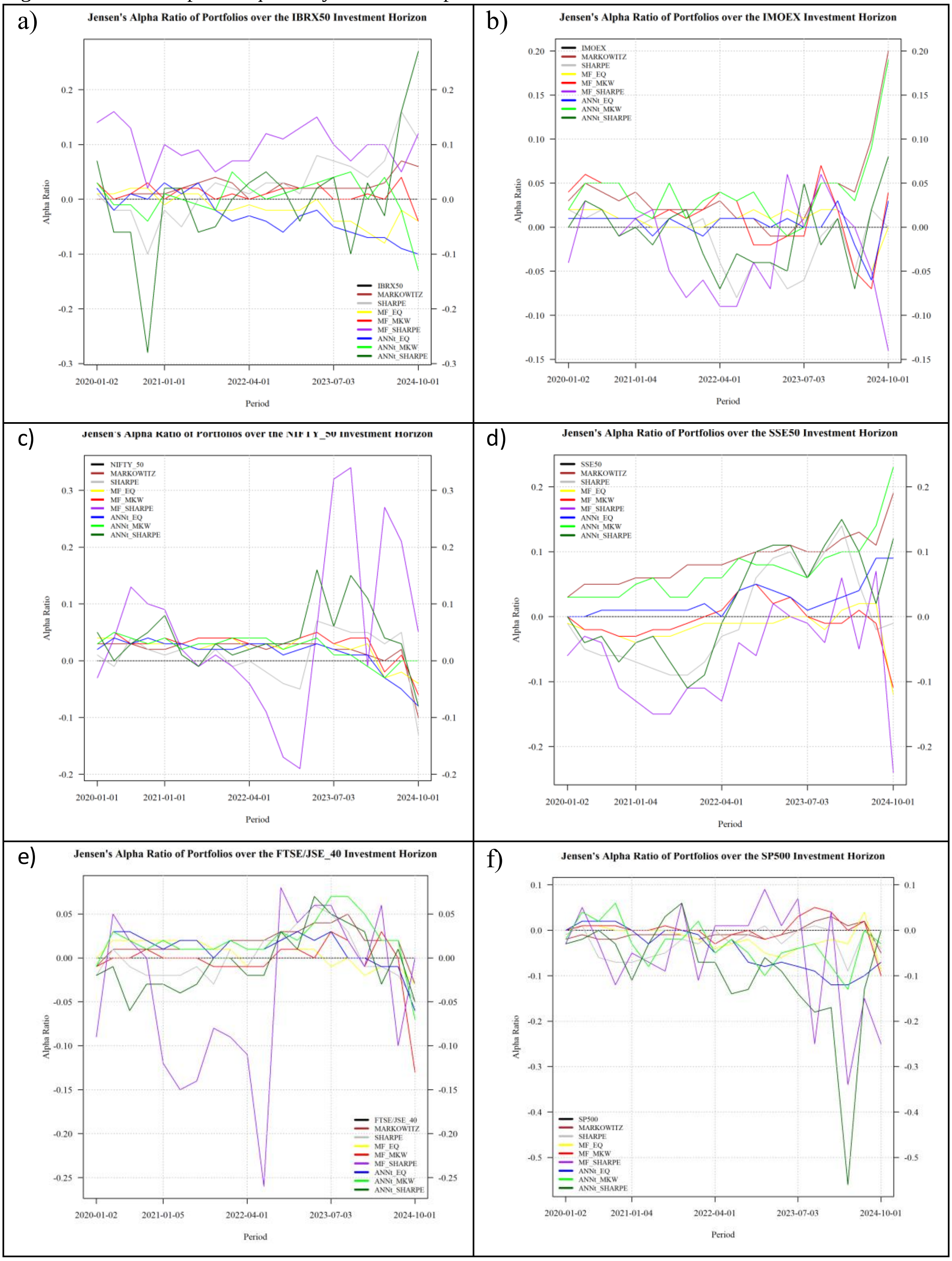
Note: Figure 4a analyzes the Brazilian market, where the MF_SHARPE portfolio reports the highest outperformance among the portfolios in 14 quarters of the 20 rebalanced. Figure 4b analyzes the Russian market, where the ANNt_MKW portfolio presents positive results in most of the series, with the exception of 3 rebalancings. Figure 4c analyzes the Indian market, where the MF_SHARPE portfolio presents an annualized return higher than the market proxy in 8 quarters of the 20 analyzed, holding the highest return observed. Figure 4d analyzes the Chinese market, where the MARKOWITZ portfolio presents the best annualized return in 12 quarters analyzed, being the best long-term portfolio option. Figure 4e analyzes the South African market, where the ANNt_MKW portfolio presents a higher annualized return in 5 quarters. Figure 4f analyzes the U.S. market. It has proven to be efficient in the long term. No other portfolio formation strategy can maintain a performance superior to the market itself, represented by the SP500.

Figure 5: Cumulative returns of quarterly rebalanced portfolios between 2020 and 2024



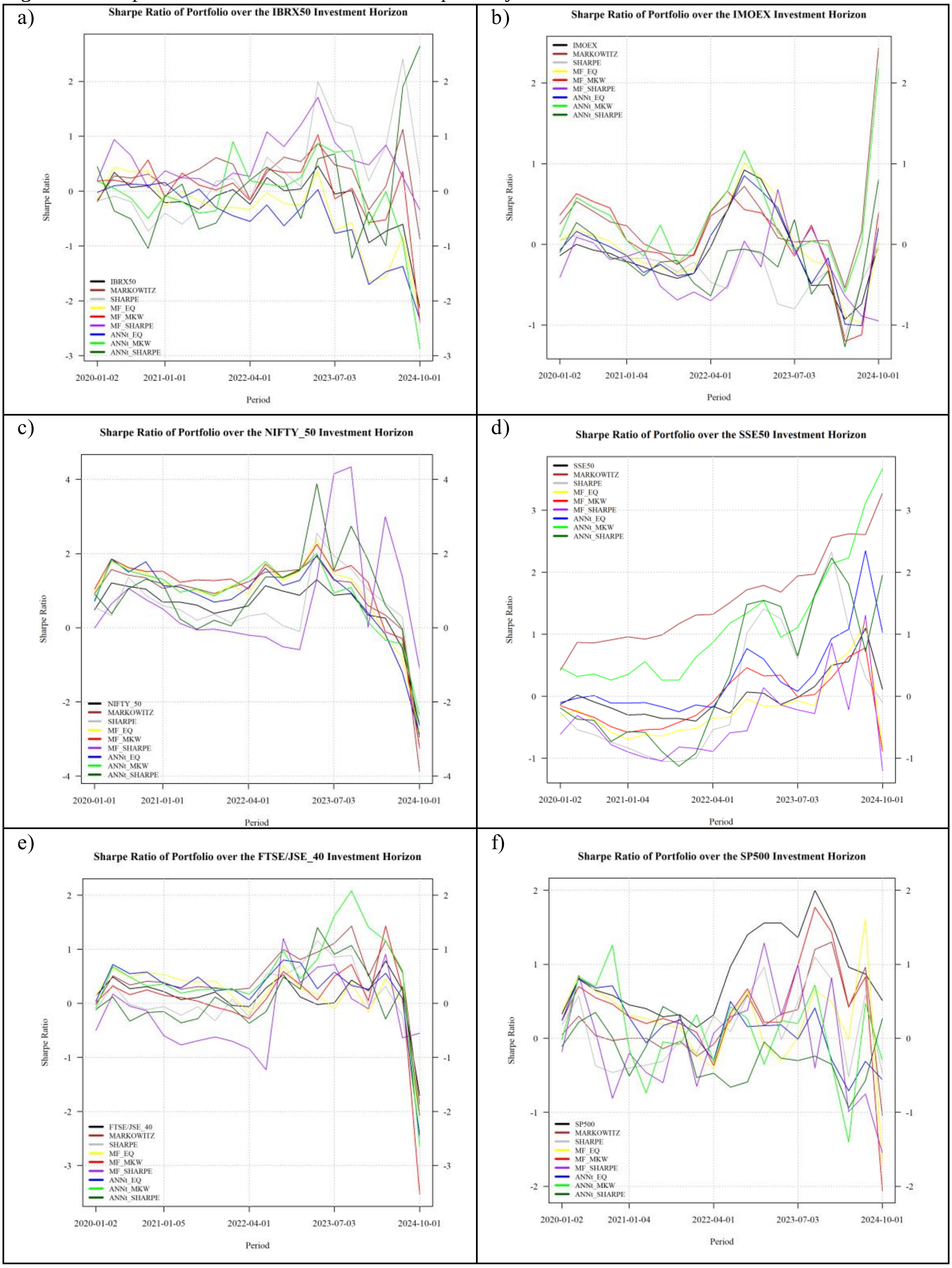
Note: Figure 5a analyzes the Brazilian market, where the MF_SHARPE portfolio is by far the best in cumulative return, being the highest value observed in 18 of the 20 periods analyzed. Figure 5b analyzes the Russian market, where the ANNt_MKW portfolio presents an outperformance in 6 quarters of rebalancing. Figure 5c analyzes the Indian market, where the MF_SHARPE portfolio has the highest observed value. Figure 5d analyzes the Chinese market, where the MARKOWITZ portfolio presents a superior cumulative return in 12 periods, being the best long-term investment option in the Chinese market. Figure 5e analyzes the South African market, where the ANNt_MKW portfolio presents the highest cumulative return in 4 quarters. Figure 5f analyzes the U.S. market, where the SP500 market presented a more stable and better performance.

Figure 6: Jensen's Alpha of quarterly rebalanced portfolios between 2020 and 2024



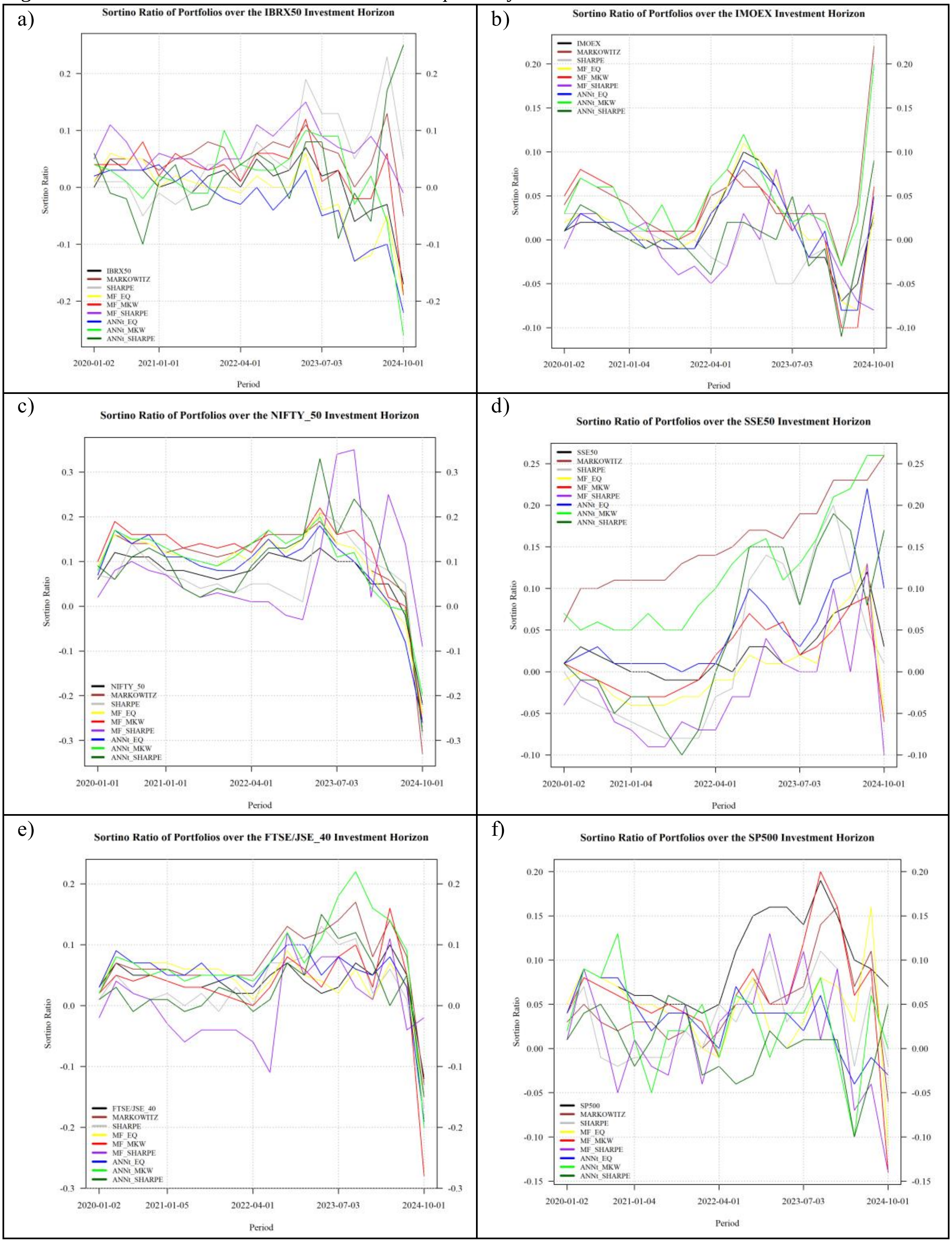
Note: Figure 6a analyzes the Brazilian market, where the MF_SHARPE portfolio has the highest Jensen's Alpha in 17 of the periods analyzed. Figure 6b analyzes the Russian market, where the ANNt_MKW portfolio provides 9 superior Alphas in the series of 20 quarters of the first half of the 20s, being the best strategy to be adopted in the Russian market. Figure 6c analyzes the Indian market, where the MF_SHARPE portfolio has a positive Jensen's Alpha in 13 periods. Figure 6d analyzes the Chinese market, where the MARKOWITZ portfolio has the highest Jensen's Alpha in 15 of the 20 quarters observed. Figure 6e analyzes the South African market, where the ANNt_MKW portfolio has the highest Jensen's Alpha in 7 quarters. Figure 6f analyzes the U.S. market, where the SP500 portfolio presents a Jensen's Alpha equal to zero because it represents the market itself.

Figure 7: Sharpe Ratio of Portfolios rebalanced quarterly between 2020 and 2024



Note: Figure 7a analyzes the Brazilian market, where the MF_SHARPE portfolio presents a positive Sharpe ratio in 19 rebalancings, which demonstrates that it is an excellent investment-optimizing tool. Figure 7b analyzes the Russian market, where the ANNt_MKW portfolio has a higher Sharpe ratio in 5 of the periods analyzed, all the others being very close to the highest value. Figure 7c analyzes the Indian market, where the MF_MKW portfolio has the highest Sharpe ratio in 7 quarters, mainly in the long term. Figure 7d analyzes the Chinese market, where the MARKOWITZ portfolio has the highest Sharpe ratio in 18 quarters. Figure 7e analyzes the South African market, where the ANNt_MKW portfolio has the highest Sharpe ratio in 4 quarters. Figure 7f analyzes the U.S. market. The SP500 itself presents the best Sharpe Ratio in most rebalancings, in 13 of the 20 portfolio rebalancings.

Figure 8: Sortino Ratio of Portfolios rebalanced quarterly between 2020 and 2024

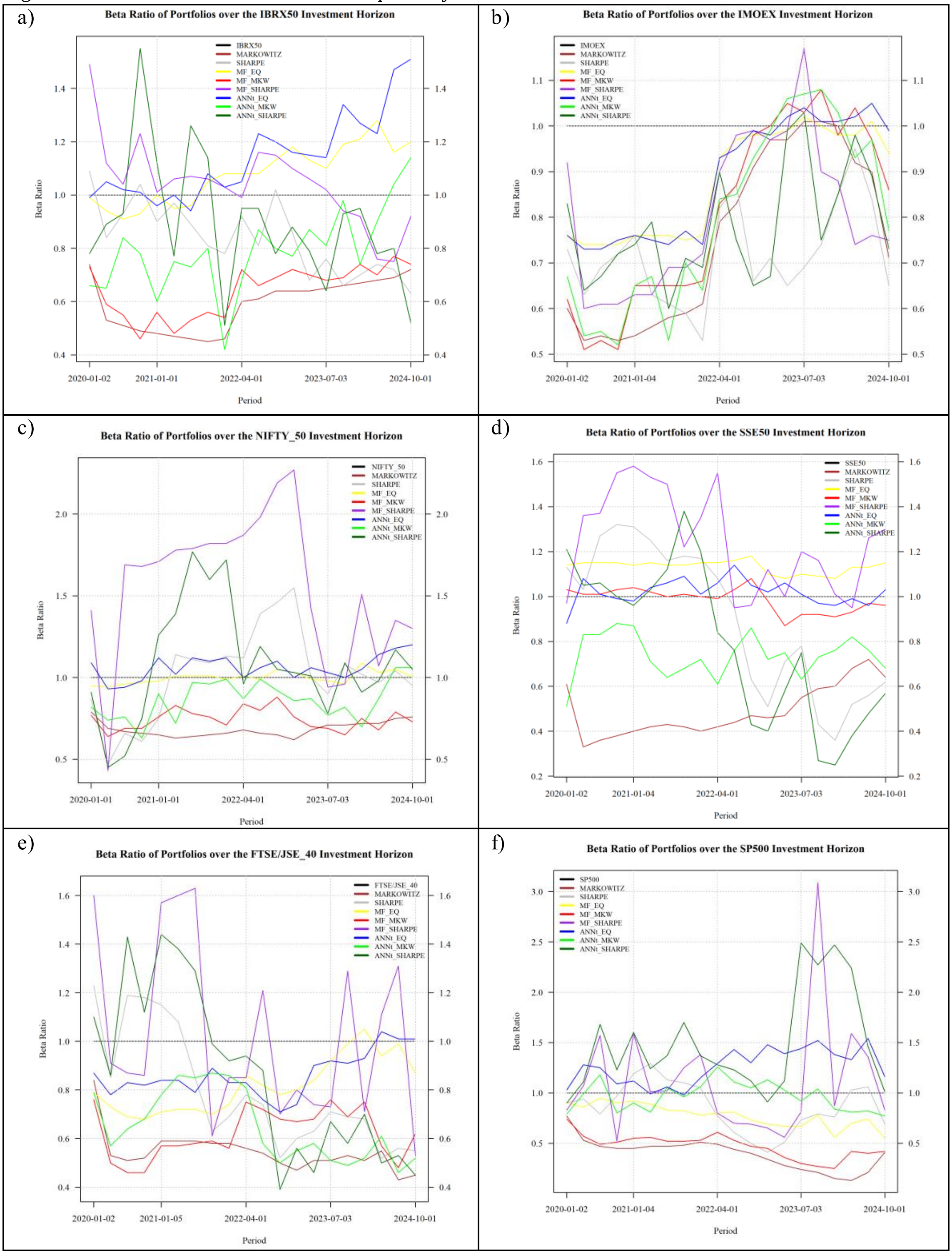


Note: Figure 8a analyzes the Brazilian market, where the MF_SHARPE portfolio presents the highest Sortino ratio in the 6 periods analyzed. Figure 8b analyzes the Russian market, where the ANNt_MKW portfolio produces the highest Sortino ratio in 7 periods, and in the others, it follows the highest value. Figure 8c analyzes the Indian market, where the MF_SHARPE portfolio has the highest Sortino ratio in 5 periods, especially in investments up to one year. Figure 8d analyzes the Chinese market, where the MARKOWITZ portfolio has the highest Sortino ratio in 18 quarters. Figure 8e analyzes the South African market, where the ANNt_MKW portfolio presents the highest Sortino ratio in 4 quarters, in investments of up to 1 year and 9 months. Figure 8f analyzes the U.S. market; the SP500 portfolio provided the highest Ratio values over the 20 quarters analyzed, in 11 of the rebalancings.

Figure 9: Treynor Ratio of Portfolios rebalanced quarterly between 2020 and 2024

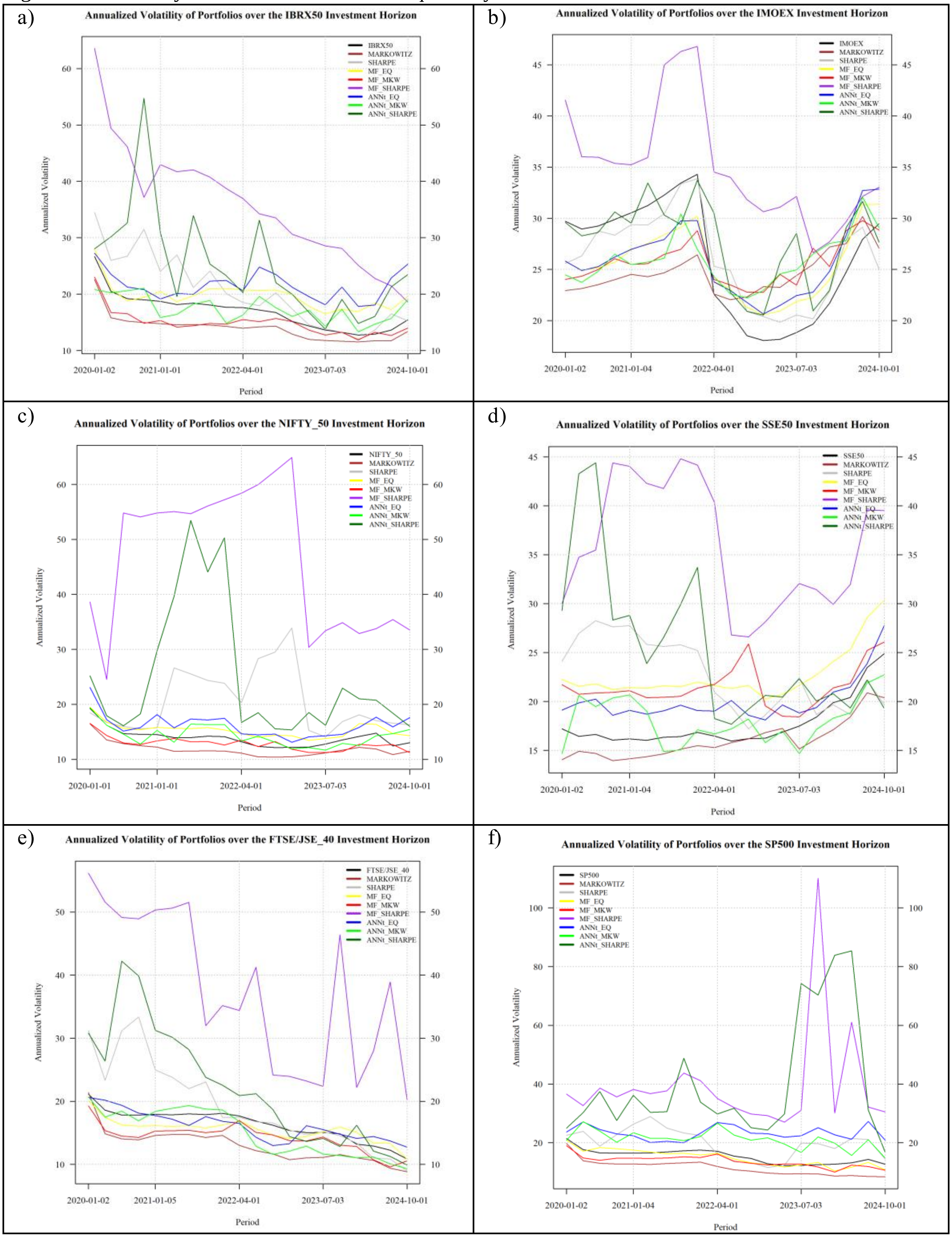
Note: Figure 9a analyzes the Brazilian market, where the MF_SHARPE portfolio is the highest performer in accumulating positive Treynor ratios, with 19 rebalancings. Figure 9b analyzes the Russian market, where the ANNt_MKW portfolio presents a Treynor ratio higher than the other portfolios in 5 moments, with the others being very close to the upper limit of the portfolios. Figure 9c analyzes the Indian market, where the MF_SHARPE portfolio presents the highest Treynor ratio in 7 periods. Figure 9d analyzes the Chinese market, where the MARKOWITZ portfolio has the highest Treynor ratio in 12 quarters. Figure 9e analyzes the South African market, where the ANNt_MKW portfolio has the highest performance in 7 quarters, which gives it the best performance of the portfolios analyzed. Figure 9f analyzes the U.S. market. The SP500 portfolio obtained the highest Treynor Ratio in 3 rebalancings: 2020/01/02, 2022/10/01, and 2024/10/01, and it was well distributed throughout the 5 years of analysis.

Figure 10: Beta of Portfolios rebalanced quarterly between 2020 and 2024



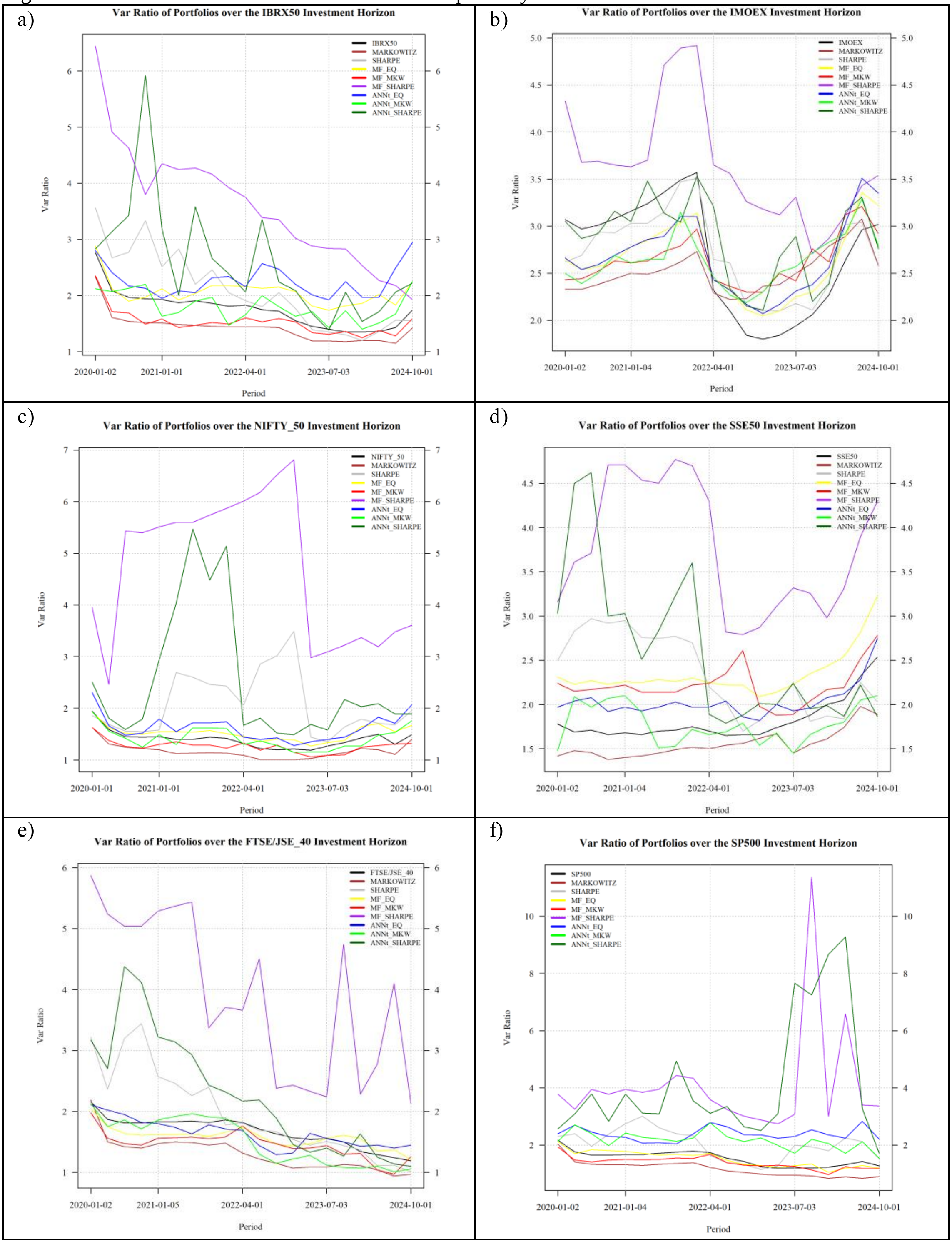
Note: Figure 10a analyzes the Brazilian market, where the MF_SHARPE portfolio is leveraged in 14 rebalancings, being of lower risk in the short term. Figure 10b analyzes the Russian market, where the ANNt_MKW portfolio is volatile, but with values that are mostly lower than the market. Figure 10c analyzes the Indian market, where the MF_SHARPE portfolio has the most leveraged Beta in the series, exceeding 2.0, and also the least leveraged, with a value less than 0.5. In 17 periods, it is leveraged, and in 3, it is unlevered. Figure 10d analyzes the Chinese market, where the MARKOWITZ portfolio has the lowest exposure to systemic risk, with the lowest Beta values in 10 quarters. Figure 10e analyzes the South African market, where the ANNt_MKW portfolio presents unlevered Betas in 20 quarters, with the lowest in the portfolio invested for 1 year and 3 months. Figure 10f analyzes the U.S. market. The Beta of the SP500 portfolio is 1, being the proxy of the U.S. market.

Figure 11: Volatility of Portfolios rebalanced quarterly between 2020 and 2024



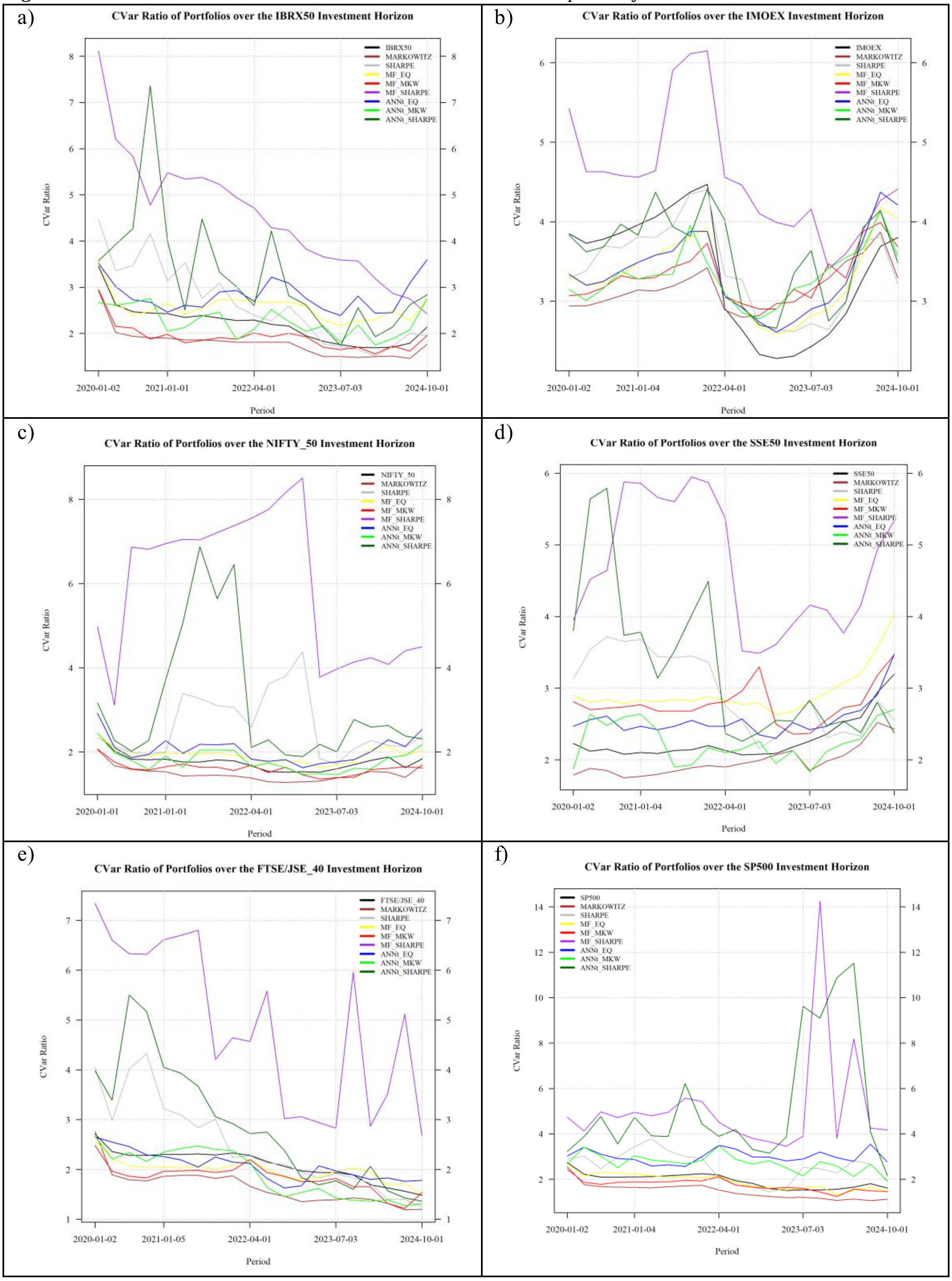
Note: Figure 11a analyzes the Brazilian market, where the MF_SHARPE portfolio is the most volatile of all, with a value greater than 60 in the 5-year investment period. Figure 11b analyzes the Russian market, where the ANNt_MKW portfolio is midway between the lowest and highest volatility portfolios. Figure 11c analyzes the Indian market, where the MF_SHARPE portfolio is the most volatile of all series, with values exceeding 60. Figure 11d analyzes the Chinese market, where the MARKOWITZ portfolio is the least volatile portfolio in 16 quarters and, therefore, the one with the lowest volatility in the entire series. Figure 11e analyzes the South African market, where the ANNt_MKW portfolio has the lowest volatility in 3 quarters. Figure 11f analyzes the U.S. market. The SP500 has low volatility, slightly above MARKOWITZ's portfolio.

Figure 12: Value at Risk of Portfolios rebalanced quarterly between 2020 and 2024



Note: Figure 12a analyzes the Brazilian market, where the MF_SHARPE portfolio presents the highest Value at Risk in 17 periods, also presenting the highest Value at Risk of the entire series analyzed. Figure 12b analyzes the Russian market, where the ANNt_MKW portfolio has a sinusoidal curve, but it is smoother, with lower maximum and minimum points. Figure 12c analyzes the Indian market, where the MF_SHARPE portfolio has the highest Value at Risk in the entire series, in the 20 periods analyzed, with the peak reaching almost 7. Figure 12d analyzes the Chinese market, where the MARKOWITZ portfolio has the lowest Value at Risk in 17 quarters. Figure 12e analyzes the South African market, where the ANNt_MKW portfolio has the lowest Value at Risk in 3 quarters, with values that fluctuate between 2.1 and 1%. Figure 12f analyzes the U.S. market; the SP500 portfolio oscillates around 1.5.

Figure 13: Conditional Value at Risk of Portfolios rebalanced quarterly between 2020 and 2024



Note: Figure 13a analyzes the Brazilian market, where the MF_SHARPE portfolio has the highest Conditional Value at Risk in 17 of the 20 periods analyzed, including the highest in the entire series, with a value greater than 8, in 5 years of investment. Figure 13b analyzes the Russian market, where the ANNt_MKW portfolio has neither the worst nor the best value, fluctuating between the worst and the highest. Figure 13c analyzes the Indian market, where the MF_SHARPE portfolio is the riskiest portfolio of all, with a wide difference from the others. Figure 13d analyzes the Chinese market, where the MARKOWITZ portfolio presents the lowest Conditional Value at Risk in 16 quarters. Figure 13e analyzes the South African market, where the ANNt_MKW portfolio presents the lowest Conditional Value at Risk in 3 quarters of the series. Figure 13f analyzes the U.S. market. The SP500 portfolio now presents a CVar that oscillates around 2.

APPENDIX

Table 13 shows that the ANNt_SHARPE portfolio outperforms the other portfolios in the Brazilian market in terms of returns. Table 15 further indicates that the returns of ANNt_SHARPE are statistically different from those of the traditional SHARPE portfolio, thereby supporting Hypothesis 1. Figure 14a illustrates the annualized returns of the portfolios and confirms the outperformance of the ANNt_MKW and ANNt_SHARPE strategies.

The portfolio strategies also capture the highly unstable conditions of the Russian financial market during the war-economy period. In the Russian market, all portfolios record negative cumulative returns. Only the SHARPE and MF_SHARPE portfolios achieve average and annualized returns slightly above zero. The ANNt_SHARPE portfolio emerges as the most defensive strategy in this market, exhibiting the lowest risk levels. However, cumulative returns are not statistically different across portfolios, while only the risk levels of the ANNt_EQ and ANNt_MKW portfolios differ significantly from the IMOEX market index. Figure 14b illustrates the annualized returns in the Russian market and reflects this unstable market environment.

In the Indian market, ANNt_SHARPE outperforms the other strategies across all return metrics, while the MARKOWITZ portfolio exhibits the lowest risk. The ANNt_SHARPE strategy therefore performs strongly, although its return advantage over the traditional SHARPE portfolio is statistically significant only at the 10% level. In terms of risk, however, the ANNt_SHARPE portfolio differs significantly from almost all other portfolios. Figure 14c illustrates the strong performance of the ANNt portfolios in the Indian market.

The Chinese market continues to exhibit a distinct pattern. During the 2025 window, the SHARPE portfolio achieves the highest returns, the highest Jensen's Alpha, and the lowest Beta. However, the statistical evidence in Table 15 indicates that no portfolio differs significantly from the others in terms of cumulative returns. For volatility, only a limited number of portfolio pairs show statistically significant differences at the 5% level. Figure 14d illustrates the annualized returns of the portfolios in the Chinese market.

In the South African market, MF_SHARPE appears to be the best-performing investment strategy in terms of returns, but this performance is accompanied by extreme volatility. Cumulative returns differ significantly from the market only for the MARKOWITZ, MF_EQ, and ANNt_MKW portfolios. In contrast, volatility does not differ significantly across portfolios. Figure 14e illustrates the outperformance of the MF_SHARPE portfolio in South Africa, as well as its substantial volatility. The result also shows that the strategy is highly concentrated, since only one asset receives a non-zero weight, implying limited diversification and a high degree of capital concentration.

In the U.S. market, Table 14 and Figure 14f show a different pattern in 2025 compared with the 2020–2024 period. Unlike the earlier sample, the U.S. market index is no longer the dominant source of returns, while the ANNt portfolios generate substantially higher returns and positive alpha. This suggests that the ANNt model captures changes in stock-price dynamics during this specific period, when the U.S. market displayed behavior more similar to less efficient and more volatile market environments. Figure 14f illustrates the outperformance of the ANNt_SHARPE and MF_SHARPE portfolios in the U.S. market.

Table 16 further shows that the cumulative returns of the ANNt_SHARPE portfolio differ significantly from those of the U.S. market index and the MARKOWITZ portfolio at the 10% significance level, and from the traditional SHARPE portfolio at the 5% level. A similar pattern is observed for the ANNt_EQ portfolio. To make the data and methodology transparent, the results can be replicated with the use of the command Investment_Horizon() of the ANNt package, with the script:

```
Investment_Horizon(Tickers = c('Current_SP500_Tickers'),
                   RM = c('^GSPC'),
                   Rf = 0,
```

```
Initial_Date = c('2018-01-03'),
Final_Date_Training = c('2019-12-31'),
Final_Date = '2025-12-31',
Frequency = 2,
Periodicity = c('daily'),
Hidden = '',
Stepmax = 2000,
Asymmetry = 'Positive',
Type_ANNt = 'T4',
N_Assets = 20,
Base = 'yahoo',
Fun = 'Out',
Specific_Dates = c( '2025-09-30','2025-06-30',
                    '2025-03-31','2024-12-31'),
Type_ANN = 'ANNt',
Import = 'No',
Download = 'Yes',
Excluded_ticket = c(‘Q’, ‘SNDK’),
Skew_t = 'No'
)
```

The script uses the USA market data. The processing time is 2 days in a conventional computer (i7 processor with 20Gb of memory). For the BRICS market, it is necessary to modify the data in the command. The same command can be used to reproduce the analysis from 2020 to 2024.

Table 13: Average of parameters in BRICS markets (best performances in bold) in 2025

Brazilian Market									
Ratio	IBRX50	MARKOWITZ	SHARPE	MF_EQ	MF_MKW	MF_SHARPE	ANNt_EQ	ANNt_MKW	ANNt_SHARPE
$\bar{r}_p$	0.128	0.115	0.070	0.100	0.140	0.055	0.053	0.158	**0.173**
r_p^A	35.823	33.588	18.085	26.565	42.980	13.538	11.443	48.668	**51.358**
r_p^C	20.105	15.180	9.753	14.548	20.718	1.820	9.738	23.635	**26.885**
α	0.000	0.040	-0.033	-0.038	**0.063**	-0.078	-0.128	0.040	0.018
S	2.415	3.0275	1.245	1.5525	**3.375**	0.6175	0.4775	2.805	2.145
$Sort$	0.193	0.258	0.108	0.128	**0.295**	0.058	0.048	0.228	0.165
Tr	0.128	0.195	0.085	0.093	**0.230**	0.038	0.035	0.175	0.143
β	1.000	**0.593**	0.808	1.090	0.633	1.010	1.428	0.940	1.270
σ_p^A	14.805	**11.058**	14.405	17.245	12.748	23.303	23.660	17.645	24.810
$VaR_{1-\gamma}$	1.408	**1.035**	1.420	1.688	1.180	2.365	2.398	1.673	2.398
$CVaR_{1-\gamma}$	1.798	**1.323**	1.800	2.143	1.518	2.975	3.023	2.135	3.045
Russian Market									
Ratio	IMOEX	MARKOWITZ	SHARPE	MF_EQ	MF_MKW	MF_SHARPE	ANNt_EQ	ANNt_MKW	ANNt_SHARPE
$\bar{r}_p$	-0.020	-0.043	0.020	-0.055	-0.095	**0.023**	-0.058	-0.083	-0.040
r_p^A	-7.125	-12.505	**3.178**	-16.573	-24.355	2.718	-16.258	-20.708	-11.450
r_p^C	-4.918	-4.730	**-2.273**	-9.970	-12.753	-3.635	-9.220	-7.870	-7.035
α	0.000	-0.033	0.033	-0.045	-0.083	**0.035**	-0.043	-0.068	-0.025
S	-0.303	-0.448	0.158	-0.615	-0.890	**0.243**	-0.615	-0.750	-0.408
$Sort$	-0.020	-0.030	0.023	-0.043	-0.070	**0.030**	-0.043	-0.058	-0.028

Tr	-0.020	-0.045	0.025	-0.073	-0.110	**0.040**	-0.070	-0.088	-0.048
β	1.000	0.818	0.705	0.848	0.918	0.768	0.850	0.865	**0.540**
σ_p^A	23.663	25.233	25.870	27.175	28.448	33.315	26.750	27.108	**22.973**
$VaR_{1-\gamma}$	2.473	2.658	2.660	2.873	3.043	3.428	2.828	2.893	**2.423**
$CVaR_{1-\gamma}$	3.093	3.323	3.343	3.593	3.790	4.308	3.533	3.608	**3.025**
Indian Market									
Ratio	NIFTY_50	MARKOWITZ	SHARPE	MF_EQ	MF_MKW	MF_SHARPE	ANNt_EQ	ANNt_MKW	ANNt_SHARPE
$\bar{r}_p$	0.048	0.025	0.008	0.053	0.043	-0.173	0.065	0.060	**0.100**
r_p^A	12.363	6.493	1.840	13.915	11.148	-36.833	16.840	15.938	**28.060**
r_p^C	6.380	3.548	-1.525	8.543	6.295	-26.840	10.238	9.968	**16.050**
α	0.000	-0.013	-0.038	0.005	0.005	-0.243	0.015	0.015	**0.060**
S	1.365	0.745	0.250	1.273	1.210	-1.133	1.470	1.478	**1.863**
$Sort$	0.138	0.075	0.025	0.123	0.118	-0.100	0.138	0.140	**0.168**
Tr	0.048	0.030	0.013	0.050	0.050	-0.130	0.060	0.063	**0.118**
β	1.000	**0.800**	0.995	1.023	0.853	1.383	1.070	0.973	0.935
σ_p^A	9.728	**8.905**	13.318	11.075	9.688	31.970	11.743	10.925	15.165
$VaR_{1-\gamma}$	0.958	**0.895**	1.373	1.098	0.960	3.488	1.155	1.073	1.470
$CVaR_{1-\gamma}$	1.218	**1.130**	1.723	1.385	1.215	4.328	1.460	1.360	1.870
Chinese Market									
Ratio	SSE50	MARKOWITZ	SHARPE	MF_EQ	MF_MKW	MF_SHARPE	ANNt_EQ	ANNt_MKW	ANNt_SHARPE
$\bar{r}_p$	0.078	0.085	**0.128**	0.078	0.040	0.075	0.065	0.088	0.093
r_p^A	20.975	22.9325	**36.6925**	21.16	9.6475	16.0925	16.485	24.005	25.055
r_p^C	12.983	13.168	**20.063**	12.633	5.445	16.988	9.298	14.483	15.518
α	0.000	0.035	**0.095**	-0.003	-0.013	-0.015	-0.003	0.028	0.070
S	1.685	2.095	**2.233**	1.428	0.815	0.628	1.350	1.885	1.633
$Sort$	0.133	0.178	**0.190**	0.120	0.078	0.063	0.118	0.165	0.150
Tr	0.078	0.140	**0.278**	0.075	0.060	0.038	0.078	0.115	0.335
β	1.000	0.623	**0.558**	1.043	0.650	1.600	0.855	0.755	0.313
σ_p^A	12.640	**11.068**	16.293	15.035	12.118	36.733	12.633	12.630	15.260
$VaR_{1-\gamma}$	1.230	**1.063**	1.560	1.483	1.215	3.733	1.245	1.220	1.485
$CVaR_{1-\gamma}$	1.568	**1.353**	1.988	1.875	1.533	4.698	1.580	1.553	1.890
South-African Market									
Ratio	FTSE/JSE40	MARKOWITZ	SHARPE	MF_EQ	MF_MKW	MF_SHARPE	ANNt_EQ	ANNt_MKW	ANNt_SHARPE
$\bar{r}_p$	0.145	0.060	0.103	0.105	0.043	**0.278**	0.090	0.048	0.083
r_p^A	42.478	-1.610	11.723	7.145	-40.938	**111.463**	11.890	-20.065	19.153
r_p^C	25.955	-5.988	-2.653	-2.303	-41.063	-571.005	**-0.488**	-22.180	-33.878
α	0.000	0.003	0.025	0.000	-0.028	**0.108**	0.000	-0.018	0.018
S	**2.688**	0.843	1.973	1.095	0.028	0.835	1.380	0.485	1.435
$Sort$	**0.210**	0.095	0.185	0.113	0.035	0.123	0.128	0.070	0.148
Tr	0.145	0.168	0.183	0.150	0.093	0.115	0.143	0.130	**0.338**
β	1.000	**0.388**	0.538	0.708	0.450	1.195	0.635	0.435	0.453
σ_p^A	16.043	44.108	**37.993**	49.173	76.573	203.223	39.508	60.338	56.460
$VaR_{1-\gamma}$	1.518	4.513	3.835	4.995	7.893	20.778	**4.008**	6.208	5.768
$CVaR_{1-\gamma}$	1.940	5.670	4.835	6.288	9.910	26.125	**5.045**	7.793	7.255

Note: $\bar{r}_p$ is the Average Return; r_p^A is the Annualized Return; r_p^C is the Accumulated Return; σ_p^A is the Annualized Volatility; $VaR_{1-\gamma}$ is the Value at Risk; $CVaR_{1-\gamma}$ is the Conditional Value at Risk; Sp is the Sharpe Ratio; $Sort$ is the Sortino Index; β is Beta; and α is Jensen's Alpha. The highest $\bar{r}_p$ and r_p^A is the MF_SHARPE portfolio, and the highest r_p^C is in the Brazilian Market, with ANNt_SHARPE. The best α are those of the MF_SHARPE with 0.108 in the South African market. The highest S is from the MF_MKW portfolio, with 3.375% in the Brazilian Market. The top Sort is also the MF_MKW portfolio in the Brazilian Market, with 0.295%. The best Tr is that of the ANNt_SHARPE portfolio with 0.338% in the South African Market. The lowest β is that of the MARKOWITZ portfolio, with a value of 0.388 in the South African Market. The portfolio with the lowest σ_p^A is the MARKOWITZ portfolio with a value of 8.905% in the Indian Market. The smallest $VaR_{1-\gamma}$ and $CVaR_{1-\gamma}$ is the MARKOWITZ Portfolio, with 0.895 and 1.353%, respectively, also in the Indian Market.

Table 14: Average of parameters in the U.S. market (best performances in bold) in 2025

U.S. Market									
Ratio	S&P500	MARKOWITZ	SHARPE	MF_EQ	MF_MKW	MF_SHARPE	ANNt_EQ	ANNt_MKW	ANNt_SHARPE
$\bar{r}_p$	0.075	0.060	0.028	0.113	0.073	0.183	0.230	0.205	**0.255**
r_p^A	19.845	15.238	6.975	31.580	19.463	53.758	72.470	63.150	**77.695**
r_p^C	12.540	8.830	6.603	19.040	13.510	21.188	34.620	30.713	**37.838**
α	0.000	0.033	-0.005	0.045	0.035	0.060	0.093	0.090	**0.100**
S	1.348	1.708	0.410	1.928	1.495	1.160	2.368	**2.445**	1.990
$Sort$	0.118	0.155	0.038	0.168	0.138	0.110	0.175	**0.190**	0.155
Tr	0.075	**0.195**	0.040	0.123	0.150	0.118	0.123	0.135	0.120
β	1.000	**0.330**	0.485	0.908	0.463	1.678	1.868	1.510	2.085
σ_p^A	15.363	**9.630**	12.303	16.850	12.580	46.360	32.123	27.393	40.058
$VaR_{1-\gamma}$	1.515	**0.943**	1.250	1.633	1.230	4.623	3.098	2.635	3.895
$CVaR_{1-\gamma}$	1.920	**1.195**	1.573	2.075	1.563	5.840	3.940	3.355	4.950

Note: $\bar{r}_p$ is the Average Return; r_p^A is the Annualized Return; r_p^C is the Accumulated Return; σ_p^A is the Annualized Volatility; $VaR_{1-\gamma}$ is the Value at Risk; $CVaR_{1-\gamma}$ is the Conditional Value at Risk; Sp is the Sharpe Ratio; $Sort$ is the Sortino Index; β is Beta; and α is Jensen's Alpha. The highest $\bar{r}_p$, r_p^A, and r_p^C is the ANNt_SHARPE, with 0.255% p.d., 77.695% p.a., and 37,838% p.p. The best α are those of the ANNt_SHARPE with 0.100. The highest S is from the ANNt_MKW, with 2.445% in the USA Market. The top Sort is also the ANNt_MKW portfolio, with 0.190%. The best Tr is that the MARKOWITZ portfolio with 0.195% in the USA Market. The lowest β is that of the MARKOWITZ portfolio, with a value of 0.330 in the USA Market. The portfolio with the lowest σ_p^A is the MARKOWITZ with a value of 9.630% in the USA Market. The smallest $VaR_{1-\gamma}$ and $CVaR_{1-\gamma}$ is the MARKOWITZ Portfolio, with 0.943 and 1.195%, respectively, also in the USA Market.

Table 15: t-test pair portfolios' p-value considering the trade-off between Cumulative Return ($RCum$) and Annualized Volatility for the BRICS Markets in 2025.

Brazil Market									
t_test	IBRX50	MARKOWITZ	SHARPE	MF_EQ	MF_MKW	MF_SHARPE	ANNt_EQ	ANNt_MKW	ANNt_SHARPE
IBOV	1.000	**0.000**	0.540	**0.039**	**0.011**	**0.010**	**0.001**	**0.042**	**0.014**
MARKOWITZ	0.393	1.000	**0.004**	**0.002**	**0.022**	**0.003**	**0.000**	**0.003**	**0.006**
SHARPE	0.121	**0.040**	1.000	**0.023**	0.057	**0.006**	**0.000**	**0.025**	**0.011**
MF_EQ	0.387	0.868	0.256	1.000	**0.003**	**0.020**	**0.001**	0.737	**0.025**
MF_MKW	0.926	0.271	0.067	0.285	1.000	**0.004**	**0.000**	**0.006**	**0.007**
MF_SHARPE	0.061	0.116	0.290	0.133	0.048	1.000	0.845	**0.025**	0.570
ANNt_EQ	0.256	0.473	0.998	0.546	0.215	0.413	1.000	**0.002**	0.621
ANNt_MKW	0.677	0.280	0.116	0.269	0.716	**0.049**	0.181	1.000	**0.028**
ANNt_SHARPE	0.396	0.125	**0.050**	0.117	0.409	**0.024**	0.095	0.715	1.000
Russian Market									
t_test	IMOEX	MARKOWITZ	SHARPE	MF_EQ	MF_MKW	MF_SHARPE	ANNt_EQ	ANNt_MKW	ANNt_SHARPE
IMOEX	1.000	0.577	0.341	0.030	0.281	0.085	**0.020**	**0.009**	0.869
MARKOWITZ	0.946	1.000	0.845	0.506	0.495	0.135	0.591	0.510	0.639
SHARPE	0.642	0.654	1.000	0.574	0.560	0.151	0.690	0.576	0.530
MF_EQ	0.284	0.249	0.251	1.000	0.755	0.208	0.742	0.957	0.355
MF_MKW	0.122	0.108	0.135	0.604	1.000	0.392	0.676	0.739	0.338
MF_SHARPE	0.874	0.890	0.881	0.471	0.315	1.000	0.185	0.204	0.104
ANNt_EQ	0.284	0.231	0.271	0.876	0.474	0.512	1.000	0.727	0.397
ANNt_MKW	0.565	0.524	0.416	0.718	0.413	0.634	0.802	1.000	0.358
ANNt_SHARPE	0.630	0.576	0.457	0.580	0.298	0.691	0.648	0.884	1.000
Indian Market									
t_test	NIFTY_50	MARKOWITZ	SHARPE	MF_EQ	MF_MKW	MF_SHARPE	ANNt_EQ	ANNt_MKW	ANNt_SHARPE
NIFTY_50	1.000	0.566	0.102	0.443	0.980	**0.007**	0.300	0.484	**0.011**
MARKOWITZ	0.287	1.000	**0.047**	0.174	0.547	**0.008**	0.128	0.187	**0.001**
SHARPE	0.192	0.365	1.000	0.275	0.090	**0.011**	0.457	0.242	0.302
MF_EQ	0.567	0.202	0.128	1.000	0.404	**0.008**	0.723	0.930	**0.031**
MF_MKW	0.974	0.265	0.194	0.541	1.000	**0.007**	0.272	0.443	**0.006**
MF_SHARPE	**0.031**	**0.040**	0.058	0.023	0.032	1.000	**0.008**	**0.008**	**0.020**
ANNt_EQ	0.400	0.169	0.098	0.734	0.383	**0.018**	1.000	0.660	0.080
ANNt_MKW	0.461	0.211	0.112	0.786	0.445	**0.018**	0.962	1.000	**0.024**
ANNt_SHARPE	0.229	0.144	0.072	0.344	0.225	**0.010**	0.468	0.456	1.000
China Market									
t_test	SSE50	MARKOWITZ	SHARPE	MF_EQ	MF_MKW	MF_SHARPE	ANNt_EQ	ANNt_MKW	ANNt_SHARPE
SSE	1.000	0.112	0.053	**0.032**	0.649	0.106	0.995	0.994	**0.045**
MARKOWITZ	0.971	1.000	**0.018**	**0.002**	0.347	0.093	0.207	0.222	**0.006**
SHARPE	0.345	0.342	1.000	0.405	**0.037**	0.147	0.060	0.062	0.514
MF_EQ	0.954	0.925	0.344	1.000	**0.035**	0.131	0.079	0.088	0.824
MF_MKW	0.135	0.088	0.080	0.201	1.000	0.101	0.700	0.708	0.037
MF_SHARPE	0.774	0.783	0.832	0.758	0.421	1.000	0.106	0.106	0.134
ANNt_EQ	0.448	0.375	0.157	0.539	0.233	0.583	1.000	0.999	0.077
ANNt_MKW	0.798	0.811	0.463	0.771	0.117	0.858	0.337	1.000	0.084

ANNt_SHARPE	0.705	0.713	0.575	0.685	0.143	0.918	0.332	0.881	1.000

South Africa Market									
t_test	FTSE/JSE40	MARKOWITZ	SHARPE	MF_EQ	MF_MKW	MF_SHARPE	ANNt_EQ	ANNt_MKW	ANNt_SHARPE
FTSE/JSE40	1.000	0.265	0.444	0.223	0.213	0.306	0.310	0.230	0.417
MARKOWITZ	**0.019**	1.000	0.856	0.871	0.492	0.373	0.875	0.669	0.808
SHARPE	0.250	0.881	1.000	0.746	0.437	0.358	0.963	0.585	0.726
MF_EQ	**0.034**	0.717	0.988	1.000	0.563	0.387	0.750	0.771	0.886
MF_MKW	0.070	0.258	0.276	0.221	1.000	0.472	0.432	0.750	0.739
MF_SHARPE	0.407	0.430	0.428	0.427	0.456	1.000	0.361	0.420	0.413
ANNt_EQ	0.084	0.668	0.926	0.888	0.208	0.426	1.000	0.579	0.737
ANNt_MKW	**0.046**	0.395	0.469	0.309	0.550	0.442	0.296	1.000	0.944
ANNt_SHARPE	0.299	0.601	0.577	0.557	0.899	0.451	0.538	0.828	1.000

Note: The lower left triangle in the table shows the pairwise t-test for differences in means of the portfolios considering the cumulative return series of the rebalanced portfolios with each of the strategies. The upper right triangle shows the pairwise t-test for differences in means of the Annualized Volatility series of the rebalanced portfolios. In bold, p-values indicate a significance level of 5%.

In the Brazilian market, regarding the traditional MF portfolios, only the MF_EQ and MF_SHARPE portfolios do not show a significant difference in $RCum$. The ANNt portfolios do not show a statistically significant difference, with the exception of the ANNt_SHARPE at 5%. Annualized Volatility, however, shows a statistically significant difference in practically all portfolio pairs.

In Russian marketing, regarding $RCum$, ANNt´s portfolios stand out compared to traditional portfolios, showing a statistically significant difference at the 5% level. However, the volatility of most portfolios does not show a significant difference.

In the Indian market, no portfolio is significantly different in terms of $RCum$, while Annualized Volatility is statistically different across virtually all portfolios.

In the Chinese market, the $RCum$ series of ANNt wallets shows a significant difference compared to traditional wallets. Similarly, most wallets have statistically significant differences from one another.

In the South African market, ANNt and MF portfolios differ from traditional portfolios in the $RCum$ series. Similarly, most portfolios exhibit a significant difference in annualized volatility.

Table 16: t-test pair portfolios' p-value considering the trade-off between Cumulative Return ($RCum$) and Annualized Volatility for the USA Market in 2025.

USA Market									
t_test	S&P500	MARKOWITZ	SHARPE	MF_EQ	MF_MKW	MF_SHARPE	ANNt_EQ	ANNt_MKW	ANNt_SHARPE
SP500	1.000	0.070	0.313	0.611	0.321	**0.028**	**0.002**	**0.013**	**0.001**
MARKOWITZ	0.430	1.000	0.271	**0.019**	0.165	**0.019**	**0.001**	**0.003**	**0.001**
SHARPE	0.369	0.683	1.000	0.119	0.906	**0.022**	**0.001**	**0.004**	**0.001**
MF_EQ	0.342	0.128	0.119	1.000	0.107	**0.033**	**0.003**	**0.019**	**0.001**
MF_MKW	0.882	0.424	0.350	0.458	1.000	**0.023**	**0.001**	**0.005**	**0.001**
MF_SHARPE	0.627	0.492	0.432	0.904	0.669	1.000	0.177	0.097	0.509
ANNt_EQ	0.077	0.055	**0.039**	0.180	0.089	0.491	1.000	0.228	0.087
ANNt_MKW	0.110	0.072	0.053	0.277	0.132	0.617	0.753	1.000	**0.020**
ANNt_SHARPE	0.082	0.062	**0.045**	0.164	0.091	0.414	0.817	0.604	1.000

Note: The lower left triangle in the table shows the pairwise t-test for differences in means of the portfolios considering the cumulative return series of the rebalanced portfolios with each of the strategies. The upper right triangle shows the pairwise t-test for differences in means of the Annualized Volatility series of the rebalanced portfolios. In bold, p-values indicate a significance level of 5%. Regarding the traditional SHARPE portfolios, the ANNt_EQ and ANNt_SHARPE portfolios show a significant difference of 5% in $RCum$. Annualized Volatility, however, shows a statistically significant difference in practically all portfolio pairs. Observed that the ANNt_EQ and ANNt_SHARPE are statistically different at 10% of the SP500, MARKOWITZ, and SHARPE portfolios.

Table 17: Neural network selection for application in the asset selection process.

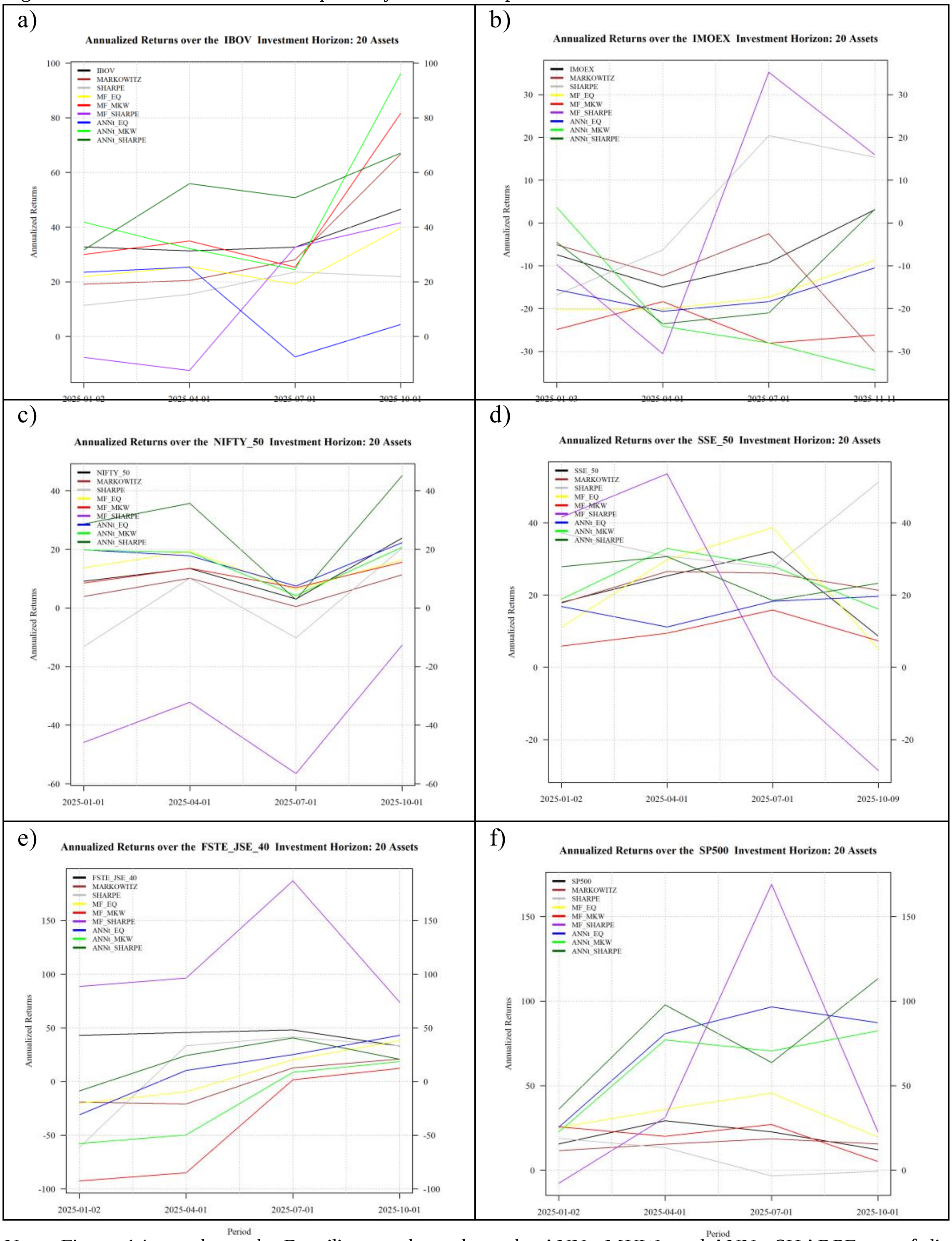
Rebalancing	Quarterly	Cumulative Return with ANN			Cumulative Return with LSTM		
		ANNt_EQ	ANNt_MKW	ANNt_SHARPE	LSTMt_EQ	LSTMt_MKW	LSTMt_SHARPE
		(1)	(2)	(3)	(4)	(5)	(6)
1	1Q 2020	-2.21	-0.32	1.88	0.20	2.02	-2.25
2	2Q 2020	-2.76	6.39	-7.65	17.20	14.10	17.38
3	3Q 2020	-9.15	-14.61	-67.16	18.68	5.09	1.70
4	4Q 2020	-3.35	-3.64	-26.33	26.39	18.56	22.18
5	1Q 2021	16.91	24.18	-17.11	34.34	33.11	41.22
6	2Q 2021	4.33	9.87	-26.97	12.83	-1.27	7.58
7	3Q 2021	12.48	13.98	-8.55	22.22	4.90	16.14
8	4Q 2021	14.52	-8.63	3.72	37.53	16.17	22.79
9	1Q 2022	15.68	20.66	-24.47	48.14	39.27	70.04
10	2Q 2022	45.55	35.43	-38.98	37.99	6.91	-3.36
11	3Q 2022	-12.82	-14.39	-27.33	19.51	8.00	7.90
12	4Q 2022	9.87	33.38	-37.74	4.27	25.01	51.47
13	1Q 2023	28.95	5.06	70.71	35.27	53.86	112.31
14	2Q 2023	25.09	7.51	69.36	22.37	14.59	105.55
15	3Q 2023	7.11	-39.95	-0.44	24.01	44.41	14.96
16	4Q 2023	42.58	-1.98	-48.58	27.43	61.30	38.60
17	1Q 2024	112.98	189.25	15.32	24.35	15.99	-34.35
18	2Q 2024	125.09	124.98	95.02	38.46	65.97	45.74
19	3Q 2024	189.65	194.71	54.70	76.85	119.35	138.44
20	4Q 2024	51.99	13.90	1.35	63.66	48.82	54.23
Mean		33.62	29.79	-0.96	29.59	29.81	36.41
		(1) - (4)	(2) - (5)	(3) - (6)	(4) - (1)	(5) - (2)	(6) - (3)
Diff.		4,04	-0,02	-37,38	-4,04	0,02	37,38
t-Test		ANNt_EQ	ANNt_MKW	ANNt_SHARPE	LSTMt_EQ	LSTMt_MKW	LSTMt_SHARPE
	ANNt_EQ	1.0000	0.8360	**0.0270**	0.7460	08540	0.8540
	ANNt_MKW	0.8360	1.0000	0.8360	0.9890	0.9990	0.7030
	ANNt_SHARPE	**0.0270**	0.0830	1.0000	0.0070	0.0120	**0.0090**
	LSTMt_EQ	0.7460	0.9890	0.0070	1.0000	0.9770	0.5200
	LSTMt_MKW	0.7770	0.9990	0.0120	0.9770	1.0000	0.5750
	LSTM_SHARPE	0.8540	0.7030	**0.0090**	0.5200	0.5750	1.0000

Note: The ANNt configuration presented in this work, originally proposed by de Oliveira et al. (2024), and the alternative neural network configuration tested, Long Short-Term Memory (LSTM) with t-distribution, an advanced type of Recurrent Neural Network (RNN) designed to process sequential data and learn long-term dependencies, overcoming the gradient fading problem. The selection criterion used real SP500 data during the sample period of the work, with 20 rebalancing. The final results consider the excess return generated by each neural network, the classification of assets based on the t-distribution, and the application of Equality, Markowitz, and Sharpe weights to the 20 selected assets. ANNt proved superior cumulative return in 2 of 3 portfolios, without a statistically significant difference from LSTM. LSTM has a difference of 1% in the SHARPE portfolio. However, the LSTM has a processing time machine that is much superior, which is very inconvenient when we use a conventional computer.

LSTM configuration: 100 stepmax, 32 hidden, MSE loss, 0.3 learning rate, 0.5 decay. Processing time: 7 days.

ANN configuration: 2000 stepmax, 487 hidden, MSE loss, 0.3 learning rate, 0.0 decay. Processing time: 2 days.

Figure 114: Annualized returns of quarterly rebalanced portfolios in 2025



Note: Figure 14a analyzes the Brazilian market, where the ANNt_MKW and ANNt_SHARPE portfolio reports the highest outperformance among the portfolios in the 2 quarters of the 4 rebalanced, respectively. Figure 14b analyzes the Russian market, where the ANNt_MKW portfolio presents positive results in most of the series, with the exception of 3 rebalancings. Figure 14c analyzes the Indian market, where the MF_SHARPE portfolio presents an annualized return higher than the market proxy in 8 quarters of the 20 analyzed, holding the highest return observed. Figure 14d analyzes the Chinese market, where the MARKOWITZ portfolio presents the best annualized return in 12 quarters analyzed, being the best long-term portfolio option. Figure 14e analyzes the South African market, where the ANNt_MKW portfolio presents a higher annualized return in 5 quarters. Figure 14f analyzes the U.S. market. It has proven to be efficient in the long term. No other portfolio formation strategy can maintain a performance superior to the market itself, represented by the SP500.