Mis-measuring Externalities: Carbon Accounting and Equilibrium Capital Allocation*

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Abstract

We study how carbon accounting rules shape capital allocation when environmental effects extend beyond the reporting boundary of an individual firm. In a two-sector equilibrium model, activities that generate equilibrium displacement raises measured emissions locally while reducing more carbon-intensive production elsewhere in the economy. As a result, the emissions metric used in financial decisions need not coincide with the welfare-relevant environmental impact. We show that accounting rules are not allocatively neutral: changing the reported emissions rule changes the signal priced by investors and therefore changes equilibrium capital allocation. When the reported displacement factor differs from the true environmental effect, markets price the wrong signal, creating a wedge between decentralized allocation and the planner benchmark. In our benchmark calibration, even recognizing only one-third of the true consequential displacement effect increases welfare by about 4% relative to firm-boundary accounting. Even when the displacement factor is correctly measured, firm-level accounting generally does not implement the first best because the planner values aggregate marginal responses rather than reported emission levels. Accounting is therefore an allocative device, but not a sufficient statistic for social efficiency.

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1 Introduction

Capital markets increasingly allocate capital using reported environmental metrics. Carbon intensities enter portfolio mandates, benchmark construction, exclusion rules, lending spreads, sustainability-linked contracts, and regulatory or reputational constraints faced by financial institutions. Once such metrics affect financing conditions, carbon accounting does more than describe firms' environmental performance. It determines the signal on which decentralized capital markets act.

This paper studies the equilibrium consequences of that signal-design problem. The central point is that the emissions object reported to investors need not coincide with the emissions object relevant for welfare. Standard emissions inventories are largely attributional: they assign emissions to a firm or activity within a reporting boundary. Welfare-relevant environmental effects are often consequential: they depend on how an activity changes emissions elsewhere in the economy through substitution, displacement, or equilibrium responses. The distinction is immaterial when firm-boundary emissions are a sufficient statistic for social impact. It is central when an activity raises its own reported emissions while reducing more carbon-intensive activity outside its boundary.

The central difficulty is that decentralized markets allocate capital using reported emissions rather than welfare-relevant emissions. To formalize this distinction, we separate three environmental objects. Realized emissions are the physical emissions generated within the firm's production and recycling activities. Reported emissions are the accounting metric disclosed to, or used by, investors under a given rule. True social emissions are the welfare-relevant externality, equal to realized emissions net of actual equilibrium displacement. These objects need not coincide. The paper asks how decentralized capital allocation changes when investors price the second object, while welfare depends on the third.

To study this mechanism, we develop a two-sector equilibrium model with circular production. Recycling provides a transparent microfoundation because it can increase emissions inside the reporting boundary while displacing more carbon-intensive virgin production elsewhere in the economy.[1] The model, however, is not a theory of waste management. The underlying mechanism is more general and arises whenever the environmental consequences of production depend on equilibrium substitution or displacement effects that extend beyond the reporting boundary of the firm. Recycling is used to discipline a broader measured-externality problem: capital markets price a reportable firm-level metric, whereas welfare depends on a system-wide counterfactual object.

The paper is closest to Landier and Lovo (2025), who study capital allocation under environmental objectives in a two-sector equilibrium framework. We build on their allocative logic but change the object being allocated on. In their setting, the environmental signal entering decentralized decisions is taken as given. In our setting, accounting determines that signal. The paper therefore shifts the focus from pricing environmental information to designing the environmental information that is priced. This distinction connects equilibrium sustainable-finance theory to the carbon-accounting literature on attributional and consequential measurement.

The analysis delivers three results. First, accounting is not allocatively neutral. Changing the accounting rule changes the reported emissions metric, changes the environmental component of perceived project payoffs, and therefore changes equilibrium capital allocation. This is a positive result: even if markets process the reported signal efficiently, the allocation depends on the accounting rule that defines the signal.

Second, accounting misalignment creates a wedge between decentralized allocation and the planner benchmark. In the model, the true consequential displacement coefficient is f, whereas the coefficient embedded in the reported metric is ϕ. When $\phi \neq f$, the environmental gradient priced by decentralized agents differs from the welfare-relevant

[1] This is not only analytically convenient but also quantitatively relevant. World Bank (2026) reports that global municipal solid waste generation reached 2.56 billion tonnes in 2022 and is projected to rise to 3.86 billion tonnes by 2050.

environmental gradient by

$$\mathcal{M}(K_1;\phi) = (f-\phi)B'(K_1). \tag{1}$$

The wedge is therefore not only an error in the level of reported emissions. It changes the marginal reallocation signal faced by capital markets. The quantitative implications are economically meaningful. In the benchmark calibration, even an accounting rule that recognizes only one-third of the true consequential displacement coefficient raises welfare by about 4% relative to firm-boundary accounting. Improving the environmental signal can therefore matter well before consequential measurement becomes perfect.

Third, even correct consequential accounting does not generally decentralize the first best. Setting $\phi = f$ aligns the reported emissions level with the true social emissions level. But decentralized investors compare firm accounting levels, whereas the planner values aggregate marginal responses. The planner's environmental condition includes equilibrium-response terms that are not contained in level-based firm accounts. Correct measurement of the level of emissions therefore removes the coefficient wedge, but not the implementation wedge.

These results contribute to the equilibrium literature on sustainable finance, including Pástor, Stambaugh, and Taylor (2021), Oehmke and Opp (2025), Berk and van Binsbergen (2025), and Landier and Lovo (2025), by showing that the environmental signal itself is an equilibrium-relevant object. They also contribute to the carbon-accounting literature on avoided emissions and consequential measurement (Weidema, 2003; Ekvall and Weidema, 2004; Brander and Wylie, 2011; WBCSD, 2025) by embedding the accounting distinction in a capital-allocation model. The paper is complementary to environmental-policy work such as Acemoglu et al. (2012), but the focus is different: we study the measurement of the environmental object entering decentralized finance, not directed technical change or optimal environmental taxation.

The contribution is a theory of equilibrium capital allocation under imperfect environmental measurement. The objective is not to study how investors interpret environmental disclosures, but how the accounting object itself changes the signal available to decentralized markets. We abstract from belief formation and heterogeneous interpretations of environmental disclosures in order to isolate the allocative role of the accounting base itself.

The broader implication is that carbon accounting is part of the market design of sustainable finance. When reported emissions enter investment mandates, lending decisions, portfolio construction, or benchmarks, accounting conventions influence which activities receive capital. The paper therefore treats accounting not as a passive disclosure layer, but as an allocative institution. Recycling is the microfoundation; the contribution is more general. Whenever welfare depends on an equilibrium counterfactual while financial decisions depend on reportable firm-level metrics, measurement rules become part of the allocation mechanism.

The remainder of the paper proceeds as follows. Section 2 presents the model. Section 3 characterizes decentralized equilibrium under alternative accounting rules. Section 4 derives the planner benchmark and identifies the accounting-induced environmental-gradient wedge. Section 5 develops the comparative statics and welfare implications. Section 6 discusses financial, empirical, and policy implications. Section 7 concludes.

2 Model

The model is designed to isolate how alternative carbon-accounting rules affect decentralized capital allocation. The circular-production structure should be viewed as a tractable microfoundation for a broader class of environments in which firms are evaluated on observable accounting objects, while the relevant social externality depends on equilibrium displacement or substitution effects outside the firm's reporting boundary. This microfoundation disciplines the interpretation of recycling, displacement, and avoided emissions, while the main results are stated in reduced-form equilibrium terms to keep the measurement channel transparent.

We consider a static two-sector economy indexed by $i \in \{1, 2\}$, with $j \neq i$. A unit mass of capital is allocated across sectors. One unit of capital activates one project, so that K_i denotes both the mass of active firms in sector i and the amount of capital allocated to that sector. Sector sizes satisfy

$$K_1 + K_2 = 1. \tag{2}$$

All firms within a sector are identical. The economy is otherwise frictionless. The only distortion studied in the paper is that decentralized agents price an accounting metric that need not coincide with the welfare-relevant externality.

From this point onward, we index all endogenous firm-level and aggregate objects by K_1, with $K_2 = 1 - K_1$. This reduced-form notation captures the fact that equilibrium prices, output, recycled-input use, and emissions depend on the sectoral allocation of capital.

2.1 Technology and Circular Production

Production is circular in the following sense. Each sector combines virgin material purchased from the other sector with recycled material recovered from its own waste. This structure creates a natural source of displacement: greater recycled-input use reduces the need for virgin production elsewhere in the economy. The model therefore links firm-level recycling activity to an economy-wide consequential emissions effect.

A representative firm in sector i produces output using a materials bundle m_i according to

$$y_i = A_i\, m_i^{\alpha_i}, \qquad A_i > 0, \qquad \alpha_i \in (0, 1). \tag{3}$$

The parameter A_i captures sector-specific production efficiency, while α_i governs the elasticity of output with respect to the materials bundle m_i and ensures diminishing returns at the project level. The materials bundle combines a virgin input purchased from the other sector with a recycled input supplied by a competitive recycling market operating in sector i:[2]

$$m_i = \left(\omega\, n_i^{\frac{\sigma-1}{\sigma}} + (1-\omega)\, r_i^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}, \qquad \omega \in (0,1), \qquad \sigma > 0, \qquad \sigma \neq 1. \tag{4}$$

Here n_i denotes virgin material used by sector i and purchased from sector $j \neq i$, while r_i denotes recycled material used by sector i and purchased from a competitive recycling market supplied out of sector-i waste. This specification captures the idea that recycled and virgin materials are imperfect substitutes. This imperfect substitutability is important because it makes recycling an economically meaningful margin of adjustment rather than a perfect one-for-one replacement technology. The elasticity parameter σ governs the strength of substitution, while ω captures the technological weight on virgin material.

Let p_i^c denote the price of sector i's output, which is the price paid by final consumers for good i, and let p_i^r denote the price of recycled input used by sector i. Since the virgin input used by sector i is good j, its price is p_j^c. Standard CES cost minimization implies that the dual price index of the materials bundle is

$$q_i = \left[\omega^{\sigma}(p_j^c)^{1-\sigma} + (1-\omega)^{\sigma}(p_i^r)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}. \tag{5}$$

The composite price q_i is the relevant marginal cost object for the firm: it embeds the substitution margin between virgin and recycled inputs and therefore governs how recycling affects production decisions in equilibrium.[3]

[2] The CES specification is close in spirit to the Armington-type aggregation used by Bongers and Casas (2022) in a circular-economy framework. In our setting, it provides a parsimonious way to model imperfect substitution between virgin and recycled inputs while keeping the equilibrium structure tractable.

[3] Appendix D collects the closed-form blocks underlying the reduced-form representation used in the main text, including household demand, sectoral profit erosion, and a benchmark welfare decomposition.

The representative firm in sector i therefore solves

$$\max_{m_i \geq 0} \; p_i^c \, A_i \, m_i^{\alpha_i} - q_i \, m_i. \tag{6}$$

Conditional on m_i, the cost-minimizing input demands are

$$n_i = m_i \, \omega^{\sigma} \left(\frac{p_j^c}{q_i} \right)^{-\sigma}, \tag{7}$$

$$r_i = m_i \, (1-\omega)^{\sigma} \left(\frac{p_i^r}{q_i} \right)^{-\sigma}. \tag{8}$$

Because recycling services are purchased from a competitive secondary-input market at price p_i^r, the resource cost of recycling is fully embedded in q_i. No separate recycling term is needed in firm profits.

Production generates waste. At the firm level, waste is proportional to output

$$\Theta_i = \theta_i \, y_i, \qquad \theta_i \in [0,1]. \tag{9}$$

At the sector level, aggregate output and aggregate waste are

$$Y_i = K_i \, y_i, \qquad \Theta_i^{agg} = \theta_i \, Y_i. \tag{10}$$

A competitive recycling sector transforms a fraction of sector i's waste into usable recycled input. In the core model, the recovery rate $\rho_i \in [0,1]$ is exogenous. Aggregate recycled-input supply therefore satisfies

$$R_i \leq \rho_i \, \theta_i \, Y_i. \tag{11}$$

The recycled-input market clears when

$$K_i \, r_i = R_i. \tag{12}$$

In a symmetric equilibrium with a binding recovery constraint, (11)–(12) imply

$$r_i = \rho_i \, \theta_i \, y_i. \tag{13}$$

Accordingly, the baseline model should be interpreted as a sector-level recycling economy: firms choose recycled-input demand, while the supply of recycled material is generated collectively from sectoral waste and cleared on a competitive secondary-input market. This formulation resolves an important modeling issue. Firms choose recycled input through standard cost minimization, while the economy-wide recycling technology determines equilibrium supply. The identity $r_i = \rho_i \, \theta_i \, y_i$ is thus an equilibrium outcome of the recycling market, not a private technological constraint imposed directly inside the firm's static problem.

This distinction is important for the interpretation of avoided emissions. Firms choose recycled-input use through standard cost minimization, while the quantity of recycled material available in the economy depends on sectoral waste generation and recycling capacity. As a result, the displacement effect associated with recycling is an equilibrium object, not a purely technological coefficient imposed at the firm level.[4]

2.2 Households and Welfare

The representative household consumes the two final goods and suffers disutility from aggregate emissions. Preferences are Cobb-Douglas over consumption and separable in the environmental externality

$$U(C_1, C_2, E^*) = \frac{C_1^{\gamma_1} \, C_2^{\gamma_2}}{(1 + E^*)^{\delta}}, \qquad \gamma_1, \gamma_2 \in (0,1), \qquad \gamma_1 + \gamma_2 = 1, \qquad \delta > 0. \tag{14}$$

[4] A schematic representation of the circular production structure is provided in Appendix D.

Here E^* denotes the true welfare-relevant externality, defined below. Let W denote aggregate household income. Standard Cobb-Douglas demand implies

$$C_i = \frac{\gamma_i W}{p_i^c}, \qquad i \in \{1, 2\}. \tag{15}$$

The environmental term shifts welfare but does not affect relative consumption demands conditional on income and prices.[5]

The paper adopts a consequential welfare criterion. Welfare depends on the true net emissions effect of economic activity once system-wide displacement is taken into account. This object need not coincide with firm-boundary emissions , and it need not coincide with the accounting metric used in decentralized allocation.

2.3 Emissions, Displacement, and Accounting

Throughout the paper, we use the term *displacement* for the equilibrium reduction in emissions generated outside the reporting firm's boundary when recycled input substitutes for more carbon-intensive virgin production. This terminology is deliberate. The model does not treat avoided emissions as an exogenous firm-level attribute. Instead, the environmental benefit arises from an equilibrium counterfactual: a change in the amount and composition of activity displaced elsewhere in the economy.

Accordingly, the paper distinguishes three objects. First, *realized emissions* are the physical emissions generated within the firm's production and recycling activities. Second, *reported emissions* are the emissions attributed to the firm under a given accounting rule and may include a displacement credit. Third, *true social emissions* are the welfare-relevant emissions object, equal to realized emissions net of the actual displaced activity in equilibrium. The term *avoided emissions* is used only when referring to accounting practice or institutional guidance; the formal object in the model is an equilibrium dis-

[5]Appendix D collects the closed-form blocks underlying the reduced-form representation used in the main text, including household demand, sectoral profit erosion, and a benchmark welfare decomposition.

placement effect.[6] The key issue is that these objects coincide only under restrictive conditions: in general, what is physically emitted, what is reported to markets, and what is socially relevant are distinct objects.

Firm-boundary emissions. Realized emissions of a representative firm in sector i are

$$e_i^R(K_1) = \varepsilon_i^{prod}\, y_i(K_1) + \varepsilon_i^{waste}\big(\theta_i\, y_i(K_1) - r_i(K_1)\big) + \varepsilon_i^{rec}\, r_i(K_1), \tag{16}$$

where ε_i^{prod} is the emissions intensity of production, ε_i^{waste} is the emissions intensity of unrecycled waste, and ε_i^{rec} is the emissions intensity of recycling. The dependence on K_1 reflects general-equilibrium effects through prices, output, and recycled-input use. Aggregate firm-boundary emissions are

$$E^R(K_1) = K_1\, e_1^R(K_1) + (1 - K_1)\, e_2^R(K_1), \tag{17}$$

Displacement. One unit of recycled input displaces virgin production elsewhere in the economy. Let $f > 0$ denote the true emissions intensity of the marginal activity displaced by one unit of recycled input. In the core model, f is treated as a reduced-form constant. Aggregate displacement capacity is

$$B(K_1) = K_1\, r_1(K_1) + (1 - K_1)\, r_2(K_1). \tag{18}$$

The corresponding aggregate displaced emissions are $f\, B(K_1)$. In the present setting, $B(K_1)$ can be interpreted as aggregate recycled-input use, that is, the economy-wide

[6] The model can be interpreted through the lens of scope-based emissions accounting. Recycling can increase emissions recorded within the reporting boundary, for instance through processing or recovery activity, while reducing emissions elsewhere by lowering demand for virgin production. The paper does not require assigning this reduction to a separate emissions scope. The formal distinction is instead between realized emissions, reported emissions, and true social emissions. In the model, displacement claims correspond to the term $fB(K_1)$ and should be interpreted as an equilibrium counterfactual rather than as a separate physical emissions inventory.

scale at which circular production displaces virgin activity.[7] The welfare-relevant displacement object is f. Let $f > 0$ denote the true consequential emissions intensity of the marginal virgin activity displaced by one unit of recycled input. This coefficient is not an accounting convention. It is the environmental primitive that determines true social emissions. Accounting enters only afterward, through the coefficient ϕ that decentralized agents can observe and price. Economically, f is a consequential parameter: it captures the emissions intensity of the marginal virgin activity displaced by recycling. Accordingly, $f\,B(K_1)$ represents displaced virgin emissions, that is, the counterfactual emissions displaced elsewhere in equilibrium, rather than an additional physical emissions inventory attached to recycling itself. Firm-boundary emissions already include the emissions generated by recycling activity through the term $\varepsilon_i^{rec}\,r_i$. The consequential adjustment therefore subtracts only the emissions of the marginal virgin activity displaced elsewhere in equilibrium. This convention rules out double counting and makes clear that the welfare object is firm-boundary emissions minus displaced virgin emissions.

True welfare-relevant externality. The consequential emissions object relevant for welfare is

$$E^*(K_1) = E^R(K_1) - f\,B(K_1). \tag{19}$$

It combines observed emissions with the emissions that are avoided elsewhere in equilibrium through displacement. With constant f, the same decomposition can be written at the firm level as

$$e_i^*(K_1) = e_i^R(K_1) - f\,r_i(K_1). \tag{20}$$

[7]Appendix E decomposes $B'(K_1)$ into a composition effect and an equilibrium-response effect. In the core model, $B(K_1)$ is aggregate recycled-input use and therefore measures the equilibrium scale of displaced virgin activity.

Using Equation (13) and Equation (16), this becomes

$$e_i^*(K_1) = y_i(K_1) \left[\varepsilon_i^{prod} + \varepsilon_i^{waste} \theta_i + \left(\varepsilon_i^{rec} - \varepsilon_i^{waste} - f \right) \rho_i \, \theta_i \right]. \tag{21}$$

This expression makes clear that recycling affects welfare through two channels at once: it changes firm-boundary emissions directly, and it changes the amount of dirtier activity displaced elsewhere in the economy. Equation (21) clarifies a point that will be central throughout the paper. Recycling enters firm-boundary emissions once, through $\varepsilon_i^{rec} r_i$. The displaced-emissions term subtracts the emissions of the marginal virgin activity displaced elsewhere in the economy. It does not subtract recycling emissions a second time. The accounting object relevant for welfare is therefore firm-boundary emissions minus displaced virgin emissions.

Nothing in the wedge identified here is conceptually specific to recycling per se. Recycling is simply the benchmark case in which the distance between realized and consequential emissions can be microfounded transparently through displaced virgin production. More generally, the same structure arises whenever reported firm-level emissions differ from the welfare-relevant externality because part of the environmental effect operates through equilibrium counterfactuals outside the reporting boundary.

Accounting rules. In practice, the coefficient embedded in the reported metric may reflect reporting conventions, standard-setting choices, or estimated avoided-emissions factors rather than the true consequential benchmark. This is consistent with the broader consequential-accounting view that avoided emissions depend on assumptions about marginal displacement and system boundaries rather than on a purely attributional inventory of firm-boundary emissions (Weidema, 2003; Ekvall and Weidema, 2004; WBCSD, 2025). Decentralized agents do not observe the true consequential coefficient f directly. Instead, they price an implementable accounting metric constructed from reportable firm-level objects. We represent accounting with a generic coefficient $\phi \in \mathbb{R}_+$ and define the emissions metric attributed to sector i as

$$e_i^{\phi}(K_1) = e_i^R(K_1) - \phi\, r_i(K_1). \tag{22}$$

Aggregating across sectors gives

$$E^{\phi}(K_1) = E^R(K_1) - \phi\, B(K_1). \tag{23}$$

This formulation nests the accounting regimes of interest:

$$\phi = 0 \quad \Rightarrow \quad \text{firm-boundary accounting,} \tag{24}$$

$$\phi = \widetilde{f} \quad \Rightarrow \quad \text{estimated social accounting,} \tag{25}$$

$$\phi = f \quad \Rightarrow \quad \text{true social accounting.} \tag{26}$$

The key distinction is that $E^*(K_1)$ is the welfare object, whereas $E^{\phi}(K_1)$ is the priced accounting object. The distinction between $\widetilde{f}$ and f is central. The coefficient $\widetilde{f}$ is the displacement factor embedded in the accounting rule that investors or regulators can actually observe and price. By contrast, f is the true consequential coefficient relevant for welfare. The two coincide only when avoided emissions are measured without error. We study the allocative consequences of the wedge between the two.

Table 1 summarizes the main emissions objects and accounting concepts used throughout the paper.

The previous subsections define the technological and environmental objects of the economy. We now turn to the decentralized pricing channel through which accounting affects capital allocation.

2.4 Decentralized Capital Allocation

One unit of capital activates one firm, so sectoral capital allocation is summarized by K_1 and $K_2 = 1 - K_1$. Investors evaluate projects using financial profitability adjusted

Table 1: Environmental objects in the model

Model term	**Accounting-practice counterpart**	**Welfare interpretation**
Firm-boundary emissions, $e_i^R(K_1)$ and $E^R(K_1)$	Emissions physically generated within the reporting boundary	Local emissions generated by production, waste treatment, and recycling activity.
Reported emissions, $e_i^\phi(K_1)$ and $E^\phi(K_1)$	Emissions metric disclosed to, or used by, investors under a given accounting rule	The environmental signal priced by decentralized agents; it may or may not coincide with the welfare-relevant object.
Displacement base, $b_i(K_1)$, $r_i(K_1)$ and $B(K_1)$	Activity measure used to compute displacement or avoided-emissions claims	Equilibrium scale of the activity that substitutes for more carbon-intensive virgin production.
Reported displacement coefficient, $\widetilde{f}$	Estimated avoided-emissions or displacement factor used in reporting practice	Implementable coefficient embedded in reported emissions; it can differ from the true marginal displaced emissions intensity.
True displacement coefficient, f	Consequential emissions intensity of the displaced marginal activity	Welfare-relevant emissions intensity of the activity displaced in equilibrium.
True social emissions, $e_i^*(K_1)$ and $E^*(K_1)$	Not directly observed in standard firm-level inventories	Welfare-relevant externality: firm-boundary emissions net of actual equilibrium displacement.
Measurement wedge, $\mathcal{M}(K_1;\phi)$	Difference between the reported signal and the consequential emissions margin	Gap between the environmental gradient priced by markets and the environmental gradient relevant for the planner.

for the priced emissions metric. Under accounting rule ϕ, the perceived payoff of a representative sector-i project is

$$\widetilde{\Pi}_i^\phi(K_1) = \pi_i(K_i) - \tau\, e_i^\phi(K_1), \tag{27}$$

where $\pi_i(K_i)$ denotes financial profit per project and $\tau \geq 0$ is the shadow value assigned to reported emissions.[8]

[8]Appendix D.3 derives the benchmark closed-form expression $\pi_i(K_i) = \frac{K_i^*}{K_i}$, which provides the reduced-form profit-erosion structure used in the main text.

The parameter τ governs the intensity with which reported emissions enter decentralized payoffs. It should not be interpreted narrowly as a carbon tax. Instead, it summarizes any financial mechanism through which reported emissions affect capital allocation, including investor preferences, portfolio constraints, lending spreads, exclusion rules, benchmark construction, or regulatory capital requirements. For a fixed accounting rule ϕ, a higher τ increases the weight placed on the reported environmental signal relative to financial profitability. The effect of τ is therefore distinct from the effect of ϕ: the coefficient ϕ determines which environmental object is reported, while τ determines how strongly that reported object is priced. Laissez-faire is the special case $\tau = 0$, in which capital is allocated solely on the basis of financial profitability. If $\tau > 0$, accounting affects capital allocation because investors price the reported object $e_i^{\phi}(K_1)$.

The microfoundation developed above determines equilibrium profits, recycled-input use, and emissions as functions of sectoral capital allocation. The next section uses this structure to characterize decentralized equilibrium through reduced-form mappings that summarize these equilibrium relationships without carrying the full system of prices and quantities.

3 Decentralized Equilibrium

This section characterizes decentralized capital allocation under a generic accounting rule ϕ. The key object is the equilibrium allocation K_1^{ϕ}, that is, the share of capital allocated to sector 1 when investors price the emissions metric induced by ϕ.

The underlying microfoundation in Section 2 implies that, for any sectoral allocation K_1, equilibrium prices, output, recycled-input use, and emissions are jointly determined by firms' input choices, household demand, and market-clearing conditions. For the allocative results of this section, it is sufficient to summarize those equilibrium relationships through reduced-form objects indexed by K_1: per-project profits $\pi_i(K_i)$, firm-level displacement bases $b_i(K_1)$,[9] and firm-level accounting emissions $e_i^{\phi}(K_1)$. These objects

[9] $b_i(K_1) := r_i(K_1)$ denotes the firm-level displacement base, that is, the reportable activity on which

should therefore be read as equilibrium mappings induced by the benchmark micro-founded economy, rather than as independent primitives.

In the benchmark circular-production economy, sectoral crowding and goods-market clearing imply that project-level profitability declines with sector size, while recycled-input use and accounting emissions vary with equilibrium output and recycling activity. The reduced-form objects used below summarize these equilibrium responses.

In particular, Appendix D.3 shows that, in the benchmark closed-form system, per-project profits satisfy

$$\pi_i(K_i) = \frac{K_i^*}{K_i}, \tag{28}$$

so profitability is strictly decreasing in sector size. Likewise, the accounting bases $b_i(K_1)$ summarize the recycling margins generated by the circular-production block, and the accounting emissions objects $e_i^\phi(K_1)$ inherit their dependence on K_1 from equilibrium output and recycled-input use.

For any allocation K_1, define

$$G(K_1, \phi) := \widetilde{\Pi}_1^\phi(K_1) - \widetilde{\Pi}_2^\phi(K_1), \tag{29}$$

that is, the difference in perceived net payoffs between sector 1 and sector 2 under accounting rule ϕ.

Equivalently,

$$G(K_1, \phi) = \pi_1(K_1) - \pi_2(1 - K_1) - \tau\Big(e_1^\phi(K_1) - e_2^\phi(K_1)\Big), \tag{30}$$

displacement credits are computed. In the benchmark circular-production model, this base is recycled-input use.

where

$$e_i^{\phi}(K_1) = e_i^{R}(K_1) - \phi\, b_i(K_1), \qquad b_i(K_1) := r_i(K_1). \tag{31}$$

The distinction is conceptual: $r_i(K_1)$ is the underlying equilibrium recycling margin, while $b_i(K_1)$ denotes the accounting base used by decentralized agents. In the benchmark model, the two coincide.

The assumptions imposed below on $\pi_i(K_i)$, $b_i(K_1)$, and $G(K_1, \phi)$ should be interpreted as reduced-form regularity conditions on the equilibrium mappings generated by the underlying microfounded economy.

Definition 1 (Decentralized equilibrium)**.** *A decentralized equilibrium under accounting rule ϕ is an allocation $K^{\phi} = (K_1^{\phi}, K_2^{\phi})$, together with prices and quantities, such that firms optimize, households optimize, goods and recycling markets clear, and investors allocate capital across sectors according to perceived payoffs.*

For an interior allocation, the equilibrium condition is

$$G(K_1^{\phi}, \phi) = 0, \qquad K_2^{\phi} = 1 - K_1^{\phi}. \tag{32}$$

To obtain a sharp characterization of decentralized allocation, we impose standard reduced-form regularity conditions: per-project profits are decreasing in sector size, and the relative-payoff function $G(\cdot, \phi)$ crosses zero exactly once for each accounting rule in the range of interest. These conditions are sufficient to deliver a unique interior equilibrium and transparent comparative statics.

Proposition 1 (Existence, uniqueness, and accounting non-neutrality)**.** *Suppose that, for each sector $i \in \{1, 2\}$, the per-project profit function $\pi_i : (0, 1) \to \mathbb{R}$ is continuously differentiable and strictly decreasing. Suppose also that, for every accounting rule ϕ in the* *range of interest, the function $K_1 \mapsto G(K_1, \phi)$ is strictly decreasing on $(0, 1)$ and satisfies*

$$\lim_{K_1 \to 0} G(K_1, \phi) > 0, \qquad \lim_{K_1 \to 1} G(K_1, \phi) < 0. \tag{33}$$

Then, for each accounting rule ϕ, there exists a unique interior decentralized equilibrium $K_1^\phi \in (0, 1)$ satisfying (32).

Moreover, if G is continuously differentiable in (K_1, ϕ), the equilibrium allocation is differentiable in ϕ, and

$$\frac{\partial K_1^\phi}{\partial \phi} = -\frac{G_\phi(K_1^\phi, \phi)}{G_{K_1}(K_1^\phi, \phi)} = -\frac{\tau\big(b_1(K_1^\phi) - b_2(K_1^\phi)\big)}{G_{K_1}(K_1^\phi, \phi)}. \tag{34}$$

In particular, since $G_{K_1}(K_1^\phi, \phi) < 0$,

$$b_1(K_1^\phi) > b_2(K_1^\phi) \qquad \Longrightarrow \qquad \frac{\partial K_1^\phi}{\partial \phi} > 0, \tag{35}$$

and the inequality is reversed when $b_1(K_1^\phi) < b_2(K_1^\phi)$.

Proposition 1 summarizes the reduced-form allocative implication of the microfounded circular-production environment developed in Section 2. The proposition abstracts from closed-form equilibrium details in order to isolate the measurement channel: changing the accounting rule changes the environmental object embedded in perceived payoffs, which in turn changes equilibrium capital allocation.

Proposition 1 formalizes the paper's first core mechanism: accounting is not neutral. The key object is the relative accounting base $b_1(K_1^\phi) - b_2(K_1^\phi)$. Sectors that receive more displacement credit per project become more attractive when the accounting rule places greater weight on displacement. If sector 1 receives more displacement credit per project than sector 2, increasing ϕ raises sector 1's relative attractiveness and shifts capital toward it.

In the underlying circular-production economy, this relative accounting base is rooted in sectoral differences in recycled-input use and therefore in displacement capacity. Proposition 1 is a positive equilibrium result: it characterizes how accounting changes decentralized allocation, not whether the resulting allocation is welfare-improving.

The decentralized allocation K_1^ϕ is always understood as the allocation solving the perceived-payoff condition $G(K_1^\phi, \phi) = 0$. The next section does not replace this equilibrium condition with an environmental first-order condition. Instead, it compares the environmental object entering G through e_i^ϕ with the true aggregate marginal externality entering the planner's problem.

4 Planner Benchmark

We now turn to the normative benchmark. Welfare depends on the true consequential externality $E^*(K_1)$, not on the accounting metric priced by decentralized investors. In that sense, our planner benchmark follows the standard public-economics logic of evaluating decentralized allocations against a welfare criterion based on the true social externality, even though our focus is not optimal taxation per se but the allocative consequences of alternative accounting bases (Pigou, 1920; Golosov et al., 2014).

For the normative analysis, we work with the reduced-form welfare object $\mathcal{W}(K_1)$ induced by the underlying equilibrium allocation. The results below require only standard regularity conditions on this object.[10]

Table 2 summarizes the main regimes considered in the paper. The key distinction is whether capital allocation is determined by decentralized investors or by the planner, and whether the relevant environmental object lies in firm-boundary accounting, estimated social accounting, or true social accounting. The last row in Table 2 is a correct-metric benchmark, not an implementation benchmark. Even when decentralized agents price the true social emissions level $E^f(K_1) = E^*(K_1)$, they need not implement the planner allocation because their decision rule is based on firm-level payoff comparisons rather than the planner's aggregate marginal condition.

As presented in Table 2, the paper compares two distinct margins. The first is the

[10]Appendix D.4 provides a closed-form welfare decomposition for the benchmark microfoundation.

Table 2: Regimes considered in the analysis

Regime	**Allocation rule**	**Environmental metric priced in decisions**
Laissez-faire	Capital is allocated according to financial profitability only ($\tau = 0$).	No environmental metric enters decentralized allocation.
Firm-boundary accounting	Capital is allocated by decentralized investors under accounting rule $\phi = 0$.	Firm-boundary estimated emissions, $E^R(K_1)$.
Estimated social accounting	Capital is allocated by decentralized investors under accounting rule $\phi = \tilde{f}$.	Estimated social emissions, $E^{\tilde{f}}(K_1) = E^R(K_1) - \tilde{f}\, B(K_1)$.
True social accounting	Capital is allocated by decentralized investors under accounting rule $\phi = f$.	True social emissions used as the priced metric, $E^f(K_1) = E^R(K_1) - f\, B(K_1)$.

allocation mechanism: decentralized capital allocation versus planner choice. The second is the environmental object entering decisions. The measurement wedge arises because decentralized agents generally respond to $E^{\phi}(K_1)$, while the planner evaluates allocations using $E^*(K_1)$.

Given the equilibrium allocation K_1, define true social welfare as

$$\mathcal{W}(K_1) := \frac{C_1(K_1)^{\gamma_1}\, C_2(K_1)^{\gamma_2}}{\big(1 + E^*(K_1)\big)^{\delta}}, \tag{36}$$

where

$$E^*(K_1) = E^R(K_1) - f\, B(K_1), \qquad B(K_1) = K_1\, b_1(K_1) + (1 - K_1)\, b_2(K_1). \tag{37}$$

The planner chooses K_1 to maximize $\mathcal{W}(K_1)$, taking into account the effect of capital allocation on output, consumption, recycling, and true aggregate emissions.

To state the planner's first-order condition compactly, define

$$\Lambda(K_1) = \gamma_1 \frac{C_1'(K_1)}{C_1(K_1)} + \gamma_2 \frac{C_2'(K_1)}{C_2(K_1)}. \tag{38}$$

This term summarizes the marginal effect of reallocating capital on the consumption component of welfare.

The next result isolates the paper's normative core: the marginal environmental gradient relevant for welfare generally differs from the one embedded in decentralized accounting-based capital allocation.

Proposition 2 (Planner FOC and the accounting-induced environmental-gradient wedge)**.** *Suppose that $\mathcal{W}$ is continuously differentiable and that the first-best allocation $K_1^{FB} \in (0,1)$ is interior. Then K_1^{FB} satisfies*

$$\Lambda(K_1^{FB}) = \delta \, \frac{E^{*\prime}(K_1^{FB})}{1 + E^{*}(K_1^{FB})}. \tag{39}$$

Moreover, true aggregate emissions satisfy

$$E^{*\prime}(K_1) = E^{R\prime}(K_1) - f \, B'(K_1), \tag{40}$$

whereas the environmental gradient induced by accounting rule ϕ is

$$E^{\phi\prime}(K_1) = E^{R\prime}(K_1) - \phi \, B'(K_1). \tag{41}$$

Hence the measurement wedge is

$$\mathcal{M}(K_1; \phi) := E^{*\prime}(K_1) - E^{\phi\prime}(K_1) = (f - \phi) \, B'(K_1). \tag{42}$$

Equation (42) identifies the paper's central wedge. The wedge $\mathcal{M}(K_1; \phi)$ is not itself the decentralized equilibrium condition. Rather, it measures the discrepancy between the environmental gradient associated with the accounting object priced in G and the welfare-relevant environmental gradient used by the planner. When $\phi \neq f$, the environmental gradient associated with the accounting object priced by decentralized agents differs from the welfare-relevant one by $(f - \phi) \, B'(K_1)$. Accounting misalignment therefore changes the marginal reallocation signal, not only the level of measured emissions. Put differently, the accounting rule determines the environmental base to which decentralized markets attach an implicit shadow price, thereby acting as an implicit Pigouvian pricing system over the reported emissions object.

The term $B'(K_1)$ is the object that makes the problem an equilibrium problem rather than a purely accounting problem. If capital is reallocated toward sector 1, aggregate displacement changes for two reasons. First, the economy mechanically changes its composition: more projects operate in sector 1 and fewer projects operate in sector 2. If sector 1 has a larger displacement base per project than sector 2, this composition effect raises aggregate displacement even if output per project were held fixed.

Second, reallocating capital changes equilibrium quantities. Sectoral output, material demand, recycled-input use, and prices all adjust when K_1 changes. These equilibrium responses affect the amount of recycled input used by each active project and therefore the amount of virgin activity displaced elsewhere in the economy. The planner internalizes both margins. A firm-level accounting metric, by contrast, records the displacement base associated with a project at the current allocation; it does not generally encode how aggregate displacement would change after a marginal reallocation of capital.[11]

The next result states the paper's central implementation result. Even true social accounting need not implement the planner allocation, because the planner's marginal condition depends on an aggregate derivative of emissions rather than on contemporaneous firm accounting levels alone.

Proposition 3 (Level-based firm accounting does not generally implement the planner allocation)**.** *Suppose decentralized agents allocate capital by equalizing perceived firm-level payoffs. Under accounting rule ϕ, the relative-payoff condition is*

$$G(K_1, \phi) = \pi_1(K_1) - \pi_2(1 - K_1) - \tau \left(e_1^{\phi}(K_1) - e_2^{\phi}(K_1) \right). \tag{43}$$

[11] Appendix E decomposes $B'(K_1)$ into these two terms.

In particular, under true social accounting $\phi = f$,

$$G(K_1, f) = \pi_1(K_1) - \pi_2(1 - K_1) - \tau \left(e_1^f(K_1) - e_2^f(K_1) \right). \tag{44}$$

Suppose that true social aggregate emissions can be written as

$$E^*(K_1) = K_1 e_1^*(K_1) + (1 - K_1) e_2^*(K_1), \tag{45}$$

with e_1^* and e_2^* differentiable. Then the planner-relevant aggregate emissions gradient satisfies

$$E^{*\prime}(K_1) = e_1^*(K_1) - e_2^*(K_1) + K_1 \frac{de_1^*(K_1)}{dK_1} + (1 - K_1) \frac{de_2^*(K_1)}{dK_1}. \tag{46}$$

Even when $\phi = f$, the environmental component of the decentralized relative-payoff condition in Equation (44) *depends on the level-based firm accounting difference*

$$e_1^f(K_1) - e_2^f(K_1). \tag{47}$$

By contrast, the planner's environmental marginal condition depends on $E^{*\prime}(K_1)$, which also includes the equilibrium-response terms

$$K_1 \frac{de_1^*(K_1)}{dK_1} + (1 - K_1) \frac{de_2^*(K_1)}{dK_1}. \tag{48}$$

Therefore, unless these response terms vanish or are exactly summarized by the level difference priced in $G(K_1, f)$, decentralized pricing of level-based firm accounting levels does not generally implement the planner allocation.

Proposition 3 identifies a structural implementation failure. The issue is not only that decentralized agents may price the wrong coefficient on displacement. More fundamentally, decentralized allocation is based on contemporaneous project accounting levels, whereas the planner's condition depends on an aggregate marginal response. Setting $\phi = f$ removes the coefficient wedge identified from Proposition 2, but it does not generally eliminate the implementation wedge, because equilibrium-response terms remain outside level-based firm accounting. They are therefore generally not sufficient statistics for the planner's derivative-based problem.

5 Comparative Statics and Welfare

The previous section identifies two wedges between decentralized accounting-based allocation and the planner benchmark: a coefficient wedge, when $\phi \neq f$, and an implementation wedge, because firm-level accounting levels need not encode aggregate marginal responses. This section derives two implications of these wedges. First, displacement can reverse environmental rankings across sectors. Second, accounting improvements raise welfare only when they move capital in the planner-preferred direction.

5.1 Ranking Reversals

The first implication of the accounting-gradient wedge is a ranking reversal: displacement can overturn environmental rankings based on realized emissions alone. What matters for marginal reallocation is not only direct emissions, but the comparison between direct emissions and marginal displacement capacity.

The next result shows that environmental rankings depend on displacement as well as firm-boundary emissions, and that the two can point in opposite directions.

Proposition 4 (Threshold for environmental ranking reversals)**.** *Fix any benchmark allocation $K_1^\circ \in (0,1)$ such that $B'(K_1^\circ) \neq 0$. Define the threshold*

$$f^\circ := \frac{E^{R\prime}(K_1^\circ)}{B'(K_1^\circ)}. \tag{49}$$

The threshold f° is the displacement coefficient at which the firm-boundary emissions *margin and the displacement margin exactly offset each other.*

Then

$$E^{*\prime}(K_1^\circ) = E^{R\prime}(K_1^\circ) - f\,B'(K_1^\circ) = B'(K_1^\circ)\big(f^\circ - f\big), \tag{50}$$

so that

$$\operatorname{sign}\big(E^{*\prime}(K_1^\circ)\big) = \operatorname{sign}\big(B'(K_1^\circ)\big)\ \operatorname{sign}\big(f^\circ - f\big). \tag{51}$$

In particular, if $B'(K_1^\circ) > 0$ corresponding to reallocations toward sector 1 increasing aggregate displacement capacity at the margin, then:

$$f > f^\circ \qquad \Longrightarrow \qquad E^{*\prime}(K_1^\circ) < 0, \tag{52}$$

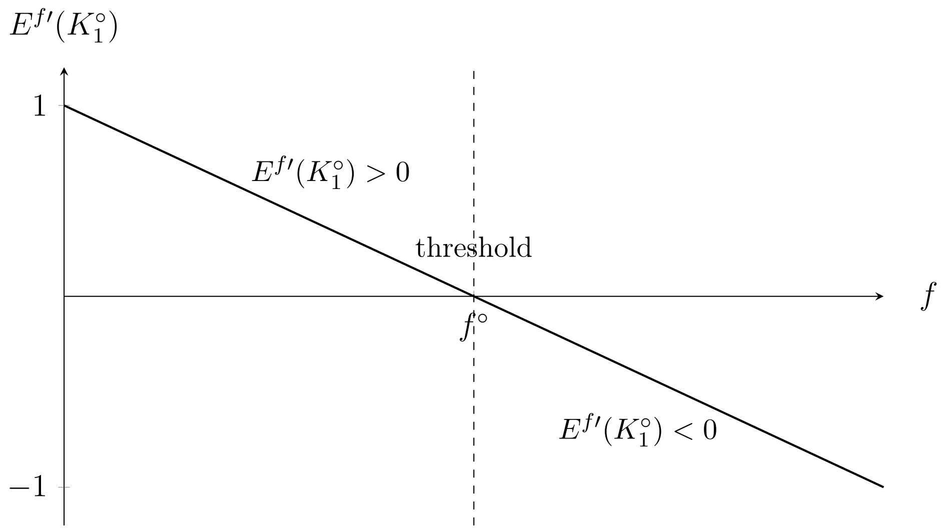
so a marginal increase in K_1 reduces true social aggregate emissions. If instead $f < f^\circ$, the environmental effect is reversed.

Proposition 4 is best understood as a ranking result. This result concerns the environmental ranking of marginal reallocations. It does not by itself characterize the decentralized equilibrium allocation, which remains governed by $G(K_1^\phi, \phi) = 0$. Under firm-boundary accounting, sectors are ranked only by the direct emissions consequences of reallocating capital. Under estimated social accounting, sectors are ranked by the net effect of reallocation once displaced emissions are taken into account. The threshold f° marks the point at which these two ranking criteria switch places. In particular, a sector can appear environmentally worse under firm-boundary emissions and yet be socially preferable once its displacement capacity is sufficiently large. The relevant comparison is therefore not firm-boundary emissions alone, but firm-boundary emissions relative to marginal displacement.

Accounting standards matter because they determine whether capital markets respond only to the firm-boundary emissions side of this comparison or to the displacement side.

Figure 1 illustrates Proposition 4. It shows how the sign of the true environmental gradient depends on the displacement factor f relative to the threshold f°. The figure makes clear that the ranking of sectors can reverse when the displacement factor is sufficiently large. Below the threshold, firm-boundary emissions dominate the environmental comparison. Above the threshold, displacement dominates and the sign of the true environmental gradient changes. This threshold logic is the key to understanding when estimated social accounting improves welfare relative to firm-boundary accounting: welfare rises only when the induced capital reallocation moves in the planner-preferred direction.

Figure 1: Threshold for environmental ranking reversals



Notes: The figure plots the true environmental gradient $E^{f\prime}(K_1^\circ)$ as a function of the displacement factor f, for the case $B'(K_1^\circ) > 0$. The threshold is $f^\circ = \frac{E^{R\prime}(K_1^\circ)}{B'(K_1^\circ)}$. Below the threshold, the gradient is positive; above it, the gradient is negative.

5.2 Accounting Misalignment and Welfare

The second implication concerns welfare. Estimated social accounting can improve welfare relative to firm-boundary accounting only when the induced reallocation moves capital in the planner-preferred direction. Because true welfare depends on the allocation K_1 but not directly on the accounting parameter ϕ, the welfare effect of changing accounting works entirely through induced capital reallocation.

The result below is deliberately local: its purpose is to identify the sign of the welfare effect of moving away from firm-boundary accounting when the accounting-induced reallocation is small.

Proposition 5 (Local welfare effect of adjusted accounting)**.** *Suppose that $\mathcal{W}$ is differentiable and that the decentralized equilibrium allocation K_1^ϕ is differentiable in ϕ around $\phi = 0$.*

Then

$$\frac{d}{d\phi}\mathcal{W}(K_1^\phi)\Big|_{\phi=0} = \mathcal{W}'(K_1^0)\,\frac{\partial K_1^\phi}{\partial \phi}\Big|_{\phi=0}. \tag{53}$$

Hence, if

$$\mathcal{W}'(K_1^0) > 0 \qquad \textit{and} \qquad b_1(K_1^0) > b_2(K_1^0), \tag{54}$$

then

$$\frac{d}{d\phi}\mathcal{W}(K_1^\phi)\Big|_{\phi=0} > 0, \tag{55}$$

so a marginal increase in the accounting weight on displacement raises true welfare. Conversely, if $\mathcal{W}'(K_1^0) < 0$ while $b_1(K_1^0) > b_2(K_1^0)$, then increasing ϕ lowers welfare locally.

Equation (53) in Proposition 5 makes clear that accounting has no direct welfare effect in the model: its welfare consequences operate entirely through the capital-allocation response it induces. Proposition 4 identifies when accounting can reverse environmental rankings; Proposition 5 shows when such accounting-induced reallocation is actually welfare-improving. Proposition 5 gives a disciplined local version of the partial-correction logic. Adjusted accounting improves welfare when it changes capital allocation in the same direction as the planner would. But this is not automatic: if displacement credits are attached to the wrong margin, or if the planner locally prefers reallocation in the opposite direction, increasing the accounting weight on displacement can lower welfare instead.

A direct local implication is that, if the accounting-induced reallocation moves capital in the planner-preferred direction at $\phi = 0$, then sufficiently small positive displacement weights improve welfare relative to firm-boundary accounting. Estimated social accounting should therefore be understood as an implementable proxy for the true consequential emissions metric $E^*(K_1)$, not as a proxy for the planner allocation itself. What is estimated is the displacement coefficient f: the emissions intensity of the marginal activity displaced by one unit of recycled input. In practice, this estimate may be produced by firms, data providers, standard setters, consultants, or regulators using engineering assumptions, life-cycle data, sectoral benchmarks, or consequential LCA methods. It can differ from f because the relevant counterfactual activity is not directly observed, depends on system boundaries and baselines, and may vary with market conditions. Thus $\widetilde{f}$ is useful precisely because it makes consequential information priceable, but it remains an imperfect approximation to the welfare-relevant coefficient.

Figure 2 illustrates the welfare consequences of changing the primitives that determine true consequential emissions.[12] The common shape of the welfare schedules reflects two forces. The first is the standard allocative component in the closed-form welfare decomposition: moving capital away from the consumption-efficient allocation lowers the consumption term. The second is the environmental component: reallocating capital toward the sector with recycling capacity changes firm-boundary emissions and displaced emissions at the same time. The resulting welfare curve is therefore not monotone in K_1. It initially rises or remains high when the allocation combines favorable consumption effects with displacement benefits, then declines as the allocative cost and direct-emissions cost dominate, and may recover slightly at high values of K_1 when displacement becomes sufficiently large.

The benchmark case (black line in Figures 2 and 3) provides the natural benchmark

[12]Figure 2 is constructed using the closed-form welfare representation derived in Proposition 7 in Appendix D.4, under the benchmark parameterization described in the notes.

because it shuts down the consequential displacement channel while leaving the rest of the equilibrium environment unchanged. In this sense, it is the closest analogue to the framework of Landier and Lovo (2025). The welfare differences relative to this benchmark therefore isolate the contribution of accounting for equilibrium displacement rather than changes in the underlying economic environment.

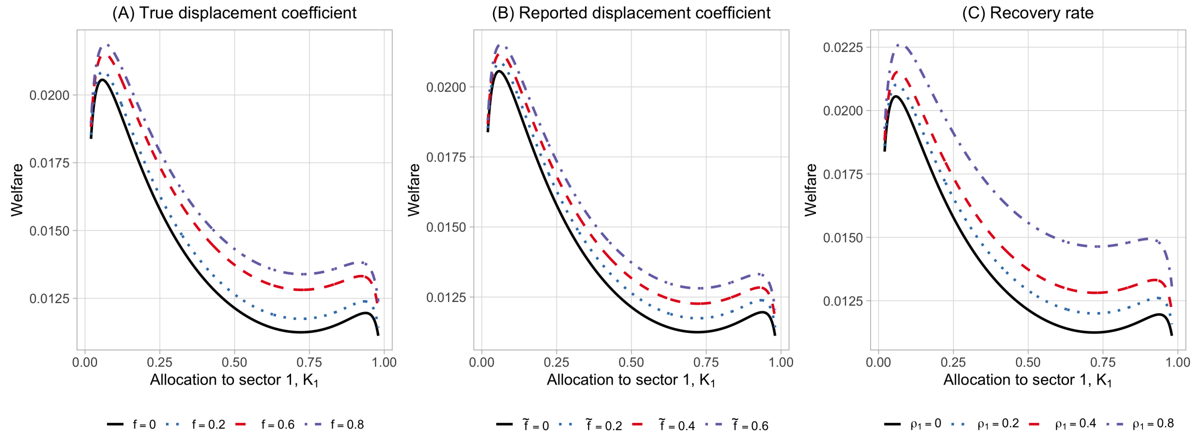
Panel A varies the true displacement coefficient f. A larger f raises the welfare value of each unit of recycled input because each unit displaces a dirtier counterfactual activity. The vertical distance between the curves is therefore an economic measure of the value of correctly recognizing displacement. Importantly, the differences are not confined to the welfare maximum. Around intermediate allocations such as $K_1 = 0.5$ or $K_1 = 0.75$, the curves remain visibly separated. These intermediate regions are empirically relevant because real-world capital allocation is unlikely to coincide with the planner optimum. Accounting quality matters precisely away from the optimum, where capital is still allocated across both high-emission and transition activities.

Panel C varies the recovery rate ρ_1. A higher recovery rate increases the displacement base $B(K_1)$, so it amplifies the welfare value of the same true displacement coefficient f. Economically, f determines the value of one unit of displacement, while ρ_1 determines how much displacement capacity is generated by production. The two parameters therefore interact: better recovery technology makes consequential accounting more important because more activity lies on the displacement margin.

Figure 3 reports the same comparative statics in relative terms. This normalization is useful because it shows that the welfare consequences are quantitatively meaningful over a broad range of allocations, not only at the maximum.

Panel A illustrates that substantial welfare improvements do not require perfect consequential accounting. In the benchmark calibration, an accounting rule that recognizes only one-third of the true consequential displacement coefficient still raises welfare by about 4% relative to firm-boundary accounting over the empirically relevant range of intermediate allocations. This result highlights an important economic point. The value

Figure 2: Welfare at Accounting-Induced Allocations



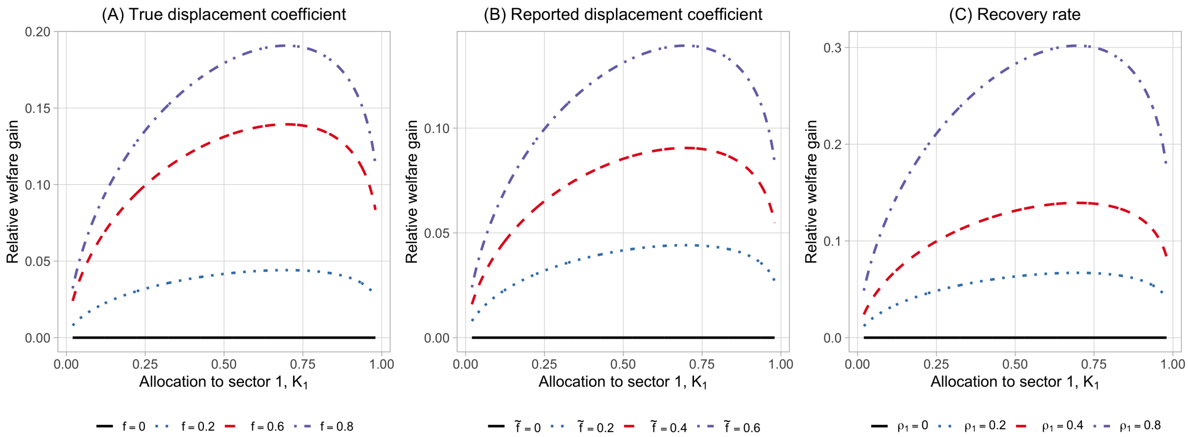
Notes: The figure reports comparative statics for true social welfare $\mathcal{W}(K_1)$ across capital allocations to sector 1, K_1, under alternative parameter values. Panel A varies the true displacement coefficient f. Panel B varies the reported displacement coefficient $\widetilde{f}$. Panel C varies the recovery rate ρ_1. In each panel, all other parameters are held at their benchmark values unless otherwise indicated: $A_1 = A_2 = 1$, $\alpha_1 = \alpha_2 = 0.5$, $\gamma_1 = \gamma_2 = 0.5$, $\theta_1 = \theta_2 = 0.5$, $\rho_1 = 0.4$, $\rho_2 = 0$, $\varepsilon_1^{prod} = 1.2$, $\varepsilon_2^{prod} = 0.2$, $\varepsilon_1^{waste} = \varepsilon_2^{waste} = 0.5$, $\varepsilon_1^{rec} = \varepsilon_2^{rec} = 0.5$, $\delta = 6$, $f = 0.6$, and $\widetilde{f} = 0.4$. The figure is included for intuition and does not represent a general analytical result.

of improving accounting does not depend on fully recovering the planner's information set. Even partial recognition of consequential effects can materially improve decentralized capital allocation. By comparison, increasing the true displacement coefficient from $f = 0.2$ to $f = 0.6$ raises welfare by roughly 6–9% over the same allocation range, while moving to $f = 0.8$ increases welfare by about 12–14%. These figures are not calibration targets but illustrate that accounting quality is economically material in the model.

Panel C shows an even stronger sensitivity to the recovery rate. When ρ_1 rises from 0.2 to 0.8, relative welfare gains exceed 20 percent over a wide intermediate range of allocations. This pattern is intuitive: a higher recovery rate expands the activity base to which the displacement coefficient applies. The model therefore predicts that accounting quality is most valuable in sectors where the displacement base is economically large.

These intermediate allocations are central for interpretation. If the economy were already at the first best, changing accounting would have limited marginal value. The relevant real-world case is different: capital remains allocated across both brown and transition activities, and accounting affects the direction and intensity of reallocation. The figures therefore support the paper's main message: imperfect but more consequential accounting can matter even when it does not fully implement the planner allocation.[13]

Figure 3: Relative welfare across capital allocations



Notes: The figure reports the relative change in true social welfare, $\frac{\mathcal{W}(K_1)}{\mathcal{W}^{\text{benchmark}}(K_1)} - 1$, across capital allocations to sector 1, K_1, under alternative parameter values. Panel A varies the true displacement coefficient f, with welfare normalized by the benchmark case $f = 0$. Panel B varies the reported displacement coefficient $\widetilde{f}$, with welfare normalized by the benchmark case $\widetilde{f} = 0$. Panel C varies the recovery rate ρ_1, with welfare normalized by the benchmark case $\rho_1 = 0$. A value of zero therefore corresponds to the benchmark specification, while positive (negative) values indicate proportional welfare gains (losses) relative to the benchmark. In each panel, all other parameters are held at their benchmark values unless otherwise indicated: $A_1 = A_2 = 1$, $\alpha_1 = \alpha_2 = 0.5$, $\gamma_1 = \gamma_2 = 0.5$, $\theta_1 = \theta_2 = 0.5$, $\rho_1 = 0.4$, $\rho_2 = 0$, $\varepsilon_1^{prod} = 1.2$, $\varepsilon_2^{prod} = 0.2$, $\varepsilon_1^{waste} = \varepsilon_2^{waste} = 0.5$, $\varepsilon_1^{rec} = \varepsilon_2^{rec} = 0.5$, $\delta = 6$, $f = 0.6$, and $\widetilde{f} = 0.4$. The figure is included for intuition and does not represent a general analytical result.

6 Implications

The model has four implications for sustainable finance. First, it clarifies how reported emissions enter financing decisions. Second, it identifies what must be estimated for consequential accounting to improve allocation. Third, it separates market pricing of carbon data from the welfare relevance of the data being priced. Fourth, it provides

[13] Additional comparative statics illustrating the sensitivity of decentralized allocation to the accounting coefficient ϕ and the pricing intensity τ are reported in Appendix F. They confirm the analytical comparative statics derived above and illustrate the distinct roles of accounting quality and environmental pricing intensity.

a disciplined interpretation of what accounting standards and financial institutions can realistically achieve.

6.1 Financial Interpretation

In the model, τ measures the intensity with which reported emissions affect decentralized payoffs. This parameter should not be interpreted narrowly as a carbon tax. It is a reduced-form representation of the many financial channels through which reported emissions affect capital allocation: exclusion screens, portfolio carbon constraints, benchmark weights, lending spreads, sustainability-linked financing terms, internal carbon budgets, regulatory constraints, or reputational costs faced by asset managers and lenders. The model only requires that some financing conditions depend on reported emissions. It does not require all investors to have social preferences.

This interpretation also explains why high-emission sectors can remain heavily financed. Reported emissions are only one component of project payoffs. Brown activities may remain profitable, may supply essential intermediate inputs, may be held by benchmark-constrained investors, or may be financed under transition mandates rather than exclusion mandates. In the model, this corresponds to the fact that $\tau e_i^{\phi}(K_1)$ enters perceived payoffs alongside financial profitability $\pi_i(K_i)$. The question is not whether capital markets fully divest from high-emission activities. The question is whether, conditional on reported emissions affecting financing terms, the accounting rule determines which environmental signal is priced.

This distinction matters for transition activities. A firm may increase its own firm-boundary emissions while reducing emissions elsewhere by displacing a dirtier activity. If financing conditions respond only to firm-boundary emissions, such a firm may be penalized even when its net environmental effect is beneficial. If financing conditions respond to a metric that credits displacement, the same activity may receive more capital. The difference is not investor sophistication or market efficiency. It is the definition of the environmental object embedded in the payoff.

The model therefore gives a finance interpretation of accounting design. Accounting rules determine the state variable that investors, lenders, and benchmark providers can condition on. When that state variable differs from the welfare-relevant externality, efficient pricing of the reported signal need not imply efficient capital allocation.

6.2 Measuring Consequential Emissions

The key object for consequential accounting is the true displacement coefficient f. In the model, f is the emissions intensity of the marginal activity displaced by one unit of recycled input. It is not directly observed in a standard emissions inventory because it is not a physical emission generated inside the reporting boundary. It is a counterfactual object: it asks which activity would have occurred elsewhere in the economy absent the activity being credited.

Several features make f difficult to estimate in practice. First, f is baseline-dependent. The relevant counterfactual may be average virgin production, marginal virgin production, imports, inventory drawdowns, delayed production, or no production at all. The appropriate baseline depends on the market environment. Second, f is system-boundary-dependent. The estimate changes with the geographic, sectoral, and temporal boundaries used to define the displaced activity. Third, f is market-mediated. If recycling expands, prices, demand, and production patterns may adjust, so the displaced activity need not be a fixed one-for-one technological substitute. Fourth, f may vary with scale. Initial units of displacement may replace high-emission production, while later units may displace cleaner marginal suppliers, or conversely may enable larger substitution effects.

These difficulties are precisely why the distinction between f and $\widetilde{f}$ matters. The true coefficient f is the welfare-relevant parameter. The reported coefficient $\widetilde{f}$ is an implementable estimate embedded in the accounting rule. It may be produced by firms, data providers, consultants, standard setters, or regulators using engineering estimates, consequential life-cycle assessment, marginal-supplier analysis, sectoral benchmarks, or scenario-based baselines. The estimate differs from f because the counterfactual is not fully observed and because market responses are only partially known.

The practical implication is not that every firm should compute a full general-equilibrium counterfactual for every activity. That would be unrealistic and would likely create excessive compliance costs. The relevant objective is more limited: reduce systematic misalignment between $\widetilde{f}$ and f where the displacement base is economically material. In the language of the model, firm-boundary accounting corresponds to $\phi = 0$. Estimated social accounting improves the priced signal when $\widetilde{f}$ is directionally closer to f than zero. Perfect consequential measurement is not required for accounting to improve allocation.

This point is important for interpreting imperfect avoided-emissions estimates. A noisy or conservative estimate of displacement is not a claim that accounting implements the first best. It is an attempt to move the priced metric closer to the welfare-relevant margin. The model therefore supports incremental improvements: disclose the baseline, state the system boundary, report uncertainty ranges when appropriate, update coefficients as markets change, and prioritize activities for which $B(K_1)$ is large enough that the accounting choice can materially affect allocation.

6.3 Implications for Empirical Finance

The model separates two empirical questions that are often conflated. The first is whether financial markets price reported environmental information. The second is whether the reported information corresponds to the welfare-relevant environmental margin. An empirical finding that markets price carbon emissions, carbon intensity, ESG scores, or avoided-emissions claims answers the first question. It does not by itself answer the second.

This distinction changes the interpretation of empirical work on carbon metrics. Regressions of returns, ownership, lending spreads, or investment flows on reported emissions identify how markets respond to a reported signal. They do not identify whether the signal is the correct environmental object. In the model, markets can price $E^{\phi}(K_1)$ efficiently and still allocate capital inefficiently if $E^{\phi}(K_1)$ differs from $E^{*}(K_1)$. Market efficiency with respect to the disclosed metric is therefore compatible with welfare inefficiency.

The model suggests that empirical work should measure not only whether carbon data are priced, but also which accounting object is priced. A useful empirical design would distinguish firm-boundary emissions from adjusted metrics that incorporate credible displacement or substitution effects. Another would examine whether changes in accounting methodologies or data-provider assumptions shift capital allocation toward firms with larger displacement bases. A third would estimate sector-specific displacement coefficients and test whether reported avoided-emissions claims correspond to actual marginal substitution.

The model also implies that disagreement across ESG ratings, carbon data vendors, or accounting methodologies can have allocative consequences. If different providers construct different E^{ϕ} objects, investors following those signals are not merely interpreting the same information differently. They may be allocating capital on the basis of different environmental objects. This gives a structural interpretation to measurement heterogeneity in sustainable finance: it is not only noise in disclosure, but variation in the signal that financial constraints and investment rules condition on.

6.4 Policy Implications

The policy implication is not simply "better accounting." The model gives a more specific prescription. Accounting standard setters should preserve the distinction between physical emissions inventories and consequential displacement claims. Firm-boundary emissions remain necessary because they measure emissions generated inside the reporting boundary. Consequential metrics are complementary because they capture substitution effects that can be relevant for capital allocation. Combining the two without clearly identifying the baseline, displacement coefficient, and system boundary would obscure rather than improve the signal (WBCSD, 2025; PCAF, 2025).

For standard setters, the realistic objective is to develop conservative, transparent, sector-level protocols for activities where displacement is material. Such protocols should specify the displacement base, the baseline activity, the relevant boundary, the coefficient used, and the uncertainty around that coefficient. The model does not imply that all firms should estimate economy-wide counterfactuals. It implies that ignoring displacement entirely can systematically distort capital allocation when displacement is economically large.

For financial institutions, the model implies that firm-boundary screens and transition-oriented metrics should be treated as distinct instruments. A portfolio rule based only on firm-boundary emissions may reduce reported portfolio emissions while discouraging activities that reduce system-wide emissions. Conversely, a transition metric that credits displacement should be tied to transparent assumptions about $\widetilde{f}$ and $B(K_1)$, rather than to unverifiable avoided-emissions claims. Financial institutions should therefore report not only the emissions level used in allocation decisions, but also the accounting rule that generates it.

For regulators and data providers, the priority should be comparability and sensitivity. Data providers should report the displacement coefficient embedded in an adjusted metric and show how reported emissions vary under alternative plausible values of that coefficient. Regulators should encourage disclosure that separates inventories from counterfactual claims, rather than forcing all environmental information into a single reported number. This would allow investors to understand whether they are pricing firm-boundary emissions, estimated consequential emissions, or a hybrid metric.

For researchers, the model identifies the missing empirical object: the mapping from reported metrics to welfare-relevant marginal emissions. Future work should estimate sector-specific displacement coefficients, identify market-mediated substitution effects, and test when accounting adjustments move capital toward activities with lower true social emissions. The relevant research agenda is therefore not only whether investors price carbon data, but whether the priced carbon data are allocatively informative.

The broader lesson is that carbon accounting is part of the allocation mechanism in sustainable finance. When reported emissions affect financing conditions, accounting determines which environmental object receives a shadow price. Better accounting can move decentralized allocation toward the welfare-relevant margin, but it cannot generally implement the first best because the planner's condition depends on aggregate marginal responses that firm-level accounting levels do not contain. Accounting is therefore an allocative device, but not a sufficient statistic for social efficiency.

7 Conclusion

We study how carbon accounting affects capital allocation when the environmental effect of production is not fully contained within the reporting firm's boundary. In the model, recycling raises realized emissions locally but displaces more carbon-intensive production elsewhere in the economy, so the environmental object priced by decentralized agents need not coincide with the welfare-relevant externality.

Three results follow. First, accounting is not allocatively neutral: changing the accounting rule changes equilibrium capital allocation because it changes the emissions object entering decentralized payoffs. Second, mismeasured displacement creates a wedge between the priced signal and the welfare-relevant one. Third, level-based firm accounting cannot generally implement the planner allocation, because the planner's condition depends on aggregate marginal responses that are not contained in contemporaneous firm-level emissions levels.

The paper thus clarifies both the role and the limit of carbon accounting in capital allocation. Better accounting improves decentralized allocation by improving the environmental signal embedded in financing decisions rather than by changing investor preferences or market efficiency. In the benchmark calibration, recognizing only one-third of the true consequential displacement effect already generates welfare gains of about 4% relative to firm-boundary accounting. More broadly, the paper highlights a general limitation of accounting-based allocation systems: whenever the welfare-relevant externality is an equilibrium object rather than a directly reportable firm-level attribute, measurement rules become part of the allocation mechanism itself.

A natural extension would be to allow the emissions intensity of displaced activity to vary with the scale of recycling. Another promising direction is to enrich the information structure. The present paper deliberately abstracts from heterogeneous beliefs in order to isolate the allocative role of accounting. Allowing investors to form beliefs about consequential emissions would make it possible to study jointly the design of accounting standards and the informational efficiency of capital markets. More generally, the framework could serve as the basis for deriving empirically implementable sufficient statistics linking firm-level accounting measures to welfare-relevant environmental effects, in the spirit of recent work using reduced-form sufficient statistics for policy evaluation (Allcott et al. 2026).

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A Notation Summary

Table 3 summarizes the main notation used in the paper.

Table 3: Summary of notation used in the paper

Notation	Interpretation	Location / role
Allocation and equilibrium		
$i \in \{1, 2\}$	Sector index.	Model setup
$j \neq i$	The other sector.	Model setup
K_i	Capital allocated to sector i; equivalently, mass of active projects/firms in sector i.	Main text, Section 2
$K_1 + K_2 = 1$	Aggregate capital endowment normalized to one.	Main text, Section 2
K_1^{ϕ}	Decentralized equilibrium allocation to sector 1 under accounting rule ϕ.	Section 3
K_1^{FB}	First-best allocation chosen by the planner.	Section 4
K_1°	Benchmark allocation used for local ranking comparisons.	Proposition 4
$\pi_i(K_i)$	Financial profit per project in sector i.	Decentralized equilibrium
$\widetilde{\Pi}_i^{\phi}(K_1)$	Investor-perceived payoff of a representative project in sector i under accounting rule ϕ.	Decentralized allocation
$\tau \geq 0$	Reduced-form shadow cost assigned by investors to measured emissions.	Decentralized allocation
$G(K_1, \phi)$	Difference in perceived net payoffs between sectors 1 and 2 under accounting rule ϕ.	Equilibrium characterization

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Notation	Interpretation	Location / role
$G_{K_1}(K_1, \phi)$	Partial derivative of G with respect to K_1.	Proposition 1
$G_\phi(K_1, \phi)$	Partial derivative of G with respect to ϕ.	Proposition 1
Technology and recycling		
A_i	Sector-specific productivity parameter.	Technology block
$\alpha_i \in (0, 1)$	Output elasticity with respect to the materials bundle; ensures diminishing returns.	Technology block
y_i	Output per project in sector i.	Technology block
$Y_i = K_i\, y_i$	Aggregate output in sector i.	Technology block
m_i	Materials bundle used by a representative firm in sector i.	Technology block
n_i	Virgin material input used by sector i, purchased from sector j.	Technology block
r_i	Recycled input used per project in sector i.	Technology / accounting base
R_i	Aggregate recycled-input supply in sector i.	Recycling market
$\omega \in (0, 1)$	CES weight on virgin material in the materials bundle.	Technology block
$\sigma > 0$	Elasticity of substitution between virgin and recycled inputs in the CES materials bundle.	Technology block
q_i	Dual price index of the materials bundle in sector i.	Technology / cost minimization
p_i^c	Consumer price of final good i.	Goods market / household demand
p_i^r	Price of recycled input used by sector i.	Recycling market

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Notation	Interpretation	Location / role
Θ_i	Waste generated per project in sector i.	Technology block
Θ_i^{agg}	Aggregate waste generated in sector i.	Technology block
$\theta_i \in [0,1]$	Waste intensity of output in sector i.	Technology block
$\rho_i \in [0,1]$	Recovery rate converting waste into recycled material in sector i.	Recycling block
Households, welfare, and planner		
C_i	Consumption of final good i.	Household side / welfare
$U(C_1, C_2, E^*)$	Household utility as a function of consumption and the true externality.	Household side
W	Aggregate household income.	Household demand / appendix closed form
$\gamma_i \in (0,1)$	Cobb-Douglas expenditure share on good i.	Household side
$\gamma_1 + \gamma_2 = 1$	Total expenditure shares sum to one.	Household side
$\delta > 0$	Weight of environmental disutility in welfare.	Household side / planner
$\mathcal{W}(K_1)$	True social welfare as a function of the capital allocation.	Section 4
$\Lambda(K_1)$	Marginal effect of reallocation on the consumption component of log welfare.	Proposition 2
$\mathcal{M}(K_1;\phi)$	Measurement wedge between the true environmental gradient and the accounting-induced gradient.	Proposition 2

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Notation	Interpretation	Location / role
Emissions and accounting		
ε_i^{prod}	Emissions intensity of production in sector i.	Emissions block
ε_i^{waste}	Emissions intensity of unrecycled waste in sector i.	Emissions block
ε_i^{rec}	Emissions intensity of recycling activity in sector i.	Emissions block
$e_i^R(K_1)$	Realized emissions per project in sector i.	Emissions block
$E^R(K_1)$	Aggregate realized emissions in the economy.	Emissions block
f	True consequential emissions intensity of the marginal activity displaced by one unit of recycled input.	Emissions / welfare benchmark
$\widetilde{f}$	Reported displacement coefficient embedded in an implementable accounting rule.	Accounting block
ϕ	Generic accounting coefficient applied to the displacement base; indexes the reported-emissions rule.	Accounting block
$e_i^\phi(K_1)$	Firm-level accounting metric priced by decentralized agents under rule ϕ.	Accounting block
$E^\phi(K_1)$	Aggregate accounting metric priced by decentralized agents under rule ϕ.	Accounting block
$e_i^*(K_1)$	True consequential emissions per project in sector i, equal to realized emissions net of true displacement.	Welfare-relevant emissions
$E^*(K_1)$	True welfare-relevant aggregate externality, equal to realized aggregate emissions net of displaced emissions.	Welfare / planner
$b_i(K_1)$	Firm-level accounting base for displacement credits; in the core model, $b_i(K_1) = r_i(K_1)$.	Decentralized equilibrium

Continued on next page

Notation	Interpretation	Location / role
$B(K_1)$	Aggregate displacement capacity, equal to aggregate recycled-input use.	Emissions / displacement
$B'(K_1)$	Marginal change in aggregate displacement capacity when capital is reallocated.	Planner wedge / threshold result
f°	Threshold value of the true displacement coefficient at which realized-emissions and displacement margins exactly offset at K_1°.	Proposition 4
Appendix closed-form objects		
X_i	Aggregate intermediate demand for good i from the other sector in the appendix closed-form block.	Appendix D.3
K_i^*	Closed-form benchmark level of aggregate sectoral profit in sector i.	Appendix D.3
Z_i	Closed-form exponent appearing in the welfare decomposition; equal to $\frac{K_i^*}{1-\alpha_i}$.	Appendix D.4
B_i	Auxiliary closed-form object linking prices, productivity, and sector size in the welfare derivation.	Appendix D.4
β	Positive constant collecting technology and preference parameters in the closed-form welfare decomposition.	Appendix D.4

B Summary of Main Results

Table 4 summarizes the propositions and lemmas in the main text and appendices.

Table 4: Summary of propositions and lemmas

Result	**Location**	**Role**
Main-text results		
Proposition 1	Section 3	Establishes existence and uniqueness of decentralized equilibrium under accounting rule ϕ, and characterizes how equilibrium allocation changes with the accounting coefficient.
Proposition 2	Section 4	Derives the planner's first-order condition and identifies the measurement wedge between the welfare-relevant environmental gradient and the accounting-induced one.
Proposition 3	Section 4	Shows that level-based firm accounting cannot generally implement the planner allocation, because the planner's marginal condition depends on equilibrium-response terms absent from project-level accounting levels.
Proposition 4	Section 5	Defines the threshold value of the true dispplacement coefficient at which realized-emissions and consequential-emissions rankings switch locally.
Proposition 5	Section 5	Characterizes the local welfare effect of increasing the accounting weight on avoided emissions through the induced capital reallocation.

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Result	Location	Role
Appendix microfoundations and closed-form blocks		
Proposition 6	Appendix D.1	Derives the CES materials block, its dual price index, Hicksian input demands, optimal materials demand, output, and per-project operating profit.
Lemma 1	Appendix D.2	Derives Marshallian demand and indirect utility for the representative household under Cobb-Douglas preferences with environmental disutility.
Lemma 2	Appendix D.3	Solves the benchmark goods-market-clearing system and delivers the reduced-form profit-erosion expression $\pi_i(K_i) = \frac{K_i^*}{K_i}$.
Proposition 7	Appendix D.4	Provides a closed-form welfare decomposition separating productivity, allocative, and environmental components.
Appendix decompositions and auxiliary results		
Lemma 3	Appendix E	Decomposes the displacement gradient into a composition effect and an equilibrium-response effect, clarifying the structure of the welfare-relevant emissions gradient.

C Proofs for the Main Text

C.1 Proof of Proposition 1

Proof. The proof proceeds in three steps.

Step 1. Existence of an interior equilibrium.

Fix an accounting rule ϕ. By the conditions stated in Proposition 1, the function $K_1 \mapsto G(K_1, \phi)$ is continuously differentiable on $(0, 1)$, hence continuous on $(0, 1)$.

Moreover, the assumption imposes the boundary sign conditions

$$\lim_{K_1 \to 0} G(K_1, \phi) > 0, \qquad \lim_{K_1 \to 1} G(K_1, \phi) < 0. \tag{56}$$

Therefore, there exist $\underline{K}, \overline{K} \in (0, 1)$ with $\underline{K} < \overline{K}$ such that

$$G(\underline{K}, \phi) > 0, \qquad G(\overline{K}, \phi) < 0. \tag{57}$$

By continuity of $G(\cdot, \phi)$, the Intermediate Value Theorem implies that there exists at least one $K_1^\phi \in (\underline{K}, \overline{K}) \subset (0, 1)$ such that

$$G(K_1^\phi, \phi) = 0. \tag{58}$$

Hence an interior decentralized equilibrium exists.

Step 2. Uniqueness.

The same conditions also imply that, for every ϕ in the relevant range, the function $K_1 \mapsto G(K_1, \phi)$ is strictly decreasing on $(0, 1)$.

Suppose, toward a contradiction, that there exist two distinct interior equilibria $K_1^a \neq K_1^b$ satisfying

$$G(K_1^a, \phi) = 0, \qquad G(K_1^b, \phi) = 0. \tag{59}$$

Without loss of generality, let $K_1^a < K_1^b$. Since $G(\cdot, \phi)$ is strictly decreasing, it follows that

$$G(K_1^a, \phi) > G(K_1^b, \phi), \tag{60}$$

which contradicts the fact that both values equal zero. Therefore, the equilibrium is unique.

This proves the first claim: for each accounting rule ϕ, there exists a unique interior equilibrium allocation $K_1^\phi \in (0, 1)$ satisfying Equation (32).

Step 3. Comparative statics with respect to the accounting rule.

Assume now that G is continuously differentiable in both arguments. Define the equilibrium correspondence implicitly by

$$G(K_1^\phi, \phi) = 0. \tag{61}$$

Because the equilibrium is interior and unique, and because

$$G_{K_1}(K_1^\phi, \phi) < 0 \tag{62}$$

by strict monotonicity, the Implicit Function Theorem applies. Hence there exists a differentiable map $\phi \mapsto K_1^\phi$ in a neighborhood of any given ϕ, and

$$\frac{\partial K_1^\phi}{\partial \phi} = -\frac{G_\phi(K_1^\phi, \phi)}{G_{K_1}(K_1^\phi, \phi)}. \tag{63}$$

It therefore remains to compute G_ϕ. Recall from Equation (30) that

$$G(K_1, \phi) = \pi_1(K_1) - \pi_2(1 - K_1) - \tau\Big(e_1^\phi(K_1) - e_2^\phi(K_1)\Big), \tag{64}$$

and that

$$e_i^{\phi}(K_1) = e_i^R(K_1) - \phi\, b_i(K_1). \tag{65}$$

Substituting into G yields

$$\begin{aligned} G(K_1,\phi) &= \pi_1(K_1) - \pi_2(1-K_1) - \tau\Big[(e_1^R(K_1) - \phi\, b_1(K_1)) - (e_2^R(K_1) - \phi\, b_2(K_1))\Big] \quad (66) \\ &= \pi_1(K_1) - \pi_2(1-K_1) - \tau\big(e_1^R(K_1) - e_2^R(K_1)\big) + \tau\,\phi\big(b_1(K_1) - b_2(K_1)\big). \quad (67) \end{aligned}$$

Differentiating with respect to ϕ gives

$$G_{\phi}(K_1,\phi) = \tau\big(b_1(K_1) - b_2(K_1)\big). \tag{68}$$

Substituting into Equation (63) yields

$$\frac{\partial K_1^{\phi}}{\partial \phi} = -\frac{\tau\big(b_1(K_1^{\phi}) - b_2(K_1^{\phi})\big)}{G_{K_1}(K_1^{\phi},\phi)}, \tag{69}$$

which is exactly Equation (34).

Since $G_{K_1}(K_1^{\phi},\phi) < 0$, the sign of $\frac{\partial K_1^{\phi}}{\partial \phi}$ coincides with the sign of $b_1(K_1^{\phi}) - b_2(K_1^{\phi})$. Therefore:

$$b_1(K_1^{\phi}) > b_2(K_1^{\phi}) \quad \Longrightarrow \quad \frac{\partial K_1^{\phi}}{\partial \phi} > 0, \tag{70}$$

and conversely if $b_1(K_1^{\phi}) < b_2(K_1^{\phi})$.

This completes the proof. □

C.2 Proof of Proposition 2

Proof. The proof proceeds in two parts. We first derive the planner's first-order condition. We then derive the measurement wedge.

Step 1. Rewriting the planner's problem in logs.

The planner chooses $K_1 \in (0, 1)$ to maximize

$$\mathcal{W}(K_1) = \frac{C_1(K_1)^{\gamma_1}\, C_2(K_1)^{\gamma_2}}{\big(1 + E^*(K_1)\big)^{\delta}}. \tag{71}$$

Because the logarithm is strictly increasing, maximizing $\mathcal{W}(K_1)$ is equivalent to maximizing

$$\log \mathcal{W}(K_1) = \gamma_1 \log C_1(K_1) + \gamma_2 \log C_2(K_1) - \delta \log\big(1 + E^*(K_1)\big). \tag{72}$$

Step 2. Differentiating the log objective.

By assumption, $\mathcal{W}$ is continuously differentiable, so all derivatives below are well defined. Differentiating Equation (72) with respect to K_1 yields

$$\frac{d}{dK_1} \log \mathcal{W}(K_1) = \frac{d}{dK_1}\left[\gamma_1 \log C_1(K_1) + \gamma_2 \log C_2(K_1)\right] - \delta \frac{E^{*\prime}(K_1)}{1 + E^*(K_1)}. \tag{73}$$

$\Lambda(K_1)$ can be defined as the following

$$\Lambda(K_1) := \frac{d}{dK_1}\left[\gamma_1 \log C_1(K_1) + \gamma_2 \log C_2(K_1)\right] \tag{74}$$

$$= \gamma_1 \frac{C_1'(K_1)}{C_1(K_1)} + \gamma_2 \frac{C_2'(K_1)}{C_2(K_1)}. \tag{75}$$

Thus, this becomes

$$\frac{d}{dK_1} \log \mathcal{W}(K_1) = \Lambda(K_1) - \delta \frac{E^{*\prime}(K_1)}{1 + E^*(K_1)}. \tag{76}$$

Step 3. First-order condition at an interior optimum.

Because K_1^{FB} is assumed to be an interior optimum, the standard first-order condition applies:

$$\frac{d}{dK_1} \log \mathcal{W}(K_1)\Big|_{K_1=K_1^{FB}} = 0. \tag{77}$$

Substituting Equation (76) gives

$$\Lambda(K_1^{FB}) = \delta \frac{E^{*\prime}(K_1^{FB})}{1 + E^*(K_1^{FB})}, \tag{78}$$

which is Equation (39).

Step 4. Derivation of the true environmental gradient.

By Equation (37), true aggregate emissions satisfy

$$E^*(K_1) = E^R(K_1) - f\,B(K_1). \tag{79}$$

Since f is constant in the core model, differentiating with respect to K_1 yields

$$E^{*\prime}(K_1) = E^{R\prime}(K_1) - f\,B'(K_1), \tag{80}$$

which is Equation (40).

Step 5. Derivation of the accounting-induced gradient.

Under accounting rule ϕ, aggregate priced emissions are

$$E^\phi(K_1) = E^R(K_1) - \phi\,B(K_1). \tag{81}$$

Again, treating ϕ as a constant coefficient, differentiation yields

$$E^{\phi\prime}(K_1) = E^{R\prime}(K_1) - \phi\,B'(K_1), \tag{82}$$

which is Equation (41).

Step 6. Measurement wedge.

By definition,

$$\mathcal{M}(K_1;\phi) := E^{*\prime}(K_1) - E^{\phi\prime}(K_1). \tag{83}$$

Substituting the previous two expressions gives

$$\mathcal{M}(K_1;\phi) = \big(E^{R\prime}(K_1) - f\,B'(K_1)\big) - \big(E^{R\prime}(K_1) - \phi\,B'(K_1)\big) \tag{84}$$

$$= (f-\phi)B'(K_1), \tag{85}$$

which is Equation (42).

This completes the proof. □

C.3 Proof of Proposition 3

Proof. The proof compares the decentralized equilibrium condition with the planner's marginal condition.

Step 1. The decentralized allocation prices a level difference.

Under accounting rule ϕ, decentralized investors compare perceived payoffs across sectors according to

$$G(K_1,\phi) = \pi_1(K_1) - \pi_2(1-K_1) - \tau\left(e_1^{\phi}(K_1) - e_2^{\phi}(K_1)\right). \tag{86}$$

An interior decentralized equilibrium satisfies

$$G(K_1^{\phi},\phi) = 0. \tag{87}$$

Under true consequential accounting, $\phi = f$, this condition becomes

$$G(K_1,f) = \pi_1(K_1) - \pi_2(1-K_1) - \tau\left(e_1^{f}(K_1) - e_2^{f}(K_1)\right) = 0. \tag{88}$$

Thus, even when the accounting coefficient is correct, the environmental component priced by decentralized agents is the contemporaneous firm-level level difference

$$e_1^f(K_1) - e_2^f(K_1). \tag{89}$$

Step 2. The planner prices an aggregate marginal response.

The planner maximizes true welfare,

$$\mathcal{W}(K_1) = \frac{C_1(K_1)^{\gamma_1} C_2(K_1)^{\gamma_2}}{(1 + E^*(K_1))^{\delta}}. \tag{90}$$

At an interior first best, the planner's first-order condition is

$$\Lambda(K_1^{FB}) = \delta \frac{E^{*\prime}(K_1^{FB})}{1 + E^*(K_1^{FB})}. \tag{91}$$

Hence the planner requires the aggregate marginal emissions response $E^{*\prime}(K_1)$, not only the contemporaneous firm-level emissions levels.

Step 3. The aggregate marginal response contains terms absent from level-based firm accounting.

Using

$$E^*(K_1) = K_1 e_1^*(K_1) + (1 - K_1) e_2^*(K_1), \tag{92}$$

differentiation gives

$$E^{*\prime}(K_1) = e_1^*(K_1) - e_2^*(K_1) + K_1 \frac{de_1^*(K_1)}{dK_1} + (1 - K_1) \frac{de_2^*(K_1)}{dK_1}. \tag{93}$$

The first term is a static level difference. The last two terms are equilibrium-response terms: they capture how sector-level emissions change when capital is reallocated.

Step 4. Comparison.

Equation (88) shows that decentralized agents, even under $\phi = f$, price the level difference

$$e_1^f(K_1) - e_2^f(K_1). \tag{94}$$

Equation (93) shows that the planner's environmental margin depends on

$$e_1^*(K_1) - e_2^*(K_1) + K_1 \frac{de_1^*(K_1)}{dK_1} + (1 - K_1) \frac{de_2^*(K_1)}{dK_1}. \tag{95}$$

When $\phi = f$, the static level component is aligned with the true net-emissions level. However, the decentralized condition still omits the equilibrium-response terms unless they are zero or exactly summarized by the level difference priced in G.

Therefore, correct consequential accounting removes the coefficient wedge but does not generally remove the implementation wedge. Level-based firm accounting is not, in general, a sufficient statistic for the planner's derivative-based problem. □

C.4 Proof of Proposition 4

Proof. The proof is immediate once the true emissions gradient is written in terms of realized emissions and displacement.

Step 1. Start from the definition of the true emissions gradient.

By Proposition 2,

$$E^{*\prime}(K_1) = E^{R\prime}(K_1) - f\,B'(K_1). \tag{96}$$

Evaluating this identity at the benchmark allocation K_1° gives

$$E^{*\prime}(K_1^\circ) = E^{R\prime}(K_1^\circ) - f\,B'(K_1^\circ). \tag{97}$$

Step 2. Define the threshold.

Since the proposition assumes $B'(K_1^\circ) \neq 0$, the ratio

$$f^\circ := \frac{E^{R\prime}(K_1^\circ)}{B'(K_1^\circ)} \tag{98}$$

is well defined.

Substituting $E^{R\prime}(K_1^\circ) = f^\circ B'(K_1^\circ)$ into Equation (97) yields

$$\begin{aligned} E^{*\prime}(K_1^\circ) &= f^\circ B'(K_1^\circ) - f B'(K_1^\circ) && (99) \\ &= B'(K_1^\circ)\big(f^\circ - f\big). && (100) \end{aligned}$$

Step 3. Derive the sign formula.

Because $E^{*\prime}(K_1^\circ)$ is the product of $B'(K_1^\circ)$ and $(f^\circ - f)$, its sign is

$$\operatorname{sign}\big(E^{*\prime}(K_1^\circ)\big) = \operatorname{sign}\big(B'(K_1^\circ)\big)\ \operatorname{sign}\big(f^\circ - f\big), \tag{101}$$

which is exactly Equation (51).

Step 4. Interpretation when $B'(K_1^\circ) > 0$.

If $B'(K_1^\circ) > 0$, then the sign of $E^{*\prime}(K_1^\circ)$ is the sign of $(f^\circ - f)$. Hence:

$$f > f^\circ \quad \Longrightarrow \quad f^\circ - f < 0 \quad \Longrightarrow \quad E^{*\prime}(K_1^\circ) < 0. \tag{102}$$

Thus a marginal increase in K_1 reduces true aggregate emissions. Conversely, if $f < f^\circ$, then $E^{*\prime}(K_1^\circ) > 0$, so the environmental effect is reversed.

This completes the proof. $\square$

C.5 Proof of Proposition 5

Proof. The proof is an application of the chain rule.

Step 1. Welfare depends on accounting only through equilibrium allocation.

The true welfare function is

$$\mathcal{W}(K_1) = \frac{C_1(K_1)^{\gamma_1}\, C_2(K_1)^{\gamma_2}}{(1 + E^*(K_1))^{\delta}}. \tag{103}$$

Importantly, the accounting parameter ϕ does not appear directly in this expression. Its only effect on welfare operates indirectly through the equilibrium allocation K_1^{ϕ}.

Therefore, the composite map of interest is

$$\phi \mapsto \mathcal{W}(K_1^{\phi}). \tag{104}$$

Step 2. Differentiate by the chain rule.

By assumption, the equilibrium allocation K_1^{ϕ} is differentiable in ϕ around $\phi = 0$. Assuming that $\mathcal{W}$ is differentiable in its argument, the chain rule gives

$$\frac{d}{d\phi}\mathcal{W}(K_1^{\phi}) = \mathcal{W}'(K_1^{\phi})\, \frac{\partial K_1^{\phi}}{\partial \phi}. \tag{105}$$

Evaluating at $\phi = 0$ yields

$$\frac{d}{d\phi}\mathcal{W}(K_1^{\phi})\Big|_{\phi=0} = \mathcal{W}'(K_1^{0})\, \frac{\partial K_1^{\phi}}{\partial \phi}\Big|_{\phi=0}, \tag{106}$$

which is Equation (53).

Step 3. Sign of the local welfare effect.

By Proposition 1,

$$\frac{\partial K_1^{\phi}}{\partial \phi} = -\frac{\tau\big(b_1(K_1^{\phi}) - b_2(K_1^{\phi})\big)}{G_{K_1}(K_1^{\phi}, \phi)}. \tag{107}$$

Evaluating at $\phi = 0$ gives

$$\frac{\partial K_1^{\phi}}{\partial \phi}\Big|_{\phi=0} = -\frac{\tau\big(b_1(K_1^{0}) - b_2(K_1^{0})\big)}{G_{K_1}(K_1^{0}, 0)}. \tag{108}$$

Since $\tau \geq 0$ and $G_{K_1}(K_1^0, 0) < 0$ by assumption, it follows that

$$b_1(K_1^0) > b_2(K_1^0) \quad \Longrightarrow \quad \frac{\partial K_1^\phi}{\partial \phi}\Big|_{\phi=0} > 0. \tag{109}$$

Hence, if in addition

$$\mathcal{W}'(K_1^0) > 0, \tag{110}$$

then both terms on the right-hand side of Equation (53) are positive, so

$$\frac{d}{d\phi}\mathcal{W}(K_1^\phi)\Big|_{\phi=0} > 0. \tag{111}$$

Therefore, a marginal increase in the accounting weight on displacement raises true welfare locally.

Likewise, if

$$\mathcal{W}'(K_1^0) < 0 \qquad \text{and} \qquad b_1(K_1^0) > b_2(K_1^0), \tag{112}$$

then

$$\frac{d}{d\phi}\mathcal{W}(K_1^\phi)\Big|_{\phi=0} < 0, \tag{113}$$

so increasing ϕ lowers welfare locally.

This proves the proposition. $\square$

D Microfoundation and Closed-Form Equilibrium Blocks

This appendix restates the closed-form equilibrium blocks underlying the reduced-form objects used in the main text. It provides the corresponding derivations for the production block, household demand, sectoral profitability, and welfare.

Figure 4 summarizes the mechanism studied in the paper.

Figure 4: Mechanism of the paper

Accounting rule
ϕ

$\downarrow$

Priced accounting metric
$E^{\phi}(K_1) = E^{R}(K_1) - \phi B(K_1)$

$\downarrow$

Investor perceived payoff
$\widetilde{\Pi}_i^{\phi}(K_1) = \pi_i(K_i) - \tau e_i^{\phi}(K_1)$

$\downarrow$

Capital allocation
K_1^{ϕ}

$\downarrow$

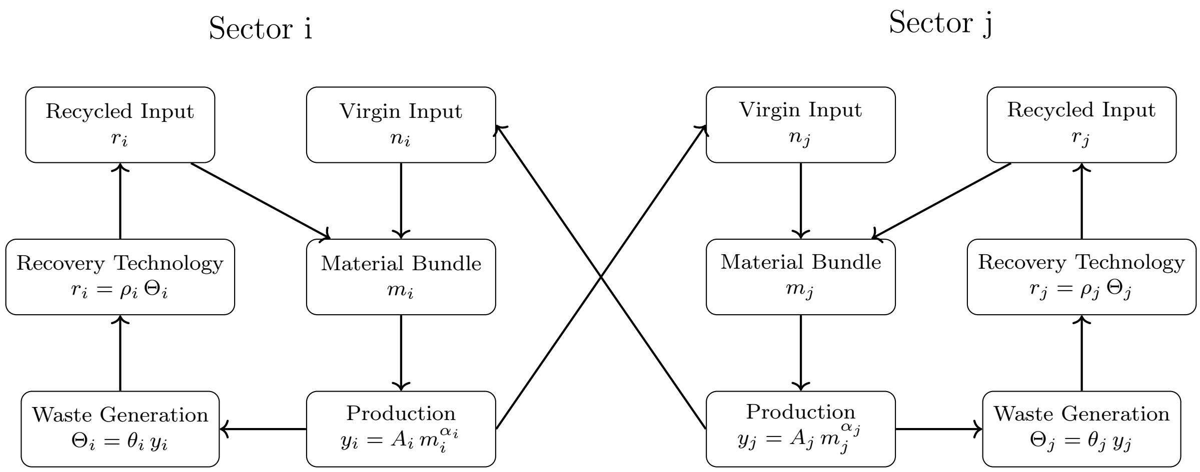
True welfare
$$\mathcal{W}(K_1) = \frac{C_1(K_1)^{\gamma_1} C_2(K_1)^{\gamma_2}}{(1 + E^*(K_1))^{\delta}}$$

Notes: The accounting rule ϕ determines the environmental object priced by decentralized investors. This priced metric affects perceived payoffs and therefore equilibrium capital allocation. Welfare, however, depends on the true consequential externality $E^*(K_1)$ rather than on the priced metric $E^{\phi}(K_1)$.

Figure 4 highlights the central wedge: decentralized agents respond to the priced accounting metric $E^{\phi}(K_1)$, whereas welfare depends on the true consequential externality $E^*(K_1)$.

Figure 5 provides a schematic overview of the circular production structure that underlies the microfoundation.

Figure 5: Circular production structure



Notes: Each sector produces a final good using a materials bundle that combines a virgin input purchased from the other sector with a recycled input sourced from a competitive recycling market. Production generates waste at the sector level, a fraction of which is recovered and supplied back as recycled material. Circularity arises because each sector both supplies the other sector's virgin input and generates the waste base from which its recycled-input market is supplied.

D.1 CES input block, composite pricing, and operating profits

We begin with the firm's static problem. The next result derives the CES materials block, the associated dual price index, and the corresponding expressions for materials demand, output, and operating profit.

Proposition 6 (CES input block and composite input pricing)**.** *Consider a representative firm in sector $i \in \{1, 2\}$ with production technology*

$$y_i = A_i \, m_i^{\alpha_i}, \qquad A_i > 0, \qquad \alpha_i \in (0,1), \tag{114}$$

where the materials bundle m_i is a CES composite of virgin input n_i and recycled input r_i:

$$m_i = \left(\omega \, n_i^{\frac{\sigma-1}{\sigma}} + (1-\omega) \, r_i^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}, \qquad \omega \in (0,1), \qquad \sigma > 0, \qquad \sigma \neq 1. \tag{115}$$

Let p_i^c denote the output price in sector i, let p_j^c denote the price of the virgin input purchased from sector $j \neq i$, and let p_i^r denote the price of recycled input used by sector i. Then:

(i) The dual price index of the materials bundle is

$$q_i = \left[\omega^\sigma (p_j^c)^{1-\sigma} + (1-\omega)^\sigma (p_i^r)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}. \tag{116}$$

(ii) Conditional on m_i, the Hicksian input demands are

$$n_i = m_i\, \omega^\sigma \left(\frac{p_j^c}{q_i}\right)^{-\sigma}, \tag{117}$$

$$r_i = m_i\, (1-\omega)^\sigma \left(\frac{p_i^r}{q_i}\right)^{-\sigma}. \tag{118}$$

(iii) The firm's optimal materials-bundle demand solves

$$\max_{m_i \geq 0} \; p_i^c A_i m_i^{\alpha_i} - q_i m_i, \tag{119}$$

and is given by

$$m_i = \left(\frac{\alpha_i\, A_i\, p_i^c}{q_i}\right)^{\frac{1}{1-\alpha_i}}. \tag{120}$$

(iv) Output is

$$y_i = A_i \left(\frac{\alpha_i\, A_i\, p_i^c}{q_i}\right)^{\frac{\alpha_i}{1-\alpha_i}}. \tag{121}$$

(v) Per-project operating profit is

$$\pi_i = (1-\alpha_i)\, q_i \left(\frac{\alpha_i\, A_i\, p_i^c}{q_i}\right)^{\frac{1}{1-\alpha_i}}. \tag{122}$$

D.2 Household demand

We next turn to the household side. The following lemma states the standard Marshallian demand system and indirect utility implied by the preference specification in the main text.

Lemma 1 (Marshallian demand and indirect utility)**.** *Consider the representative household with preferences*

$$U(C_1, C_2, E^*) = \frac{C_1^{\gamma_1} C_2^{\gamma_2}}{(1+E^*)^{\delta}}, \qquad \gamma_1, \gamma_2 \in (0,1), \qquad \gamma_1 + \gamma_2 = 1, \tag{123}$$

and budget constraint

$$p_1^c C_1 + p_2^c C_2 \leq W, \tag{124}$$

where W denotes aggregate household income. Taking E^* as given, the Marshallian demand for good $i \in \{1, 2\}$ is

$$C_i = \frac{\gamma_i W}{p_i^c}. \tag{125}$$

Indirect utility is

$$U^*(W; p_1^c, p_2^c, E^*) = \frac{\left(\frac{\gamma_1}{p_1^c}\right)^{\gamma_1} \left(\frac{\gamma_2}{p_2^c}\right)^{\gamma_2}}{(1+E^*)^{\delta}} W. \tag{126}$$

D.3 Goods market clearing and profit erosion

The next result derives the closed-form relation between goods-market clearing and sectoral profitability. In particular, it yields the benchmark profit-erosion expression used to motivate the reduced-form profit functions in the main text.

Lemma 2 (Goods market clearing and closed-form profit erosion)**.** *Suppose that all firms* *in sector $i \in \{1,2\}$ are identical, with production technology*

$$y_i = A_i\, m_i^{\alpha_i}, \qquad \alpha_i \in (0,1), \tag{127}$$

and let sector sizes satisfy

$$K_1 + K_2 = 1. \tag{128}$$

Let Y_i denote aggregate output of good i, C_i aggregate final consumption of good i, and let sector $j \neq i$ use good i as its virgin material input. Then goods-market clearing requires

$$Y_i = C_i + K_j\, n_j. \tag{129}$$

In the closed-form benchmark inherited from the microfounded two-sector block, per-project profit in sector i can be written as

$$\pi_i(K_i) = \frac{K_i^*}{K_i}, \tag{130}$$

where

$$K_i^* := \frac{(\gamma_i + \alpha_j\, \gamma_j)(1-\alpha_i)}{1-\alpha_i\alpha_j}, \qquad j \neq i. \tag{131}$$

Hence aggregate sectoral profit is constant and equal to K_i^*, while per-project profit is strictly decreasing in sector size.

D.4 Closed-form welfare decomposition

We finally combine the previous building blocks to obtain a closed-form welfare decomposition. The resulting expression separates the productivity, allocative, and environmental components of welfare.

Proposition 7 (Closed-form welfare decomposition)**.** *Suppose that the closed-form equilibrium representation of Lemma 2 holds. Let sector sizes satisfy $K_1 + K_2 = 1$, and let true firm-level consequential emissions be*

$$e_i^*(K_1) = y_i(K_1)\left[\varepsilon_i^{prod} + \varepsilon_i^{waste}\,\theta_i + \left(\varepsilon_i^{rec} - \varepsilon_i^{waste} - f\right)\rho_i\,\theta_i\right]. \tag{132}$$

Then aggregate consequential emissions are

$$E^*(K_1) = K_1\,e_1^*(K_1) + (1 - K_1)\,e_2^*(K_1). \tag{133}$$

Under the closed-form benchmark, true welfare can be written as

$$\mathcal{W}(K_1) = \beta\,\frac{A_1^{Z_1}\,A_2^{Z_2}}{(1 + E^*(K_1))^{\delta}}\left(\frac{K_1}{K_1^*}\right)^{K_1^*}\left(\frac{K_2}{K_2^*}\right)^{K_2^*}, \tag{134}$$

where

$$Z_i := \frac{K_i^*}{1 - \alpha_i}, \tag{135}$$

and $\beta > 0$ collects technological and preference parameters.

This decomposition separates welfare into three components: a productivity term, a misallocation term, and an environmental term driven by the true consequential externality $E^*(K_1)$.

Together, these results provide the closed-form benchmark underlying the reduced-form representation used in the main text.

D.5 Proofs

D.5.1 Proof of Proposition 6

Proof. The proof proceeds in two steps. We first solve the firm's inner cost-minimization problem for the composition of the materials bundle. We then solve the firm's outer problem for total materials demand, output, and operating profit.

Step 1. Inner CES cost minimization and composite input pricing.

Fix a target materials bundle $m_i > 0$ and input prices $(p_j^c, p_i^r) \gg 0$. The representative firm in sector i solves

$$\min_{n_i, r_i \geq 0} \; p_j^c\, n_i + p_i^r\, r_i \qquad \text{s.t.} \qquad \left(\omega\, n_i^{\frac{\sigma-1}{\sigma}} + (1-\omega)\, r_i^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \geq m_i. \tag{136}$$

By monotonicity of the CES aggregator, the constraint binds at the optimum. Let

$$\rho := \frac{\sigma - 1}{\sigma}, \qquad \rho < 1, \qquad \sigma \neq 1.$$

Then the problem can be rewritten as

$$\min_{n_i, r_i \geq 0} \; p_j^c\, n_i + p_i^r\, r_i \qquad \text{s.t.} \qquad \omega\, n_i^\rho + (1-\omega)\, r_i^\rho = m_i^\rho. \tag{137}$$

The Lagrangian is

$$\mathcal{L} = p_j^c\, n_i + p_i^r\, r_i + \lambda \left[m_i^\rho - \omega\, n_i^\rho - (1-\omega)\, r_i^\rho\right], \qquad \lambda > 0. \tag{138}$$

The first-order conditions are

$$\frac{\partial \mathcal{L}}{\partial n_i} = p_j^c - \lambda\,\omega\,\rho\,n_i^{\rho-1} = 0, \tag{139}$$

$$\frac{\partial \mathcal{L}}{\partial r_i} = p_i^r - \lambda\,(1-\omega)\,\rho\,r_i^{\rho-1} = 0, \tag{140}$$

$$\omega\,n_i^{\rho} + (1-\omega)\,r_i^{\rho} = m_i^{\rho}. \tag{141}$$

Dividing Equation (139) by Equation (140) yields

$$\frac{p_j^c}{p_i^r} = \frac{\omega}{1-\omega}\left(\frac{n_i}{r_i}\right)^{\rho-1}. \tag{142}$$

Since

$$\rho - 1 = -\frac{1}{\sigma},$$

we obtain

$$\frac{p_j^c}{p_i^r} = \frac{\omega}{1-\omega}\left(\frac{n_i}{r_i}\right)^{-\frac{1}{\sigma}}. \tag{143}$$

Next, multiply Equation (139) by n_i and Equation (140) by r_i, then add the two expressions:

$$p_j^c\,n_i + p_i^r\,r_i = \lambda\,\rho\left[\omega\,n_i^{\rho} + (1-\omega)\,r_i^{\rho}\right]. \tag{144}$$

Using the binding constraint Equation (141), this becomes

$$p_j^c\,n_i + p_i^r\,r_i = \lambda\,\rho\,m_i^{\rho}. \tag{145}$$

Let q_i denote the unit expenditure function associated with one unit of the materials bundle:

$$q_i := \min_{n_i, r_i \geq 0} \left\{ p_j^c \, n_i + p_i^r \, r_i : \left(\omega \, n_i^{\frac{\sigma-1}{\sigma}} + (1-\omega) \, r_i^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} = 1 \right\}. \tag{146}$$

By homogeneity of degree one of the CES aggregator, total cost is $q_i \, m_i$, so Equation (145) implies

$$q_i \, m_i = \lambda \, \rho \, m_i^{\rho} \qquad \Longrightarrow \qquad \lambda = \frac{q_i}{\rho} \, m_i^{1-\rho}. \tag{147}$$

Substituting Equation (147) into Equation (139) yields

$$p_j^c = \omega \, q_i \, m_i^{1-\rho} \, n_i^{\rho-1}. \tag{148}$$

Using again $\rho - 1 = -\frac{1}{\sigma}$, we obtain

$$\frac{p_j^c}{q_i} = \omega \left(\frac{n_i}{m_i} \right)^{-1/\sigma} \qquad \Longrightarrow \qquad \frac{n_i}{m_i} = \omega^{\sigma} \left(\frac{p_j^c}{q_i} \right)^{-\sigma}. \tag{149}$$

Hence

$$n_i = m_i \, \omega^{\sigma} \left(\frac{p_j^c}{q_i} \right)^{-\sigma}. \tag{150}$$

Similarly, substituting Equation (147) into Equation (140) gives

$$\frac{p_i^r}{q_i} = (1-\omega) \left(\frac{r_i}{m_i} \right)^{-\frac{1}{\sigma}} \qquad \Longrightarrow \qquad \frac{r_i}{m_i} = (1-\omega)^{\sigma} \left(\frac{p_i^r}{q_i} \right)^{-\sigma}, \tag{151}$$

so

$$r_i = m_i \, (1-\omega)^{\sigma} \left(\frac{p_i^r}{q_i} \right)^{-\sigma}. \tag{152}$$

To derive the dual price index, use the identity

$$q_i\, m_i = p^c_j\, n_i + p^r_i\, r_i,$$

and substitute Equations (150)–(152):

$$q_i\, m_i = p^c_j\, m_i\, \omega^\sigma \left(\frac{p^c_j}{q_i}\right)^{-\sigma} + p^r_i\, m_i\, (1-\omega)^\sigma \left(\frac{p^r_i}{q_i}\right)^{-\sigma} \tag{153}$$

$$= m_i\, q_i^\sigma \left[\omega^\sigma (p^c_j)^{1-\sigma} + (1-\omega)^\sigma (p^r_i)^{1-\sigma}\right]. \tag{154}$$

Dividing by $m_i\, q_i^\sigma$ gives

$$q_i^{1-\sigma} = \omega^\sigma (p^c_j)^{1-\sigma} + (1-\omega)^\sigma (p^r_i)^{1-\sigma}, \tag{155}$$

hence

$$q_i = \left[\omega^\sigma (p^c_j)^{1-\sigma} + (1-\omega)^\sigma (p^r_i)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}. \tag{156}$$

This proves items (i) and (ii).

Step 2. Outer problem: optimal materials demand, output, and profit.

Given the dual unit cost q_i, the representative firm solves

$$\max_{m_i \geq 0}\ \ p^c_i\, A_i\, m_i^{\alpha_i} - q_i\, m_i. \tag{157}$$

The first-order condition is

$$p^c_i\, A_i\, \alpha_i\, m_i^{\alpha_i - 1} = q_i. \tag{158}$$

Rearranging gives

$$m_i^{1-\alpha_i} = \frac{\alpha_i\, A_i\, p_i^c}{q_i}, \tag{159}$$

so optimal materials demand is

$$m_i = \left(\frac{\alpha_i\, A_i\, p_i^c}{q_i}\right)^{\frac{1}{1-\alpha_i}}. \tag{160}$$

This proves item (iii).

Substituting Equation (160) into the production function yields

$$y_i = A_i\, m_i^{\alpha_i} = A_i \left(\frac{\alpha_i\, A_i\, p_i^c}{q_i}\right)^{\frac{\alpha_i}{1-\alpha_i}}, \tag{161}$$

which proves item (iv).

Finally, profit is

$$\pi_i = p_i^c\, y_i - q_i\, m_i. \tag{162}$$

Using Equation (158), we have

$$q_i\, m_i = p_i^c\, A_i\, \alpha_i\, m_i^{\alpha_i} = \alpha_i\, p_i^c\, y_i,$$

so

$$\pi_i = (1-\alpha_i)\, p_i^c\, y_i. \tag{163}$$

Substituting Equation (161) gives

$$\pi_i = (1-\alpha_i) p_i^c\, A_i \left(\frac{\alpha_i\, A_i\, p_i^c}{q_i}\right)^{\frac{\alpha_i}{1-\alpha_i}} \tag{164}$$

$$= (1-\alpha_i)\, q_i \left(\frac{\alpha_i\, A_i\, p_i^c}{q_i}\right)^{\frac{1}{1-\alpha_i}}, \tag{165}$$

which proves item (v).

This completes the proof. □

D.5.2 Proof of Lemma 1

Proof. Consider the representative household's problem:

$$\max_{C_1, C_2} \frac{C_1^{\gamma_1}\, C_2^{\gamma_2}}{(1+E^*)^{\delta}} \qquad \text{s.t.} \qquad p_1^c\, C_1 + p_2^c\, C_2 \le W. \tag{166}$$

The Lagrangian is

$$\mathcal{L} = \frac{C_1^{\gamma_1}\, C_2^{\gamma_2}}{(1+E^*)^{\delta}} + \lambda \left(W - p_1^c\, C_1 - p_2^c\, C_2\right). \tag{167}$$

The first-order conditions are

$$\frac{\partial \mathcal{L}}{\partial C_1} = \frac{\gamma_1\, C_1^{\gamma_1 - 1}\, C_2^{\gamma_2}}{(1+E^*)^{\delta}} - \lambda\, p_1^c = 0, \tag{168}$$

$$\frac{\partial \mathcal{L}}{\partial C_2} = \frac{\gamma_2\, C_1^{\gamma_1}\, C_2^{\gamma_2 - 1}}{(1+E^*)^{\delta}} - \lambda\, p_2^c = 0, \tag{169}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = W - p_1^c\, C_1 - p_2^c\, C_2 = 0. \tag{170}$$

Dividing Equation (168) by Equation (169) gives

$$\frac{\gamma_1}{\gamma_2}\frac{C_2}{C_1} = \frac{p_1^c}{p_2^c}, \tag{171}$$

which implies

$$C_2 = \frac{\gamma_2 p_1^c}{\gamma_1 p_2^c} C_1. \tag{172}$$

Substituting Equation (172) into the budget constraint Equation (170), we obtain

$$p_1^c\, C_1 + p_2^c \left(\frac{\gamma_2\, p_1^c}{\gamma_1\, p_2^c} C_1 \right) = W, \tag{173}$$

that is,

$$p_1^c C_1 \left(1 + \frac{\gamma_2}{\gamma_1} \right) = W. \tag{174}$$

Since $\gamma_1 + \gamma_2 = 1$, we have

$$1 + \frac{\gamma_2}{\gamma_1} = \frac{\gamma_1 + \gamma_2}{\gamma_1} = \frac{1}{\gamma_1}.$$

Therefore

$$p_1^c\, C_1\, \frac{1}{\gamma_1} = W, \tag{175}$$

so

$$C_1 = \frac{\gamma_1\, W}{p_1^c}. \tag{176}$$

By symmetry,

$$C_2 = \frac{\gamma_2\, W}{p_2^c}. \tag{177}$$

This proves the Marshallian demand system.

We now derive indirect utility. Substitute Equations(176)–(177) into the utility function:

$$U^*(W; p_1^c, p_2^c, E^*) = \frac{\left(\frac{\gamma_1 W}{p_1^c}\right)^{\gamma_1} \left(\frac{\gamma_2 W}{p_2^c}\right)^{\gamma_2}}{(1+E^*)^{\delta}} \tag{178}$$

$$= \frac{\gamma_1^{\gamma_1}\, \gamma_2^{\gamma_2}\, W^{\gamma_1+\gamma_2}}{(p_1^c)^{\gamma_1}\, (p_2^c)^{\gamma_2}\, (1+E^*)^{\delta}}. \tag{179}$$

Using $\gamma_1 + \gamma_2 = 1$, this simplifies to

$$U^*(W; p_1^c, p_2^c, E^*) = \frac{\left(\frac{\gamma_1}{p_1^c}\right)^{\gamma_1} \left(\frac{\gamma_2}{p_2^c}\right)^{\gamma_2}}{(1+E^*)^{\delta}}\, W. \tag{180}$$

This completes the proof. □

D.5.3 Proof of Lemma 2

Proof. **Step 1. Goods-market clearing.**

Let Y_i denote aggregate output of sector i, C_i aggregate final consumption of good i, and let sector $j \neq i$ use good i as virgin material input. Goods-market clearing requires

$$Y_i = C_i + K_j\, n_j. \tag{181}$$

Under the benchmark closed-form block, virgin material demand by a representative firm in sector j is proportional to output through cost minimization and the firm's first-order condition. In particular, the aggregate intermediate demand for good i by sector j is

$$X_i := K_j\, n_j = \alpha_j\, \frac{p_j^c\, Y_j}{p_i^c}. \tag{182}$$

Using household demand from Lemma 1,

$$C_i = \frac{\gamma_i\, W}{p_i^c}. \tag{183}$$

Substituting Equations (182)–(183) into Equation (181) yields

$$Y_i = \frac{\gamma_i W}{p_i^c} + \alpha_j \frac{p_j^c Y_j}{p_i^c}. \tag{184}$$

Similarly,

$$Y_j = \frac{\gamma_j W}{p_j^c} + \alpha_i \frac{p_i^c Y_i}{p_j^c}. \tag{185}$$

Step 2. Sectoral revenues.

Define sectoral revenues

$$Z_i := p_i^c Y_i, \qquad Z_j := p_j^c Y_j. \tag{186}$$

Multiplying Equation (184) by p_i^c and Equation (185) by p_j^c gives

$$Z_i = \gamma_i W + \alpha_j Z_j, \tag{187}$$

$$Z_j = \gamma_j W + \alpha_i Z_i. \tag{188}$$

In the closed-form benchmark, aggregate household income equals aggregate profits. Since only relative allocations matter here, we normalize

$$W = 1.$$

Then Equations (187)–(188) become

$$Z_i = \gamma_i + \alpha_j Z_j, \tag{189}$$

$$Z_j = \gamma_j + \alpha_i Z_i. \tag{190}$$

Substitute the second equation into the first:

$$Z_i = \gamma_i + \alpha_j(\gamma_j + \alpha_i\, Z_i) \tag{191}$$

$$= \gamma_i + \alpha_j\, \gamma_j + \alpha_i\, \alpha_j\, Z_i. \tag{192}$$

Hence

$$(1 - \alpha_i\, \alpha_j)\, Z_i = \gamma_i + \alpha_j\, \gamma_j, \tag{193}$$

so

$$Z_i = \frac{\gamma_i + \alpha_j\, \gamma_j}{1 - \alpha_i\, \alpha_j}. \tag{194}$$

By symmetry,

$$Z_j = \frac{\gamma_j + \alpha_i\, \gamma_i}{1 - \alpha_i\, \alpha_j}. \tag{195}$$

Step 3. Per-project profits.

From Proposition 6, per-project operating profit in sector i is

$$\pi_i = (1 - \alpha_i)\, p_i^c\, y_i. \tag{196}$$

Aggregate profit in sector i is therefore

$$K_i\, \pi_i = (1 - \alpha_i)\, p_i^c\, K_i\, y_i = (1 - \alpha_i)\, p_i^c\, Y_i = (1 - \alpha_i)\, Z_i. \tag{197}$$

Using Equation (194), aggregate sectoral profit is

$$K_i\, \pi_i = (1 - \alpha_i)\frac{\gamma_i + \alpha_j \gamma_j}{1 - \alpha_i \alpha_j}. \tag{198}$$

Define

$$K_i^* := \frac{(\gamma_i + \alpha_j \gamma_j)(1-\alpha_i)}{1-\alpha_i\alpha_j}, \qquad j \neq i. \tag{199}$$

Then aggregate sectoral profit is exactly K_i^*, so

$$K_i \pi_i = K_i^*. \tag{200}$$

Dividing by K_i yields per-project profit

$$\pi_i(K_i) = \frac{K_i^*}{K_i}. \tag{201}$$

Step 4. Monotonicity.

Since $K_i^* > 0$ is constant with respect to sector size, it follows immediately that

$$\frac{d\pi_i(K_i)}{dK_i} = K_i^* \frac{d}{dK_i}\left(K_i^{-1}\right) = K_i^* \left(-K_i^{-2}\right) \tag{202}$$

$$= -\frac{K_i^*}{K_i^2} < 0. \tag{203}$$

Hence per-project profit is strictly decreasing in sector size.

This completes the proof. □

D.5.4 Proof of Proposition 7

Proof. The proof derives the closed-form welfare representation from the household block, the closed-form benchmark for sectoral revenues, and the profit representation of Lemma 2.

Step 1. Start from indirect utility.

From Lemma 1, true welfare is

$$\mathcal{W}(K_1) = \frac{\left(\frac{\gamma_1}{p_1^c}\right)^{\gamma_1}\left(\frac{\gamma_2}{p_2^c}\right)^{\gamma_2}}{(1+E^*(K_1))^{\delta}} W. \tag{204}$$

Thus the key task is to express the price term $(p_1^c)^{-\gamma_1}\,(p_2^c)^{-\gamma_2}$ in terms of fundamentals and sector sizes.

Step 2. Relate prices to sector sizes through profits.

From Proposition 6,

$$\pi_i = (1-\alpha_i)\,q_i\,\left(\frac{\alpha_i\,A_i\,p_i^c}{q_i}\right)^{\frac{1}{1-\alpha_i}}. \tag{205}$$

Equivalently,

$$\pi_i = (1-\alpha_i)\,(A_i\,p_i^c)^{\frac{1}{1-\alpha_i}}\,\alpha_i^{\frac{\alpha_i}{1-\alpha_i}}\,q_i^{-\frac{\alpha_i}{1-\alpha_i}}. \tag{206}$$

Rearranging,

$$\frac{p_i^c}{q_i^{\alpha_i}}\,A_i = \left(\frac{\pi_i}{(1-\alpha_i)\,\alpha_i^{\frac{\alpha_i}{1-\alpha_i}}}\right)^{1-\alpha_i}. \tag{207}$$

Using Lemma 2, $\pi_i(K_i) = \frac{K_i^*}{K_i}$, we obtain

$$\frac{p_i^c}{q_i^{\alpha_i}}\,A_i = \left(\frac{K_i^*}{K_i\,(1-\alpha_i)}\right)^{1-\alpha_i}\frac{1}{\alpha_i^{\alpha_i}}. \tag{208}$$

Define

$$B_i := \frac{p_i^c}{q_i^{\alpha_i}}\,A_i = \left(\frac{K_i^*}{K_i\,(1-\alpha_i)}\right)^{1-\alpha_i}\frac{1}{\alpha_i^{\alpha_i}}. \tag{209}$$

Similarly,

$$B_j := \frac{p_j^c}{q_j^{\alpha_j}}\,A_j = \left(\frac{K_j^*}{K_j\,(1-\alpha_j)}\right)^{1-\alpha_j}\frac{1}{\alpha_j^{\alpha_j}}. \tag{210}$$

In the benchmark two-sector block, the materials-bundle price of sector i depends on the output price of sector j, so we can write the system

$$p_i^c = \frac{B_i}{A_i}\,(p_j^c)^{\alpha_i}, \tag{211}$$
$$p_j^c = \frac{B_j}{A_j}\,(p_i^c)^{\alpha_j}. \tag{212}$$

Step 3. Solve for the equilibrium price terms.

Substitute Equation (211) into Equation (212):

$$p_j^c = \frac{B_j}{A_j}\left(\frac{B_i}{A_i}(p_j^c)^{\alpha_i}\right)^{\alpha_j} \tag{213}$$
$$= \frac{B_j}{A_j}\left(\frac{B_i}{A_i}\right)^{\alpha_j}(p_j^c)^{\alpha_i\,\alpha_j}. \tag{214}$$

Hence

$$(p_j^c)^{1-\alpha_i\alpha_j} = \frac{B_j}{A_j}\left(\frac{B_i}{A_i}\right)^{\alpha_j}, \tag{215}$$

so

$$p_j^c = \left[\frac{B_j}{A_j}\left(\frac{B_i}{A_i}\right)^{\alpha_j}\right]^{\frac{1}{1-\alpha_i\,\alpha_j}}. \tag{216}$$

Similarly,

$$p_i^c = \left[\frac{B_i}{A_i}\left(\frac{B_j}{A_j}\right)^{\alpha_i}\right]^{\frac{1}{1-\alpha_i\,\alpha_j}}. \tag{217}$$

Therefore

$$(p_i^c)^{-\gamma_i}\,(p_j^c)^{-\gamma_j} = \left(\frac{A_i}{B_i}\right)^{\frac{\gamma_i+\alpha_j\,\gamma_j}{1-\alpha_i\,\alpha_j}}\left(\frac{A_j}{B_j}\right)^{\frac{\gamma_j+\alpha_i\,\gamma_i}{1-\alpha_i\,\alpha_j}}. \tag{218}$$

Define

$$Z_i := \frac{\gamma_i + \alpha_j\,\gamma_j}{1 - \alpha_i\,\alpha_j}, \qquad Z_j := \frac{\gamma_j + \alpha_i\,\gamma_i}{1 - \alpha_i\,\alpha_j}. \tag{219}$$

Then

$$(p_i^c)^{-\gamma_i}\,(p_j^c)^{-\gamma_j} = \left(\frac{A_i}{B_i}\right)^{Z_i} \left(\frac{A_j}{B_j}\right)^{Z_j}. \tag{220}$$

Step 4. Substitute into welfare.

Using Equation (204) and Equation (220),

$$\mathcal{W}(K_1) = \frac{\gamma_1^{\gamma_1}\,\gamma_2^{\gamma_2}\,W}{(1+E^*(K_1))^{\delta}} \left(\frac{A_1}{B_1}\right)^{Z_1} \left(\frac{A_2}{B_2}\right)^{Z_2}. \tag{221}$$

Now substitute Equations (209)–(210):

$$\left(\frac{A_i}{B_i}\right)^{Z_i} = A_i^{Z_i} \left[\left(\frac{K_i^*}{K_i(1-\alpha_i)}\right)^{1-\alpha_i} \frac{1}{\alpha_i^{\alpha_i}}\right]^{-Z_i} \tag{222}$$

$$= A_i^{Z_i} \left(\frac{K_i}{K_i^*}\right)^{Z_i(1-\alpha_i)} (1-\alpha_i)^{Z_i(1-\alpha_i)}\,\alpha_i^{Z_i\alpha_i}. \tag{223}$$

Since

$$K_i^* = Z_i(1-\alpha_i),$$

therefore

$$\left(\frac{A_i}{B_i}\right)^{Z_i} = A_i^{Z_i} \left(\frac{K_i}{K_i^*}\right)^{K_i^*} (1-\alpha_i)^{K_i^*}\,\alpha_i^{Z_i\alpha_i}. \tag{224}$$

Applying the same expression to sector 2 and substituting into Equation (221), we obtain

$$\mathcal{W}(K_1) = \frac{A_1^{Z_1}\,A_2^{Z_2}}{(1+E^*(K_1))^{\delta}} \left(\frac{K_1}{K_1^*}\right)^{K_1^*} \left(\frac{K_2}{K_2^*}\right)^{K_2^*} \tag{225}$$

$$\times\, \gamma_1^{\gamma_1}\,\gamma_2^{\gamma_2}\,W\,\alpha_1^{Z_1\alpha_1}\,(1-\alpha_1)^{K_1^*}\,\alpha_2^{Z_2\alpha_2}\,(1-\alpha_2)^{K_2^*}. \tag{226}$$

Collect all terms independent of K_1 and $E^*(K_1)$ into the positive constant β:

$$\beta := \gamma_1^{\gamma_1}\, \gamma_2^{\gamma_2}\, W\, \alpha_1^{Z_1 \alpha_1}\, (1-\alpha_1)^{K_1^*} \alpha_2^{Z_2 \alpha_2}\, (1-\alpha_2)^{K_2^*}. \tag{227}$$

Then welfare can be written as

$$\mathcal{W}(K_1) = \beta\, \frac{A_1^{Z_1}\, A_2^{Z_2}}{(1+E^*(K_1))^{\delta}} \left(\frac{K_1}{K_1^*}\right)^{K_1^*} \left(\frac{K_2}{K_2^*}\right)^{K_2^*}. \tag{228}$$

This is exactly the claimed closed-form decomposition. It separates welfare into a productivity component, an allocative component, and an environmental component driven by the true consequential externality $E^*(K_1)$.

This completes the proof. □

E Decomposition of the Displacement Gradient

This appendix unpacks the marginal displacement term appearing in the main text. In particular, it shows that the derivative of the displacement base $B'(K_1)$ contains both a composition effect and an equilibrium-response effect through output adjustments.

Lemma 3 (Decomposition of the displacement gradient)**.** *Let*

$$B(K_1) = K_1\,\rho_1\,\theta_1\,y_1(K_1) + (1 - K_1)\,\rho_2\,\theta_2\,y_2(K_1). \tag{229}$$

Then

$$B'(K_1) = \underbrace{\rho_1\,\theta_1\,y_1(K_1) - \rho_2\,\theta_2\,y_2(K_1)}_{\textit{composition effect}} + \underbrace{K_1\,\rho_1\,\theta_1\,\frac{dy_1}{dK_1} + (1 - K_1)\,\rho_2\,\theta_2\,\frac{dy_2}{dK_1}}_{\textit{equilibrium-response effect}}. \tag{230}$$

Moreover, since

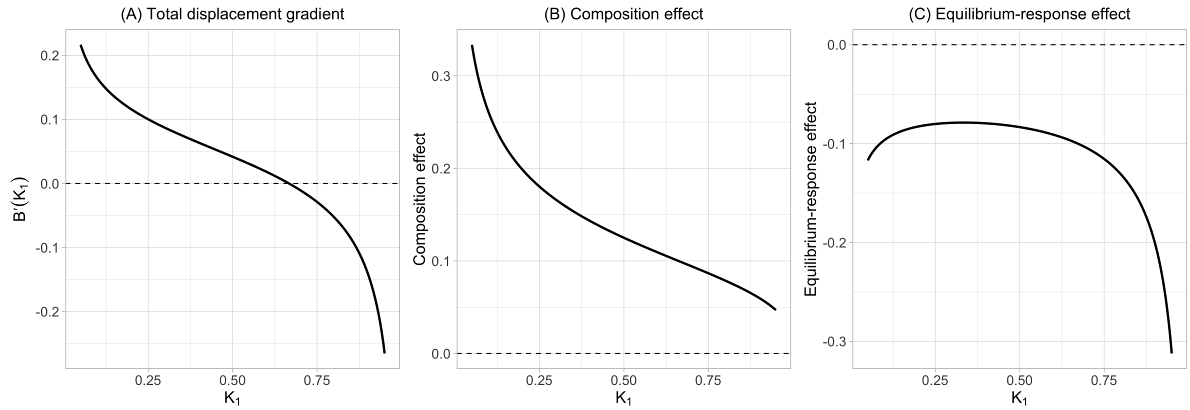
$$E^*(K_1) = E^R(K_1) - f\,B(K_1), \tag{231}$$

it follows that

$$E^{*\prime}(K_1) = E^{R\prime}(K_1) - f\,B'(K_1). \tag{232}$$

Figure 6 provides a benchmark illustration of the decomposition of the displacement gradient developed in Lemma 3. It separates the marginal displacement term into the total displacement gradient, a composition effect, and an equilibrium-response effect.

Figure 6: Decomposition of the displacement gradient



Notes: Panel A plots the total displacement gradient $B'(K_1)$. Panel B plots the composition effect, that is, the direct effect of reallocating capital across sectors while holding per-firm output responses fixed. Panel C plots the equilibrium-response effect, which captures how sectoral output adjustments contribute to displacement capacity. The figure is based on the benchmark parameterization $A_1 = A_2 = 1$, $\alpha_1 = \alpha_2 = 0.5$, $\gamma_1 = \gamma_2 = 0.5$, $\theta_1 = \theta_2 = 0.5$, $\rho_1 = 0.5$, $\rho_2 = 0$, $\varepsilon_1^{prod} = 1.2$, $\varepsilon_2^{prod} = 0.2$, $\varepsilon_1^{waste} = \varepsilon_2^{waste} = 0.5$, $\varepsilon_1^{rec} = \varepsilon_2^{rec} = 0.5$, $\delta = 6$, $f = 0.6$, and $\widetilde{f} = 0.35$. The figure is included to visualize the logic of Appendix E and does not represent a general analytical result.

E.1 Proof

Proof. By definition,

$$B(K_1) = K_1\,\rho_1\,\theta_1\,y_1(K_1) + (1 - K_1)\,\rho_2\,\theta_2\,y_2(K_1). \tag{233}$$

Differentiating with respect to K_1 yields

$$B'(K_1) = \rho_1\,\theta_1\,y_1(K_1) - \rho_2\,\theta_2\,y_2(K_1) + K_1\,\rho_1\,\theta_1\,\frac{dy_1}{dK_1} + (1 - K_1)\,\rho_2\,\theta_2\,\frac{dy_2}{dK_1}. \tag{234}$$

This gives the decomposition into a composition effect and an equilibrium-response effect.

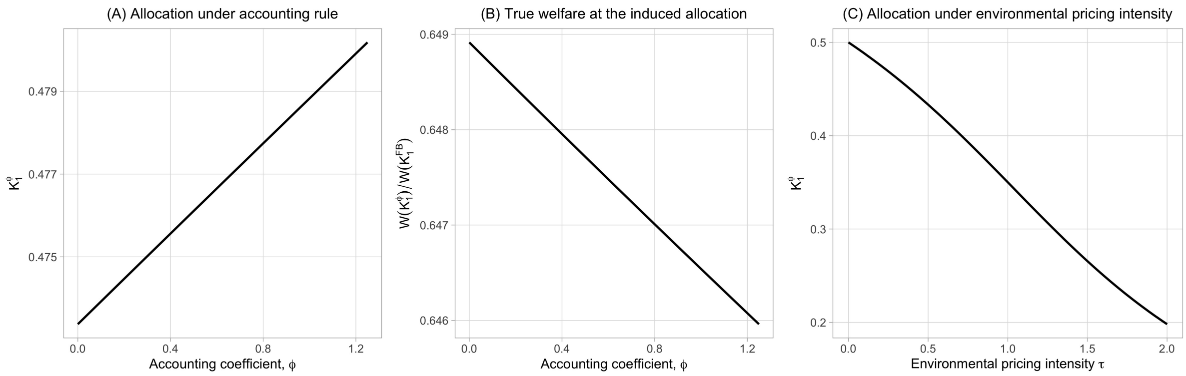
The second claim follows immediately by differentiating

$$E^*(K_1) = E^R(K_1) - f\,B(K_1). \tag{235}$$

$\square$

F Sensitivity to Accounting and Pricing Parameters

Figure 7: Sensitivity of capital allocation and welfare to accounting and pricing intensity



Notes: Panel A plots the decentralized equilibrium allocation to sector 1, K_1^{ϕ}, as a function of the accounting coefficient ϕ, holding the environmental pricing intensity τ fixed at its benchmark value. Panel B plots the corresponding true welfare at the induced allocation, normalized by first-best welfare, $W(K_1^{\phi})/W(K_1^{FB})$, as a function of ϕ. Panel C plots the decentralized equilibrium allocation K_1^{ϕ} as a function of the environmental pricing intensity τ, holding the accounting coefficient fixed at the benchmark reported-displacement value $\widetilde{f}$. In all panels, the other parameters are held at their benchmark values.

Figure 7 provides additional comparative statics for the decentralized equilibrium. Panel A illustrates Proposition 1: increasing the accounting coefficient ϕ reallocates capital toward the sector with the larger displacement base. Panel B reports the corresponding welfare evaluated at the induced decentralized allocation. Consistent with Proposition 5, accounting affects welfare only through the capital reallocation it induces. Panel C illustrates the distinct role of the environmental pricing intensity τ. Whereas ϕ determines which environmental signal is priced, τ determines how strongly decentralized agents respond to that signal. The figure therefore separates the accounting margin from the pricing margin.