Bank Lending Practices and Nonlinearity of Firms' Investment*

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Abstract

How can credit-market frictions generate heterogeneous investment responses across firms of different sizes? We develop a micro-founded macroeconomic model in which lenders choose whether to incur auditing costs when lending to firms whose idiosyncratic productivity shocks are otherwise unobservable. The lender's optimisation problem gives rise to an endogenous capital threshold that separates firms into different credit-treatment regimes. As aggregate conditions change, movements in this threshold alter firms' access to credit and generate non-uniform investment responses across the size distribution. The model provides a financial-frictions channel through which medium-sized firms may experience disproportionately larger investment declines during recessions. Its predictions are qualitatively consistent with evidence from UK firms during the financial crisis.

Keywords: *Bank Lending Practices, SME vs Corporate Banking, Credit Discrimination, Nonuniform Private Investment, Investment Specific Technology Shocks*

JEL Classificaition: E44, G21

*This is a work in progress paper. The model presented in this version of the paper, is currently being developed further to address comments that were received recently following research presentations in March and June 2026.

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1 Introduction

Most of the literature discussing the variation in performance between firms based on their size, have focused on dividing firms into two groups, Small and Medium Enterprises (SMEs) as a single block and large firms. Little work, at least within economics, have tried to theoretically discover the variation within SMEs. It is important to investigate this variation, since more than 95 per cent of firms around the world belong to this category. More than half of the workers in emerging economies are employed by firms with fewer than 100 employees (Ayyagari et al. (2011)). SMEs are equally important in developed economies as well. For instance, according to the UK's Department for Business , Energy and Industrial Strategy, DBEIS (2017), at the start of 2017 SMEs accounted for 99.9 per cent of the businesses population, 60 per cent of private sector employment and 52 per cent of private sector turnover.[1] The importance of SMEs extends beyond the absolute numbers and proportions they account for. Evidence on a negative relationship between firm size and net growth rates was found using US firms' employment dynamics datasets in Haltiwanger et al. (2010) and Neumark et al. (2011).

Many other studies reflect a similar pattern of difference in performance between small and large firms, especially during the recent recession and the following recovery. Small firms suffered proportionately more and exhibited a slower recovery (see Aliprantis and Burgen (2012); Tasci and Burgen (2012); Sahin et al. (2011); Krueger and Charnes (2011); CBO (2012); Fort et al. (2013)). The main explanation provided for the aforementioned pattern, is the severe reduction in credit extended by banks to small firms. Those firms are typically more reliant on bank lending, and hence are more affected by banking distress. Those greater financial limitations faced by SMEs as opposed to large firms have translated to different real activity. Using US data, Duygan-Bump et al. (2015) have found that small firms' financial limitations played an important role in contributing to the high unemployment during the recent recession, where small firms' workers were more likely to become unemployed. Benito et al. (2010) have used UK data and showed more generally that the contraction in credit supply during the financial crisis, has caused a sharp fall in business investment.

However, all those empirical studies have either reported aggregate phenomena with no breakdown of firms' behaviour based on pre-recession characteristics such as size, or made no distinction between firms within the subgroup of SMEs. In this paper I focus

[1]There is no universally agreed standard on classifying firms based on their size or other criteria such as their revenue or net worth. Definitions differ across countries and sometimes even between institutions within the same country. Among the standards used for classifying firms are workforce size, assets and turnover (Beck (2013)). For instance, most UK government departments including the Office for National Statistics (ONS) consider small firms as having 49 or less employees and medium firms to be the ones with 50 to 249 employees. However, the UK Companies Act consider a medium firm to be one with significant net worth of £4 millions and annual sales of £15 millions (ICAEW (2014)).

my attention on the empirical regularities reported in Crawford et al. (2013), where a large sample of data between 1997 and 2009 on UK firms and individuals has been used. Their study examines the extent to which firms' labour productivity, investment and profits have diverged during the Great Recession from their pre-recession linear trends. They report their results distinguishing between firms' responses based on their size as being small, medium or large.[2] I further restrict my attention to their results on firms' investment based on their size and try to provide a theoretical framework that can explain how such empirical regularities may have occurred. Figure 1 below summaries the results of Crawford et al. (2013) with regards to firms' investment responses during the recession relative to their pre-crisis trend. Their findings are consistent with Benito et al. (2010), where firms' total real investment has fallen sharply during the recession. However, unlike Benito et al. (2010) their investigation into those investment responses highlights a great dispersion based on firms' size, with medium-sized firms exhibiting the sharpest decline.

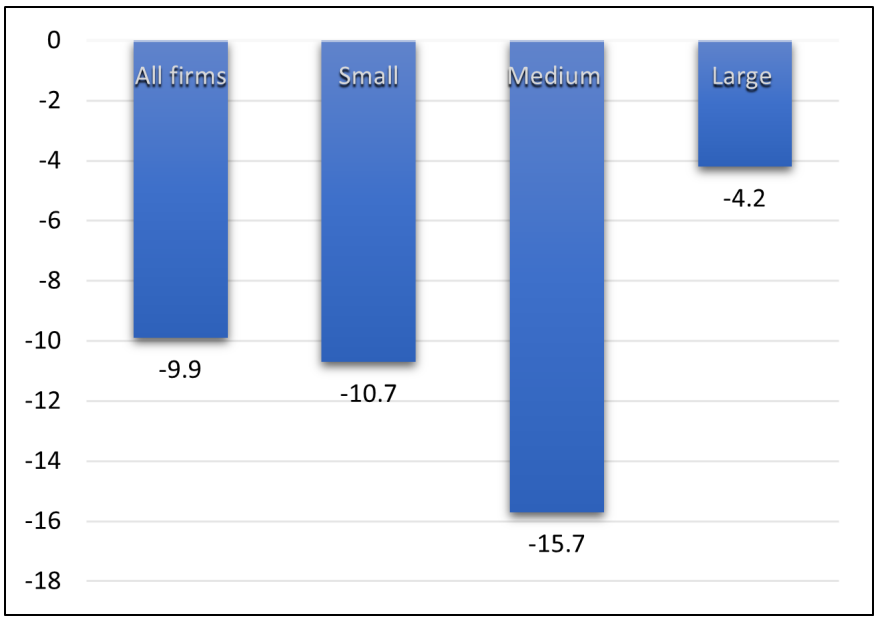

Figure 1: The recession's percentage impact on firms' investment in the UK relative to pre-crisis linear trend (source: Crawford et al. (2013)).

Their findings contradict with the intuition provided in the financial frictions' literature, where based on access to credit, investment declines are expected to be the sharpest in small firms followed by medium firms and then large firms. It is worth noting that such non-linear behaviour based on size is not unique to the empirical findings of this study. Audretsch and Elston (2002) have used German data from The Bonn Database and found results that are in line with Crawford et al. (2013) findings. Moreover, although such empirical investigations that look more into SMEs sub-classification are not widespread, recent

[2]They follow the ONS classification of firms' sizes, which depends on the size of a firm's workforce.

reports by market research firms suggest that medium-sized firms have outperformed both small and large firms post the recent recession.[3]

The model in this paper tries to explain the empirical regularities presented in Figure 1, based on bank lending practices. In the vast majority of commercial banks, lending activities to firms are split into two main divisions, each dealing with firms of a certain size. Usually firms' turnover or net worth is used as a proxy by banks to determine in which banking division firms will be dealt with. In order to motivate the assumptions of the theoretical framework, further empirical evidences on such banking practices are presented in the literature review. The model endogenously generates this cutting threshold of firms' net worth, $\hat{k}$, where the threshold can shift due to changes in the economy's exogenous parameters. Out of those economic parameters, I am most interested in the effects of neutral productivity shocks, z, and investment specific technology shock, q (see Fisher (2006)).

Given the theoretical framework presented, each of the two shocks will generate different responses between small, medium and large firms. Each shock causes those different responses based on a certain channel. In terms of the empirical regularities given in the UK, the model predicts that two channels must have been acting at the same time. However, just having the two channels working at the same time (two shocks), will not be sufficient in explaining how the empirical regularity came into place. What also matters is the relative size of the shocks to each other. The model predicts that the effect of the neutral productivity shock, z, on $\hat{k}$ should have been stronger than the effect of the investment specific technology shock, q, for medium firms to respond by more than other firms on average. In the same time, when comparing small and large firms' investment responses, the effect of the channel of q must have been stronger than the channel of z, such that small firms' investment would respond by more than the large firms' investment.

The paper proceeds in the following order: Section 2 presents the literature that is most relevant to the theoretical framework and provides the necessary background to motivate the model's environment. Section 3 presents the theoretical model. Section 4 solves for the equilibrium and discusses its properties. In Section 5, I amend the model by adding Investment. Section 6 presents the model results and discusses them. Finally, Section 7 concludes and suggests some policy recommendations and possible paths for future research.

[3]For example, check: BDO (2017, January). "Mid-sized companies revealed as the engine of the economy as they outperform UK plc," BDO United Kingdom, 30 January 2017, available at https://www.bdo.co.uk/en-gb/news/2017/mid-sized-companies-revealed-as-engine-of-uk

2 Literature Review

The literature review section of this paper presents a background on a number of different areas. Those areas are not directly connected, however, combined they provide the necessary literature background that is most relevant to the two questions being addressed and the motivation behind them. The topics covered in the literature review are: medium sized firms, bank lending practices, credit discrimination and financial frictions in macroeconomics, and the role of investment specific technology shocks.

2.1 Medium Sized Firms

Little work, at least within economics, have tried to discover the variation within SMEs. Medium-sized firms' performance and characteristics are different from small firms, which makes general statements about SMEs very broad brush. Nevertheless, the huge scale of the very large firms and their capabilities, as well as the view that the smallest firms are more deserving and adaptive to changes, mean that mid-sized firms are largely ignored in research and policy. In addition to the results of Crawford et al. (2013) and Audretsch and Elston (2002), and due to the little academic research on mid-sized firms, I also present in this subsection information that was obtained from market research firms and government agencies reports and surveys.

According to the UK's Department for Business, Energy & Industrial Strategy (DBEIS (2017)), there were around 5.7 million private firms at the start of 2017, with SMEs representing 99.9 per cent of the total population. This is not industry sector dependent, as SMEs represent at least 99.5 per cent of businesses in each of the main sectors. Figure 2 presents the contribution of different sized firms to the total population, employment and turnover at the start of 2017.

As can be seen in the figure above, although medium sized firms represent less than 1 per cent of the total UK business population, they punch way above their weight in terms of contribution to total private sector employment and turnover. BDO (2017) found medium sized firms to be the driving force behind UK exports in the five years following the Great Recession. Medium sized firms have grown at a faster rate and generated more jobs and growth in profits in 2016 than small and large firms. According to the classification that BDO uses,[4] those firms represented only 1.5 per cent of all UK businesses in 2016, yet they have contributed with a one third of the UK's turnover. Moreover, Livesey and Thompson (2013), Grant Thornton (2015) and OUBS (2011), report similar qualitative results where medium sized firms have outperformed small and large firms. Other

[4]BDO classifies small businesses as those with a turnover of less than £10 million. Medium-sized businesses are those with a revenue of £10 million to £300 million. While large businesses are those on the FTSE 350.

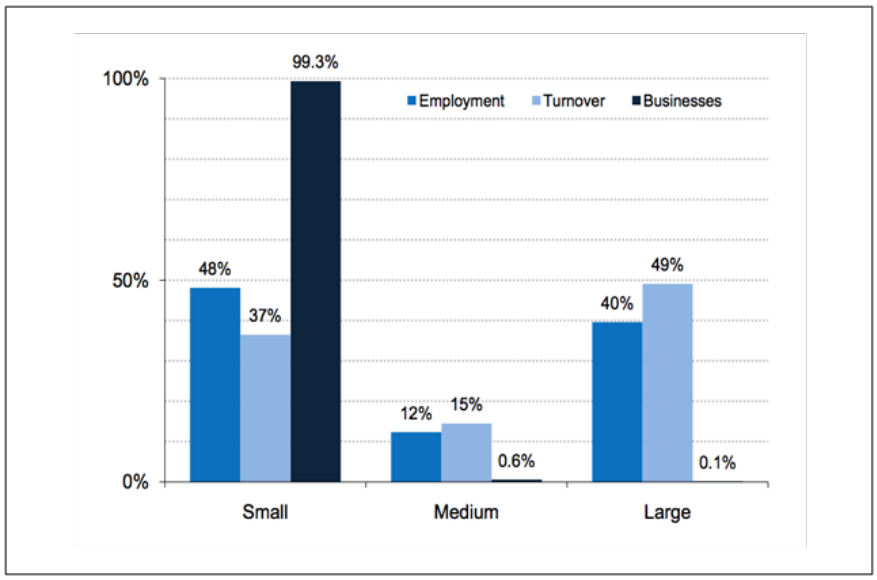

Figure 2: The contribution of different sized businesses to total population, employment and turnover, at the start of 2017 (Source: DBEIS (2017)).

surveys stress that the distinction between medium sized businesses and other SMEs, also extends to their entrepreneurial behaviour and attitudes towards growth. OUBS (2009) has constructed an entrepreneurial index and found that larger SMEs were more entrepreneurial. Additionally, in a SME survey conducted by the Institute for Chartered Accountants in England and Wales (ICAEW (2014)), it was found that growth appetite was strongly correlated with firms' size. While around half of small businesses in the survey were looking to grow, nearly 90 per cent of medium-sized businesses did. Furthermore, in a survey conducted by Driver (2007) that reports on the relation between business optimisms and investment, medium sized manufacturing firms' investment where found to be more sensitive to changes in the real interest rate.

Crawford et al. (2013), used a large sample of data between 1997 and 2009 on UK firms and individuals. As shown in Figure 1, they found that firms' investment based on size have exhibited a non-linear behaviour, where medium sized firms had the sharpest contraction in their investments relative to pre-crisis trend. They suggest that the sharper investment decline exhibited by SMEs in general is consistent with those firms facing tougher credit condition during the financial crisis. However, they do not provide an explanation to the greatest investment decline exhibited by medium firms. A similar result was also found by Audretsch and Elston (2002) who used data on German firms between 1961 and 1989 from The Bonn Database.[5] They examined the relationship between firms' investments and their liquidity constraints. They found that mid-sized firms faced greater

[5]They follow a different size criterion than Crawford, et al. (2013), with two mid-sized groups (small firms have 500 employees or less, small-mid 501 to 1300 employees, large-mid 1301 to 5500 employees and large 5501 employees and more).

liquidity constraints during economic downturns, thus, affecting their investments to a greater extent than small firms followed by large firms. Using survey data on firms in 90 countries, Aterido et al. (2009) have found that during adverse economic conditions, micro firms with less than 10 employees are less affected compared to bigger SMEs. They conclude that their results might be due to finance transmission channels affecting firms differently based on their size within the large SMEs population.

Financial constraints are faced by firms of all sizes. However, it is widely understood that the severity of those constraints is primarily size dependent, implying that the smaller is a firm the tougher are the credit conditions it faces. Consequently, the more severe credit conditions faced by medium firms compared to small and large firms during economic downturns, presents a puzzle. I explain this through the way banks perform their lending activities to firms, which is explained in the following section.

2.2 Bank Lending Practices

In the literature that studies lender-borrower relationships, banks are mostly treated as a single entity without a thorough investigation of their internal structures and characteristics. Conversely, the majority of commercial banks follow a segment approach, where they split their lending activities to firms between two main divisions. Each of those divisions deals with firms of a certain size category. Usually firms' turnover or net worth is used as a proxy by banks to determine in which division a firm will be dealt with. Here and after, I call those two bank divisions, SME banking and corporate banking. SME banking refers to financial services that are primarily catered for firms that fall below a certain threshold of firm size, while corporate banking provides services to firms that are considered large. This division of lending activities extends to different resource allocation, product differentiation and even delivery and client evaluation techniques carried out by banks (Beck (2013)).

In what follows I present the characteristics of those two main bank divisions, where I depend on various literature sources. However, I particularly make extensive use of information obtained from Albareto et al. (2011) who conducted a survey on behalf of the Bank of Italy on banks conducting lending activities to firms. Their sample covers 82.2 per cent of outstanding loans made by Italian banks to non-financial firms in 2007. The survey reports the differences in banking practices between SME and corporate banking. In assessing their business customers, banks use a mixture of quantitative techniques and soft information on their clients to determine customers' credit worthiness. However, the balance between quantitative methods and soft information gathering will differ between banks departments depending on the firm segment they deal with.

In corporate banking divisions, banks tend to employ more qualified staff and dedicate

more time and effort to study the quality of large firms' projects and financials. Moreover, rigid credit scoring techniques and statistical models are given less relative importance. On the other hand, qualitative information especially on the borrower's organisational structure, management credentials, quality of investment projects and the extensiveness of the borrower's banking relations with other competing banks, are given more importance. This allows for a judgemental decision making in extending loans (Albareto et al. (2011)). Additionally, when credit facilities are approved by banks they are usually granted with little collateral requirements, when compared to credit facilities extended to small firms. This is due to a number of reasons; the bargaining power of large firms that have larger network of banking relationships, allows them to substitute between lenders. Also, unlike small firms, large firms can get access to the commercial paper market, which remains largely a restricted domain for them. Furthermore, the in-depth analysis that banks conduct on large firms, allows them to identify good projects, and thus reducing the risk of defaults (Cressy and Olofsson (1997)).

The aforementioned is consistent with some of the relationship lending theories predictions, although those theories have not specifically approached the topic from the angle of bank lending practices and their division. The premises on which those theories depend is the existence of asymmetric information between borrowers and lenders, which results in heterogenous treatments that produce two main predictions. The first is that the cost to borrowers of switching lenders, declines as the borrowers' transparency improves. Transparency should improve with size, as large firms generally have better prepared financial data and better reputational record. Second, the cost of asymmetric information will depend negatively on the borrower's size, if the bigger is the loan size, the lower is the per unit cost of monitoring (Chodorow-Reich (2014)). Furthermore, Bester (1985) and Villas-Boas and Schmidt-Mohr (1999) build theoretical models where banks can screen borrowers. In their models, under asymmetric information, credit screening becomes a substitute for credit rationing, where rationing occurs if some borrowers cannot obtain loans although they accept higher collaterals and borrowing costs.

In SME banking, less effort and resources are dedicated to study the quality of firms' projects. This is due to a number of reasons, which include the large number of potential clients. Although, the total number of potential SME customers will be more than corporate customers, banks make less profit per SME banking relationship. Moreover, technological progress which significantly helped in reducing the cost of processing data, has led banks to depend more on credit scoring models and statistical techniques when selecting borrowers within the large segment of SME clients. This in return has allowed banks to direct more resources to auditing and screening large business borrowers (Albareto et al. (2011)). The decreasing dependence on soft information in SME banking is also due to the amount of available information on those firms. The availability and qual-

ity of information is strongly linked with firm size. Petersen and Rajan (1994) have used data from the American National Survey of Small Business Finances and found that small firms are more likely to face credit rationing. The lack of sufficient information means that lenders are less able to screen out potential defaulting SME borrowers. Furthermore, in a survey on US banks, Cowan and Cowan (2006) found that the less dependence on soft information when considering SME customers, meant that banks depended more on traditional creditworthiness criterions such as the availability of collaterals. Credit rationing due to lack of information has been addressed in the theoretical literature. For example, Jaffee and Russell (1976) built a theoretical model in which lenders ration extended credit when they cannot distinguish the quality of borrowers. A similar outcome will arise in the model presented in this paper, where some borrowers may not always be able to obtain their optimal borrowing demands due to uncertainty about the quality of their projects.

Binks and Ennew (1997), used data on six thousand UK small businesses to study the practices followed by banks in dealing with their SME customers. Banks use a combination of income and capital gearing approaches in evaluating loan applications, though the balance between both approaches vary with firm size. The income approach is preferable to the capital gearing approach, since it evaluates loan applications from a cash flow to loan size view instead of collateral to loan size. The earlier view depends on evaluating firms' current and expected future performance, and hence their ability to repay debt. However, this approach requires banks to have an in-depth understanding of a firm's business and the market it operates in, as well as the provision of such information and financial data by the borrowing firm. This is less likely in the case of SMEs. Additionally, it is also the case that such evaluations are too costly for banks to carry, and in the case of SMEs will render unprofitable given the loans sizes. Unlike equity finance where equity investors share both the up and downside, banks only share the risk of debt default, which in turn further reduces banks' incentive to follow the income evaluation approach when dealing with SME customers. Furthermore, banks also have less incentive for providing more flexible treatments, since SMEs are less likely to substitute between lenders. In a research conducted by the UK Competition and Market Authority (CMA (2015)), it was found that for most small business, once a business banking account have been established with a provider, there was little evidence of those businesses switching their bank accounts elsewhere. In another report (CMA (2016)), it was found that only 4 per cent of SME customers substitute to other banking providers in any year. This is usually due to SMEs being restricted by their geographical location as well as the search costs associated with finding alternative banking services. Therefore, banks will depend on the traditional capital gearing method, asking for high collaterals against credit provided. Both Fort et al. (2013) and Sahin et al. (2011) have found evidence that US states which suffered more severe fall in house prices were also the ones with greater decline in small businesses

borrowing. This is due to the small business owners who used their homes as a collateral for obtaining credit to finance their businesses' operations.

The phenomena of dividing banking activities for firms between two departments having the aforementioned characteristics, are more pronounced in larger banks. In the survey carried by Albareto et al. (2011), 70 per cent of large banks where internally organised in the said structure. They also applied more rigid quantitative methods and capital gearing approaches, in evaluating SME loan applications. It is worth noting that their survey did not cover subsidiaries of foreign banks with presence in Italy. In other words, such phenomenon is expected to be stronger in those banks, as they are international and typically larger. Indeed, Beck et al. (2011) confirm this hypothesis, using data on 91 banks across 45 countries. Furthermore, Small banks are less centralized and more efficient in collecting and processing soft information when dealing with small businesses. This is due to the geographical proximity between firms and the decision maker in a bank. This is rarely the case in large banks, where final decisions to grant loans are not made in branch offices (Berger and Udell (1995); Stein (2002)). Both Sapienza (2002) and Berger et al. (2005) have used US data and obtained results that are consistent with this prediction. Canales and Nada (2012) used Mexican data and found that less centralised lenders have provided relatively greater loan amounts to small firms. Furthermore, Presbitero et al. (2014) have used a sample of Italian SMEs and found that the effect of the financial crisis was stronger in regions that had a larger share of non-local and larger banks, where this organisation phenomena is more pronounced.

Bank concentration and centralisation is stronger in the UK and US. The deregulations of the 1970s and 1980s in the US, have resulted in the merger of many small banks into larger nationwide banks (Beck (2013)). In the UK the largest and oldest banks, which all have strong international presence, hold the majority of business accounts in the retail market. This high concentration by few very large banks, means that they do not have to compete hard to retain their SME customers (CMA (2016)). This had a profound effect on the ability of SMEs to obtain loans with more flexible terms and less rigid credit requirements. Indeed, as David Young, who was the UK Prime Minister's adviser on enterprise and small business in 2014, puts it: *"The real part of the problem is that banks' expenses are too high for small loans. If you are running a bank and you have got a choice between doing a £500,000 loan and £25,000 loan, the cost of doing both is the same so they tend to make the larger loan. I do not think it is deliberate. It is a fact of life"*.[6] Although bank lending practices play a central role in the model developed in this paper, I follow a 'reduced form' approach, which abstracts from modelling the financial intermediation process. Thus the

[6]Check: Hopkins, K. and M. Costello (2014). "Banks' high costs discriminate against small firms," The Times, 28 January 2014, available at https://www.thetimes.co.uk/article/banks-high-costs-discriminate-against-small-firms-0hv5265rm8p.

lenders and the borrowers interact directly, with the lenders following the aforementioned credit-screening practices in their lending decisions.

2.3 Credit Discrimination and Financial Frictions in Macroeconomics

The large macroeconomic literature on financial frictions deals with informational frictions between lenders and borrowers. Such informational frictions will result in problems of adverse selection or moral hazard, which in turn will lead to feedback and propagation channels that amplify initial real or monetary shocks. Bernanke et al. (1996, 1999) have named the changes in credit market conditions, caused by informational frictions which propagate initial shocks, as the financial accelerator. The amplification of initial shocks will cause more severe business cycles by affecting investment and production on several rounds. The numerous models in the literature differ greatly in their predictions and the ingredients that they use. This is due to the huge disagreement about which informational frictions are of paramount relevance and soundness. However, the models in the literature are divided between two main feedback and propagation channels; the net worth (balance sheet) channel and the bank-equity (bank lending) channel. In what follows, I present some models that employ those two feedback and propagation channels, then I discuss credit discrimination in theoretical literature more generally.

Models that employ the net worth channel are more established and they address the demand side of credit markets.[7] Essentially, those models depend on high state verification costs, to be incurred by the lenders in order to verify the state of the borrowers. Accordingly, lenders are discouraged from performing the required audits, and instead will either ration credit or require firms to pledge collaterals against credit extended to them. However, the limited wealth of firms will create contracting problems, which propagate adverse shocks to the economy (Bernanke and Gertler (1989); Bernanke et al. (1999); Carlstrom and Fuerst (1997)).[8] Kiyotaki and Moore (1997) built a dynamic model in which borrowers' net worth, represents a binding constraint on their ability to obtain credit. Moreover, the model incorporates endogenous procyclical changes in borrowers' assets values, thus affecting the amount of collateral they can pledge with the lenders, and consequently affecting their ability to borrow and invest. Hart and Moore (1994) develop a model in which borrowers use their assets as both factors of production and

[7] Those models are also called the internal equity models of credit frictions.

[8] To illustrate, starting from an adverse aggregate shock (for example, productivity, demand, etc), this shock will result in a decrease in firms' net worth. Accordingly, lenders will be less willing to lend, and thus aggregate investment and production drops. Consequently, firms' net worth will decrease even further, resulting in a further round of the same chain and so on. By symmetry, similar reasoning will apply when starting from a good aggregate shock. The key aspect of the informational frictions is that good investment projects are not funded, purely because of low net worth. In other words what determines if projects will be financed, is not whether projects have positive net present value (NPV) or not, but rather if there is sufficient net worth to pledge as a collateral against obtaining credit.

collaterals in order to obtain credit. In their model, adverse economic shocks lower the value of those assets, which in turn reduce credit extended by lenders. Consequently, the reduction in spending by borrowers will deteriorate the economy further resulting in a further lower value of their assets. Other papers have developed static models with adverse selection that focus on the properties of contracts between the lender and the borrower. The degree of access to credit and its effect on contracting efficiency and factor utilisation are presented in Farmer (1988a,b). Also, Diamond (1984) and Holmstrom and Tirole (1997) have developed microeconomic models that link the borrower's wealth level to the intermediation process and properties of the contracts they can obtain.

Models that employ the Bank equity channel are less established and they address the supply side of credit markets.[9] However, they also present mechanisms in which macroeconomic shocks are amplified.[10] Den Haan et al. (2003), develop a dynamic equilibrium model in which households' loanable funds are channelled to borrowers through intermediaries, via long term banking relationships. However, intermediaries will need to have a minimum amount of funds in order to induce firms to produce and not choose their outside option. Adverse shocks which make households invest less in the credit market, will result in some of the relationships to severe since some of the intermediaries will end up with funds below the required threshold that is necessary to induce borrowers to produce. With less existing relationships in the credit market, returns on household investment will drop. This in turn induces households to invest even less, resulting in more relationships breaking up. If the initial shock is sufficiently large, then it can lead to a complete collapse of the credit market by destroying enough number of credit relationships.

A number of empirical studies have investigated the variation between large and small firms' responses to adverse economic shocks, through the financial frictions channels. Gertler and Gilchrist (1994) have used US data on manufacturing firms to study the response to monetary contractions. They found that small firms have exhibited significant disproportionate response compared to large firms. This has mainly occurred through a decline in small firms' demand for inventories due to the financial limitations they face. Crouzet and Mehrotra (2017) have also used data on manufacturing firms from the US Census Bureau, and they found that small firms' turnover, inventories and investment

[9]Those models are also called the liquidity-flows models.

[10]To illustrate, according to the Basel accords which regulate banks' lending activities, bank equity constraints bank ability to issue or renew loans. Therefore, starting from an adverse shock, such as some firms defaulting on their loans, will result in a decrease in banks' equity. Accordingly, banks will supply less loans, preventing firms from obtaining sufficient financing. Consequently, firms scale back their production plans, which will result in the economy deteriorating even further. This in turn will result in more firms defaulting on their debt obligations, triggering a further round of the same chain and so on. By symmetry, similar reasoning will apply when starting from a good aggregate shock that increases banks' equity. Again, in a completely frictionless world, all that matters is that firms' projects have positive NPV for banks to be able to extend credit to them.

were more procyclical than large firms. Moreover, they also found that independent of firm size, indebted firms were more sensitive to adverse shocks. This led them to conclude that the excess procyclicality of small firms is due to the tougher credit conditions they face. On the contrary, Kudlyak and Sanchez (2017) findings challenge the predictions of the financial frictions theoretical literature. They found that large firms turnover and short-term debt have contracted proportionately more than small firms during the recent recession.[11]

Some of the theoretical models in the literature, especially within the net worth channel, have tried to explain this variation in the response between small and large firms, through firms' credit having varying vulnerability to the adverse shocks affecting the economy. Small firms in general are not just a scaled down version of large firms. Their financial characteristics differ from large firms. For example, they have lower ratios of fixed to total assets, higher liabilities to total assets ratios, and they are less productive than large firms due to economies of scale (Cressy and Olofsson (1997)). To the best of my knowledge, all the theoretical models generate discriminating credit treatments and contracts, by imposing exogenous heterogeneity between small and large firms in one or more dimensions, other than their initial condition, which is their size. Firms can be of multiple types in those models. Types can take the form of differing abilities in repaying loans with high and low risk borrowers, such as in Bester (1985), Besanko and Thakor (1987) and Broecker (1990). Additionally, types may reflect different production technologies, where one type of firms is more productive than the other type, and enjoys higher success rates, such as in Becsi et al. (2005). Moreover, some models impose that firms have access to different investment projects, that are either of different rates of success or returns for different firms, such as in Caminal and Matutes (1997) and Villas-Boas and Schmidt-Mohr (1999). Other models depend on lenders having different market power to generate heterogenous credit treatments, such as in Petersen and Rajan (1995). Finally, some models abstract from imposing different characteristics between firms that lead to credit discrimination, and simply assume the heterogenous credit treatment exogenously, such as in Bernanke et al. (1999).

[11] Their dataset covered the period between 1958Q4 and 2014Q2. The data they used is aggregated into eight asset classes, with those classes remaining constant through the entire sample. However, they define small firms to be the ones below the twenty-fifth percentile of firms' turnover distribution in each period. Thus, as the distribution of firms' sales continues shifting to the right with time, the asset classes in which data is aggregated into, remains constant. For example, in the early years of their sample, firms with total assets of $5 million and under were small firms, while in the end of their sample (precisely by the start of the recession and onward) firms with total assets of $1000 million and under were considered to be small firms. I find this very implausible, nevertheless, I later show that my model can generate such results if I follow their definition of small firms. The implication of their definition of small firms on my model, is that medium firms will be grouped with large firms, where medium firms exhibit the largest proportionate contraction in debt and output, following adverse shocks.

2.4 The Role of Investment Specific Shocks

Investment specific technology (IST) shocks play an important role in the model presented later in this paper. In this section of the literature review, I provide a brief overview of the theoretical and empirical literature on this type of shocks. Unlike neutral technology (total factor productivity) shocks, which affect the production of all goods homogenously, investment shocks impact only the production of investment and capital goods. Technological advancements such as improved transportation, telecommunication and equipment, have made the production of other investment goods relatively cheaper over time, which in turn caused higher accumulation of capital. Therefore, the trend has been a reduction in the relative price at which those goods are sold at (Fisher (2006)). The literature on investment shocks considers two types of those shocks; investment specific technology (IST) shocks and marginal efficiency of investment (MEI) shocks. IST shocks affect the rate at which investment goods are produced from consumption goods, while MEI shocks affect the rate at which investment goods can be transformed to productive capital (Justiniano et al. (2011)). In my model, I consider IST shocks and implicitly assume that there are no shocks to the marginal efficiency of investment.

The theoretical literature on IST shocks argues that they represent an important alternative to neutral productivity shocks as a source of business cycles in a general equilibrium setup. They have a key role in explaining short run macroeconomic fluctuations, as well as the long-term movements of some key indicators such as labour productivity in growth models (check Greenwood et al. (1988); Greenwood et al. (1997); Fisher (1997); Christiano and Fisher (1998); Campbell (1998)). More recent studies include Greenwood et al. (2000) who develop a variation of the model in Greenwood et al. (1997). They have used US data on the relative price of new equipment to calibrate their model and found that IST shocks can account for 30 per cent of the fluctuations in output. This is very significant when considering that investment in new equipment represent only 7 per cent of the US GNP. Moreover, both Smets and Wouters (2007) and Justiniano et al. (2010) have developed DSGE models, which are extended versions of the New Neoclassical Synthesis model and use Bayesian estimation. In both studies, in the short run, IST shocks explain a significant proportion of the variation in labour hours and output, and their estimates are comparable to the structural vector autoregression (SVAR) estimations in Fisher (2006).

Fisher (2006) adopts the same competitive growth model as in Greenwood et al. (1997), however, he has expressed it with a social planner problem and simplified by incorporating one instead of two capital goods.[12] The simplification did not affect the analysis. When compared to the neutral productivity shock, he found that most of the variation in short run labour hours and output are explained by IST shocks. In Fisher (2006) the

[12]The welfare theorem holds in the competitive model, hence why Fisher (2006) has expressed it with a social planner problem.

accumulation of capital in the model is given by:

$$\begin{aligned} k_{t+1} &\leq (1+\delta)k_t + \nu_t i_t \\ \nu_t &= \exp(\theta + \varepsilon_{\nu t})\ \nu_{t+1}, \quad \theta \geq 0 \end{aligned} \tag{1}$$

where k is the level of capital from a period to another as indicated by the time subscript, i_t is the amount of investment in capital, ν_t is the level of the investment specific shock, and $\varepsilon_{\nu t}$ denotes the shock to the investment-specific technology at time t. Therefore, in a competitive economy, IST is equivalent to the inverse of the price of investment relative to the consumption good. I follow the specification given in Equation (1) to introduce IST shocks later in my model. The models in the literature use different channels, through which shocks transmit and propagate to create large variations in the macroeconomic variables.

Fisher (2006) extends on his work by conducting a SVAR estimation and use the theoretical model to derive the identifying assumptions. He uses US data and splits the sample at 1982, due to the significant changes in the conduct of monetary policy, regulations and the rate of decline in real equipment price. Moreover, three econometrical specifications were considered and in the most preferred specification, he found that neutral productivity shocks and IST shocks account for 73 per cent and 44 per cent of the variation in hours and output, respectively before 1982. Furthermore, they have accounted for 38 per cent and 80 per cent of the variation in the same variables, post 1982. He has also found that IST shocks were behind most of the variation in labour hours and output. All the previously mentioned studies are based on the US. To the best of my knowledge, Bakhshi and Larsen (2001) is the only paper that is based on the UK. They have used a DSGE model for the UK, where the investment-specific technological change is limited to the advancements in the information and communications technology (ICT) sector. They arrived at similar qualitative conclusions as the previously mentioned studies.

In the model presented in the next section, I have used both investment specific and neutral productivity shocks in a financial frictions framework. I study the effects of both shocks within the context of bank lending practices, and how this map to variations in firms' investment responses depending on their sizes and access to credit.

3 The Model

The model seeks to answer the following questions: *(i)* why banks offer different credit treatments to firms of different sizes but share the same production technology, *(ii)* why medium sized firms have exhibited the largest percentage decline in investment in the

2008 recession in the UK, followed by small firms and then large firms as presented in Crawford et al. (2013), and *(iii)* why large firms have exhibited larger decline in their sales (output) and debt relative to small firms in the 2008 recession in the US as presented by Kudlyak and Sanchez (2017).

3.1 The Model's Environment

I consider a one period economy in which there is one good that may be used for consumption, investment and contracting, and its price is normalized to one. The consumption good can be transferred to capital at a rate q, where q is a technology parameter for transferring the final good to capital.[13] Thus, $1/q$ is the number of consumption good units that must be exchanged to get a unit of the capital good.[14] The agents in this economy are: *(i)* a unit mass of identical households, indexed by n, that I also call the lenders; and *(ii)* a unit mass of entrepreneurs, indexed by j, whom I also call the borrowers. In other words, since there is a unit mass of each agent, a one to one matching is assumed at the start of the economy, which eliminates the search problem and hence I can ignore the agents' indices.[15] At the start of the economy, each household (lender) is endowed with the production factor E.[16] The household's endowment is sufficiently large, such that the resource constraint in lending to the entrepreneur will not bind. Let us suppose that he receives an endowment of E equals to X , where X is a constant.

Each household maximizes $u(c)$, where $u(c) = c$, thus being risk neutral. The household owns a production technology $g(E)$, that depends only on the production factor E, and yields one to one in terms of the final consumption good. Each household can also lend part of his endowment of E to the entrepreneur he is matched with, where the entrepreneur uses it as an input in one of her two production technologies. When the household lends, he is payed back in terms of the final consumption good. Lending takes the form of a contract with a Nash Bargaining solution. Finally, I assume that the production factor E fully depreciates when used in production, regardless of the production technology it is used in.

Each entrepreneur also maximizes $u(c)$, where $u(c) = c$, thus being risk neutral. The assumption that both agents are risk neutral simplifies the analysis and ensures that the

[13] I follow Fisher (2006) in the way he has included the investment specific shocks in the RBC model that he used to derive the identifying assumptions used in the empirical estimation. However, unlike Fisher (2006) where those shocks are persistent, I assume those shocks to be a white noise since the model presented here is static.

[14] Therefore, as in Fisher (2006), in a competitive economy $1/q$ will be the real price of the capital good.

[15] I abstract from introducing a banking sector and a financial intermediation process in the model. Instead, I follow a reduced form approach, where I assume that the lenders and the borrowers are matched directly, with the lenders performing the credit screening that is typically performed by the banking sector.

[16] It can be thought of as a raw material or a working capital that enters into the production of the final good.

equilibrium is tractable. Entrepreneurs are endowed with physical capital k which is fully observable, however, their endowments of capital differ at the start of the economy. Accordingly, the aggregate capital stock in the economy is given by:

$$K = \int_0^1 k_j \, \mathrm{d}j \tag{2}$$

Each entrepreneur owns a firm with two production technologies, both producing the final consumption good, albeit at different productivities. The first production technology is $h(k)$, where $h(k) = (1/q)k$. This production technology depends only on capital and converts the capital good to the final consumption good. Capital also fully depreciates when used in this production technology. The second production technology is $f(k, x) = zk^{\alpha}x^{1-\alpha}$, where $\alpha \in (0, 1)$ and z is the standard neutral productivity shock.[17] The latter is the production technology used when lending occurs. Production using the second technology requires two inputs; the entrepreneur's capital k, and x which is the borrowed amount from the household endowment of the production factor E. In other words, both agents can stay in autarky, or engage in lending and borrowing, where the entrepreneur uses her second production technology $f(k, x)$. I follow Quadrini (2011) in assuming multiple sectors, where the final good can be produced $g(E)$, $h(k)$ and $f(k, x)$, although at differing productivities. Finally, I assume that a fraction $\delta \in (0, 1)$ of capital k, depreciates when used in the second production technology $f(k, x)$.

My assumption that borrowing is used for working capital needs rather than investment in physical capital is supported by a number of empirical studies. Kashyap et al. (1994) have studied firms' inventories behaviours using micro US data and found that bank-leveraged firms have significantly cut their inventories relative to non-bank-dependent firms in response to adverse macroeconomic conditions. Moreover, Paravisini et al. (2015) have used Peruvian data to study the effect of credit shocks on firms' export behaviour during the recent financial crisis. They found that bank-dependent firms have reduced their exports by more relative to non-bank-dependent firms, where this occurred mostly through raising firms' variable production costs rather than affecting their stock of fixed capital. Although firms use credit for a number of reasons, the primary use of short-term debt is facilitating working capital needs rather than replacing exiting physical capital or facilitating expansions (Cressy and Olofsson (1997)).

3.2 The Uncertainty Structure and Contractual Friction

The uncertainty structure in the economy is as follows: *(i)* an aggregate neutral productivity shock, z, that is symmetric across all firms; *(ii)* an investment specific technology shock,

[17] As is Fisher (2006), z can be thought of as a number of factors that affect the production possibilities.

where q can possibly vary and it is also symmetric across all firms; and *(iii)* an idiosyncratic shock for each entrepreneur that is specific to her technology $f(k, x)$. More specifically, when using the technology $f(k, x)$, the entrepreneur's production can either be successful, generating positive output with a prior probability $P \in [0, 1]$, or unsuccessful with a prior probability $1 - P$, which generates zero output and thus wastes the borrowed amount x.

Accordingly, the lender-borrower relationship functions under uncertainty due to the idiosyncratic shocks. At the start of the economy, all shocks are realized. The realisations of the aggregate neutral productivity shock and the investment specific shock are observable. However, an entrepreneur will not learn the realisation of her idiosyncratic shock until she borrows and starts the production using the technology $f(k, x)$, or when being audited by the household (lender) before producing. The household has an audit technology, but he incurs a high fixed cost in using it, $A > 0$. Using audit reveals the realisation of the entrepreneur's idiosyncratic shock and it becomes a common knowledge. When audit is used by the lender and the entrepreneur's production project is found to be successful, the household lends without imposing collateral requirements. This is because the entrepreneur is able to pay back her debt using output from the successful production. However, if audit reveals that the entrepreneur's project is unsuccessful, then no loan will be extended by the household. Accordingly, both of the matched household and entrepreneur, will stay in autarky and use their production technologies $g(E)$ and $h(k)$, respectively.

On the other hand, if the lender chooses not to audit, then he still lends to the entrepreneur, yet he fully secures the loan to ensure the repayment of x, by using the entrepreneur's undepreciated capital stock as a collateral. In other words, an entrepreneur who is not audited can borrow a maximum amount $x \leq (1/q)(1 - \delta)k$. Consequently, a borrower with a small capital stock k, may not be able to borrow her optimal demand, especially when q is very high. This is in line with the discussion presented earlier in the literature review. Since banks do not incur the high audit costs when dealing with small firms, uncertainty will be present on whether those firms will be able to satisfy their debt obligations or not. Accordingly, banks require firms to pledge high collaterals as a precaution against defaulting debt.

As will be shown later in the model's equilibrium, a lender will only choose to pay the high fixed audit cost A, if the entrepreneur's capital stock $k \geq \hat{k}$, where $\hat{k}$ is a capital threshold for auditing that arise through the lender's optimisation problem.

3.3 The Lender-Borrower Relationship

When lending occurs, the matched household and entrepreneur engage in production and share output. According to the assumed structure of the credit contracts, the borrowed

amount x, must be paid back regardless of the production process outcome. Moreover, the division of output between the lender and borrower is agreed through a Nash Bargaining solution. I let $\gamma \in [0, 1]$ be the lender's bargaining weight, while $1 - \gamma$ is the entrepreneur's bargaining weight. Nash Bargaining is widely used in macro-labour economics literature. It is also used in macroeconomic financial frictions literature in setting credit contracts (check Den Haan et al. (2003)).[18]

3.4 Timing within the Period

Although a period is expressed as a single point in time, I add a timing description within the period to facilitate understanding the model. The timing within a period is in the following order: *(i)* aggregate and idiosyncratic shocks are realized, however, the realisation of the idiosyncratic shock is unobservable; *(ii)* the household forms the decision whether to lend the entrepreneur he is matched with or not; *(iii)* if the household decides to lend, then he also decides whether to audit the entrepreneur or not; *(iv)* agents agree the division of output through a Nash Bargaining solution; *(v)* agents form their consumption plans, more specifically, the entrepreneur declares her demand for depending on whether she is audited or not; *(vi)* the outcome of the auditing process is declared for the audited entrepreneur, more precisely, agents find out whether the realisation of their idiosyncratic shock is 0 or 1; *(vii)* credit is extended to the unaudited entrepreneur and to the successful audited entrepreneur; *(viii)* production starts. This is when the unaudited entrepreneur will learn the realisation of her idiosyncratic shock; *(ix)* output from production is realized and the division of output takes place as agreed;[19] *(x)* agents consume.

4 The Equilibrium

I now solve for the equilibrium of the model. This consists of the Nash Bargaining, the entrepreneur's problem, the household's problem and obtaining the capital threshold for auditing, $\hat{k}$. I also discuss the properties of this equilibrium by conducting comparative statics, and I discuss other conditions that are necessary for the equilibrium. In all what follows below, the budget constraints are expressed in terms of the consumption good.

4.1 Nash Bargaining

The lender and borrower negotiate and agree the credit contract. Credit contracts are assumed to be written for one period only. The division of the match surplus is agreed

[18] I have attempted to address the research questions in a general equilibrium set up, however, an equilibrium does not always exist. A detailed discussion of those attempts is presented in the Appendix A.

[19] i.e. contracts are enforced.

through a Nash Bargaining solution, where $zk^{\alpha}x^{1-\alpha}$ is the output of production that the lender and borrower bargain over. As mentioned earlier, γ and $1-\gamma$ are the lender's and borrower's bargaining weights, respectively. I let w be the payoff to the lender, and therefore $zk^{\alpha}x^{1-\alpha}-w$ is the entrepreneur's payoff. Moreover, x and k/q are the lender's and borrower's outside option, respectively. It is the amount of the consumption good that they get when they choose to stay in autarky, using the production technologies $g(e)$ and $h(k)$. Accordingly, the surplus of the match is $zk^{\alpha}x^{1-\alpha}-x-(k/q)$. The Nash Bargaining maximises the weighted combination of the lender's and borrower's shares:

$$\max_{w}(w-x)^{\gamma}\Big(zk^{\alpha}x^{1-\alpha}-w-(k/q)\Big)^{1-\gamma} \tag{3}$$

I take logs and then the first-order condition. Thus, the lender's share from the production surplus is given by:

$$w=\gamma\Big(zk^{\alpha}x^{1-\alpha}-x-(k/q)\Big)+x \tag{4}$$

while the entrepreneur's share is given by:

$$zk^{\alpha}x^{1-\alpha}-\gamma\Big(zk^{\alpha}x^{1-\alpha}-x-(k/q)\Big)-x \tag{5}$$

4.2 The Entrepreneur's Problem

The entrepreneur maximizes $u(c)=c$. Thus, the entrepreneur's problem is to maximise her profits, which are given in terms of the final consumption good c. Moreover, since an entrepreneur can be subjected to two different credit treatments, depending on whether she is being audited or not, there will be two versions of the optimisation problem.

4.2.1 The Audit Case

The entrepreneur knows her share from the production surplus. I use the Nash bargaining solution as given in Equation (5) to substitute into the optimisation problem. The entrepreneur maximises her share from the production surplus, and thus her profits, by choosing her optimal demand for x:

$$\max_{x}\pi=P\left(zk^{\alpha}x^{1-\alpha}-\gamma\Big(zk^{\alpha}x^{1-\alpha}-x-\frac{k}{q}\Big)-x+\frac{(1-\delta)k}{q}\right)+(1-P)\frac{k}{q} \tag{6}$$

To illustrate more, the audited entrepreneur faces two prospects in the audit process, either her project is found to be successful or not. Hence, when her project is found to be unsuccessful with probability $1-P$, she will use her entire capital endowment to produce the consumption good using the technology $h(k)$. Furthermore, since only a fraction δ

of capital depreciates when using the technology $f(k,x)$, she also use her undepreciated capital $(1-\delta)k$ to produce the consumption good using the technology $h(k)$, when her project is found to be successful under audit. Taking the first order condition and solving for the optimal demand x for under audit:[20]

$$x_A^* = \Big((1-\alpha)z\Big)^{\frac{1}{\alpha}} k \tag{7}$$

4.2.2 The No Audit Case

The unaudited entrepreneur does not know the realisation of her idiosyncratic shock. Nevertheless, the lender still lends but uses the entrepreneur's undepreciated capital as a collateral, to ensure the repayment of x in case the realisation of the idiosyncratic shock is equal to 0. The entrepreneur takes this uncertainty into account in her optimisation problem:

$$\begin{aligned}\max_x \pi =& P\left(zk^\alpha x^{1-\alpha} - \gamma\Big(zk^\alpha x^{1-\alpha} - x - \frac{k}{q}\Big) - x + \frac{(1-\delta)k}{q}\right) \\ &+ (1-P)\left(-x + \frac{(1-\delta)k}{q}\right)\end{aligned} \tag{8}$$

To illustrate, if the entrepreneur finds her production under the technology $f(k,x)$ to be unsuccessful, she will use her undepreciated capital $(1-\delta)k$ to produce the consumption good using the technology $h(k)$. However, she also needs to pay back the borrowed amount x. Taking the first order condition and solving for the optimal demand for x:[21]

$$x_{NA}^* = \left(\frac{(1-\alpha)Pz(1-\gamma)}{1-P\gamma}\right)^{\frac{1}{\alpha}} k \tag{9}$$

where the subscript NA indicates the optimal demand under no audit.

4.2.3 Non-Binding vs Binding Borrowing Constraint for The Unaudited Entrepreneur

Since the unaudited entrepreneur can borrow a maximum amount $x \leq (1/q)(1-\delta)k$, an entrepreneur with a small capital stock k, may not be able to borrow her optimal demand. This is specifically when q is very high. To find this threshold level of q, I compare the optimal demand x_{NA}^* and the maximum borrowing amount $(1/q)(1-\delta)k$:

[20] The function is concave; thus, the second order condition is satisfied.
[21] The function is concave; thus, the second order condition is satisfied.

$$\begin{array}{ccc} \text{unaudited entrepeneur optimal} & > & \text{maximum amount the unaudited} \\ \text{demand } x^*_{NA} & & \text{entrepreneur can borrow} \\ \left(\frac{(1-\alpha)Pz(1-\gamma)}{1-P\gamma}\right)^{\frac{1}{\alpha}} k & > & (1/q)(1-\delta)k \end{array}$$

I solve the above equation for q and obtain:

$$\bar{q} = (1-\delta)\left(\frac{(1-\alpha)Pz(1-\gamma)}{1-P\gamma}\right)^{-\frac{1}{\alpha}} \tag{10}$$

where $\bar{q}$ is the value of q at which the unaudited entrepreneur's borrowing constraint binds.

4.3 The Household's Problem

The household's utility maximisation problem consists of two problems. The first problem is whether to lend or not. To solve this, I compare the household's utility under autarky versus the case of lending. The second problem is when lending is found to be optimal compared to autarky, then should the household audit the borrowing entrepreneur or not. Accordingly, I obtain the household's indirect utility for all the said cases and compare between them.

4.3.1 Autarky

When the household choose not to lend, then he uses all of his endowment of the production factor E, in producing the consumption good using the production technology $g(E)$. Therefore, the household utility is given by:

$$u(c) = X \tag{11}$$

4.3.2 Lending with Audit

This is the case where the household chooses to incur the audit cost A to reveal the realisation of the borrower's idiosyncratic shock. When audit is used, then with prior probability P the entrepreneur's production project is found to be successful, and hence lending occurs. When lending occurs, the household lends x, which is part of his endowment of the production factor E, and uses the remaining of it after deducting the audit cost $X - A - x$, in the other production technology that he owns, $g(E)$. Otherwise, with prior probability $1 - P$ the entrepreneur's project is found to be unsuccessful and both agents

stay in autarky. The household uses his endowment $X - A$ to produce the consumption good using the technology $g(E)$. Accordingly, the household's expected utility when using audit is given by:

$$EU_A^H = P(X - A - x + w) + (1 - P)(X - A) \tag{12}$$

where the superscript H indicates the household and the subscript A indicates the case where audit is used. I substitute for w and x using the Equations (4) and (7), respectively and obtain the indirect utility under audit:

$$EU_A^H = X - A + kP\gamma \left((z)^{\frac{1}{\alpha}} \left((1-\alpha)^{\frac{1-\alpha}{\alpha}} - (1-\alpha)^{\frac{1}{\alpha}} \right) - (1/q) \right) \tag{13}$$

4.3.3 Lending with No Audit

When the lender does not use the audit technology, he will still lend to the entrepreneur. However, he will secure the loan to ensure the repayment of x by using the entrepreneur's undepreciated capital stock as a collateral. The lender will be willing to lend the entrepreneur a maximum amount $x \leq (1/q)(1-\delta)k$. Since q can either be greater or less than $\bar{q}$, two cases will emerge under lending with no audit. Additionally, the entrepreneur uses the remainder of her endowment of E, $X - x$, to produce the consumption good using the technology $g(E)$. Accordingly, the household's expected utility when lending under no audit is given by:

$$EU_{NA}^H = P(X - x + w) + (1 - P)(X - x + x) \tag{14}$$

When $q \leq \bar{q}$, the entrepreneur can obtain her optimal demand, x_{NA}^*. I substitute for w and x using the equations (4) and (9), respectively. Therefore, the household's expected utility when lending under no audit, and the entrepreneur's borrowing constraint is non-binding, is given by:

$$EU_{NA1}^H = X + kP\gamma \left[(z)^{\frac{1}{q}} \left(\left(\frac{(1-\alpha)P(1-\gamma)}{1-P\gamma} \right)^{\frac{1-\alpha}{\alpha}} \right) - \left(\left(\frac{(1-\alpha)P(1-\gamma)}{1-P\gamma} \right)^{\frac{1}{\alpha}} - \frac{1}{q} \right] \right. \tag{15}$$

where the subscript $NA1$ indicates the case where audit is not used and $q \leq \bar{q}$. On the other hand, when $q > \bar{q}$ the entrepreneur can borrow a maximum amount $x = (1/q)(1-\delta)k$, which is strictly less than x_{NA}^*. I use the later expression to substitute for x and equation (4) to substitute for w, in equation (14). The household's expected utility when lending under no audit, and the entrepreneur borrowing constraint is binding, is therefore

given by:

$$EU^{H}_{NA2} = X + kP\gamma\left(z\left(\frac{1-\delta}{q}\right)^{1-\alpha} - \frac{1-\delta}{q} - \frac{1}{q}\right) \tag{16}$$

where the subscript $NA2$ indicates the case where audit is not used and $q > \bar{q}$.

4.4 Obtaining the Audit Threshold $\widehat{k}$

When lending without incurring the audit cost, A, the household obtains x at least from the entrepreneur, by seizing the collateral. To clarify, the household's utility when lending under no audit and the realisation of the idiosyncratic shock is 0 will be X. If the realisation is equal to 1, then it will be strictly greater than X, such that in expectation it is strictly greater than his utility under Autarky. However, this may not always be the case, and I need additional conditions on the model's parameters to be satisfied, to ensure that I am not in the autarky equilibrium.[22] Those conditions are discussed later in section 4.6. For now, I assume that those conditions are satisfied. Accordingly, household will never choose autarky over lending under no audit. As will be shown below, the household's expected utility when using audit will only be greater than the case of no audit, when the entrepreneur's capital is sufficiently large. Also, since q can either be greater or less than $\bar{q}$, two cases will emerge.

4.4.1 The Non-Binding Borrowing Constraint Case

When $q \leq \bar{q}$, the unaudited entrepreneur can obtain her optimal demand, x^{*}_{NA}. The household will be indifferent between auditing and not, when:

$$EU^{H}_{A} = EU^{H}_{NA1} \tag{17}$$

I use equations (13) and (15) to substitute in equation (17), and I solve for $\hat{k}$:

$$\hat{k} = \frac{A}{P\gamma(z)^{\frac{1}{\alpha}}\left(\left((1-\alpha)^{\frac{1-\alpha}{\alpha}} - (1-\alpha)^{\frac{1}{\alpha}}\right) - \left(\left(\frac{P(1-\alpha)(1-\gamma)}{1-P\gamma}\right)^{\frac{1-\alpha}{\alpha}} - \left(\frac{P(1-\alpha)(1-\gamma)}{1-P\gamma}\right)^{\frac{1}{\alpha}}\right)\right)} \tag{18}$$

where $\hat{k}$ is the endogenous audit threshold for the level of capital that arise through the lender's optimisation. It will imply different credit treatments to firms depending on their level of capital stock. Accordingly, firms with a capital stock $k \geq \hat{k}$ are considered large

[22] The production technology $f(k, x)$ must be sufficiently productive compared to the technologies $g(E)$ and $h(k)$, for agents to choose not to be in the autarky equilibrium.

firms by the lenders and thus will be audited, while firms with a capital stock $k < \hat{k}$ are considered small and will not be audited.

4.4.2 The Binding Borrowing Constraint Case

On the other hand, when $q > \hat{q}$ the unaudited entrepreneur can borrow a maximum amount $x = (1/q)(1-\delta)k$ which is strictly less than x^*_{NA}. The household will be indifferent between auditing and not, when:

$$EU^H_A = EU^{NA2} \tag{19}$$

I use equations (13) and (16) to substitute in equation (19), and I solve for k:

$$\hat{k} = \frac{A}{P\gamma\left((z)^{\frac{1}{\alpha}}\left((1-\alpha)^{\frac{1-\alpha}{\alpha}} - (1-\alpha)^{\frac{1}{\alpha}}\right) - \left(z\left(\frac{1-\delta}{q}\right)^{1-\alpha} - \frac{1-\delta}{q}\right)\right)} \tag{20}$$

The intuition behind why audit occurs above a certain threshold of capital is as follows: For a given level of capital, the entrepreneur's borrowing is greater under audit compared to the case of no audit. This means that the division of output, w , which is the lender's revenue is bigger in absolute under auditing. However, the lender also incurs a fixed audit cost under auditing, while lending without audit is costless. Consequently, the wedge between the division of output that the lender gets back under audit and the no audit case (i.e. the difference in revenue between the two cases), will not be sufficient to offset the auditing cost at low levels of capital.

Nevertheless, this wedge in the lender's revenues between the two cases is not constant in capital, unlike the auditing cost, A. The entrepreneur's borrowing demand depends positively on her capital stock, and the difference between those demands grows larger with capital, since $(\partial x^*_A/\partial k) > (\partial x^*_{NA}/\partial k)$. This implies that the difference in the lender's revenue between the two cases depends positively on the entrepreneur's capital. Accordingly, there will exist a certain level of capital, $\hat{k}$, where above it the wedge between the lender's revenue under the two cases will surpass the fixed audit cost, and hence making it worthwhile to audit the entrepreneur.

4.5 Comparative Statics

In this section I look at the equilibrium properties, by conducting comparative statics on the borrowing demands given in Equations (7) and (9). This is also done for the auditing thresholds that are given in Equations (18) and (20) where I discuss how changes in the model's exogenous parameters cause $\hat{k}$ to shift either to the left or right within

the distribution of firms' capital that was given in Equation (2). When $\hat{k}$ shifts in either direction, some firms will experience changes in the credit treatment they are receiving as shown in Figure 3.

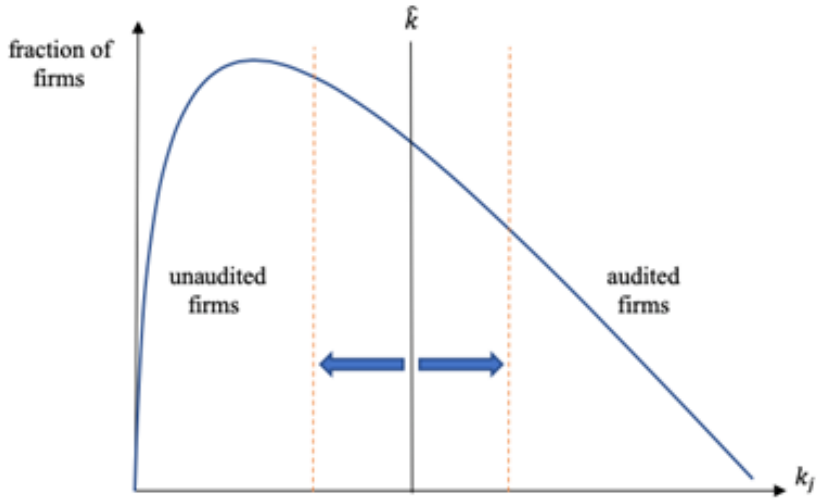

Figure 3: Shifts in the auditing threshold, $\hat{k}$, due to changes in the economy's exogenous parameters.

4.5.1 Optimal Borrowing Demands

When comparing x_A^* and x_{NA}^*, I find that the entrepreneur's borrowing demand is lower in the case where she is unaudited. This is due to the uncertainty she faces, where there is a possibility that part or all of her stock of undepreciated capital gets seized by the lender, due to a bad realisation of her idiosyncratic shock. In addition, the unaudited entrepreneur's demand,x_{NA}^* , is positively related to the prior probability of the firm being successful, P. This is intuitive since the possibility of the unaudited entrepreneur being under a situation where all or part of her capital stock gets confiscated is less likely, she would demand more of x. On the contrary, x_{NA}^* depends negatively on γ. The higher is γ, the lower is the unaudited entrepreneur's share from the expected production surplus, and thus the lower is her incentive to borrow. This is due to the unaudited entrepreneur facing the uncertainty where the possibility of her undepreciated capital being seized exists. This is not the case with the audited entrepreneur, and hence why x_A^* does not depend on γ.

4.5.2 $\widehat{k}$ with Non-Binding Borrowing Constraint

As can be seen in Equations (18) and (20), $\hat{k}$ is a function of exogenous economic parameters. Thus, as economic conditions change, so does $\hat{k}$. The direction in which $\hat{k}$ shifts will determine whether some firms will start getting better credit treatment or worse. The

sign of some of the evaluated derivatives cannot be determined more generally without assigning values to the economic parameters. Therefore, for those derivatives I used the same values that are used later to generate the numerical results.

When considering the effect of changes in the fixed audit cost, I find that $\left(\partial\hat{k}/\partial A\right) > 0$. This will also apply in the binding borrowing constraint case. As the audit cost increases, the difference in the lender's revenue between the case of audit and no audit that is required to offset the fixed audit cost, increases. Since this difference in revenues between the two cases depends positively on the entrepreneur's capital stock, the auditing threshold shifts to the right as shown in Figure 3. Accordingly, some of the firms which used to be audited will fall below the audit threshold, receiving a less favourable credit treatment than what they used to enjoy before the increase in A.

When s increases, both x_A^* and x_{NA}^* also increase. However, the increase in the audited entrepreneur demand is bigger, since $(\partial x_A^*/\partial z) > (\partial x_{NA}^*/\partial z)$. This implies that an increase in z will cause the gap between the lender's revenue between the case of audit and no audit to increase. Consequently, the difference in revenue between the two cases, that is required to offset the fixed audit cost will occur at a lower level of k than before, and hence $\hat{k}$ will shift to the left. The difference in the lender's revenue between the two cases will increase even further when the unaudited entrepreneur's borrowing constraint is binding.

An increase in the entrepreneur's prior probability of success P, means that the possibility the lender's audit will reveal a favourable realisation of the idiosyncratic shock, increases. This in turn will increase the difference between the lender's expected revenue under audit, versus no audit, for a given level of the entrepreneur's capital, k. In other words, as P increases the difference in the expected revenue between the two cases, that is required to offset the fixed audit cost will occur at a lower level of k than before, and hence $\hat{k}$ will shift to the left.

Increases in the lender's bargaining power, γ, will increase his share from the production surplus. This occurs both under the case of audit and no audit. However, his revenue from lending under audit, in absolute, will increase further than the case of no audit. To clarify, it was shown earlier that x_{NA}^* depends negatively on γ, while x_A^* is independent of it. Therefore, as the lender's bargaining power increases, a drop in x_{NA}^* will partly offset the increase in the share paid to the lender due to γ increasing. This in turn implies that the difference in lender's revenue between the two cases, that is required to offset the fixed audit cost will occur at a lower level of k than before, and hence $\hat{k}$ will shift to the left.

4.5.3 $\widehat{k}$ with Binding Borrowing Constraint

In this case, the effects of the parameters A, z, P and γ are the same on $\hat{k}$ as in the case where the entrepreneur's borrowing constraint is non-binding. The parameters δ and

q do not affect $\hat{k}$ when the entrepreneur's borrowing constraint is non-binding, as her borrowing is not constrained by the maximum amount $x = (1/q)(1-\delta)k$. However, they do affect $\hat{k}$ in this case, as shown in equation (20).

An increase in the depreciation rate, δ, decreases the maximum amount that the unaudited entrepreneur can borrow. This in turn increases the difference in the lender's expected revenue between the case of audit and no audit, given the same level of k. This in turn implies that the difference in revenue between the two cases, that is required to offset the fixed audit cost will occur at a lower level of k than before, and hence $\hat{k}$ will shift to the left. This reasoning also applies when considering the effect of an increase in q on $\hat{k}$. Table 1 below, summarises the effect of changes in the model's parameters on $\hat{k}$.

Derivative	**Sign**
$\partial\hat{k}/\partial A$	+
$\partial\hat{k}/\partial z$	-
$\partial\hat{k}/\partial P$	-
$\partial\hat{k}/\partial\gamma$	-
$\partial\hat{k}/\partial\delta$	-
$\partial\hat{k}/\partial q$	-

Table 1: The effects of changes in the model's parameters on the auditing threshold, $\hat{k}$.

4.6 Conditions on the Model's Parameters

In order for the household and the entrepreneur to engage in lending and borrowing instead of choosing to stay in autarky, the production technology $f(k,x)$ must be sufficiently productive compared to the technologies $g(E)$ and $h(k)$. In what follows, I discuss those required conditions for the household and the entrepreneur. When generating the numerical results, I calibrate the model to ensure that those conditions are satisfied.

Since q can either be greater or less than $\bar{q}$, there will be a set of conditions required for each of the two cases. When $q \leq \bar{q}$, the unaudited entrepreneur prefers borrowing and producing using the technology $f(k,x)$, when her expected utility without being audited is greater than her utility under autarky. This will occur iff:

$$P\left(zk^{\alpha}x^{1-\alpha} - \left(zk^{\alpha}x^{1-\alpha} - x - \frac{k}{q}\right) - x + \frac{(1-\delta)k}{q}\right) + (1-P)\left(-x + \frac{(1-\delta)k}{q}\right) > \frac{k}{q}$$

I substitute for x using equation (9). Thus, the condition on the parameters is:

$$(1-\gamma)Pz\left(\frac{(1-\alpha)Pz(1-\gamma)}{1-P\gamma}\right)^{\frac{1-\alpha}{\alpha}}-(1-P\gamma)\left(\frac{(1-\alpha)Pz(1-\gamma)}{1-P\gamma}\right)^{\frac{1}{\alpha}}+\frac{P\gamma-\delta}{q}>0 \quad (21)$$

When $q > \bar{q}$, I use $(1/q)(1-\delta)k$ to substitute for x and the condition becomes:

$$(1-\gamma)Pz\left(\frac{1-\delta}{q}\right)^{1-\alpha}+\frac{2P\gamma-P\gamma\delta-1}{q}>0 \quad (22)$$

When $q \leq \bar{q}$, the household will prefer lending over staying in autarky when his expected utility as given in equation (15) is strictly greater than X. I simplify, and thus the condition on the parameters is:

$$(z)^{\frac{1}{\alpha}}\left(\left(\frac{(1-\alpha)Pz(1-\gamma)}{1-P\gamma}\right)^{\frac{1-\alpha}{\alpha}}-\left(\frac{(1-\alpha)Pz(1-\gamma)}{1-P\gamma}\right)^{\frac{1}{\alpha}}\right)-\frac{1}{q}>0 \quad (23)$$

On the other hand, when $q > \bar{q}$ then the household's utility as given in equation (16) must be strictly greater than X. Accordingly, the condition on the parameters is given by:

$$z\left(\frac{1-\delta}{q}\right)^{1-\alpha}-\frac{2-\delta}{q}>0 \quad (24)$$

5 Adding Investment to the Model

The model in its current setup, can address the first question of this research. However, it cannot provide an answer to why medium sized firms have exhibited the largest percentage decrease in investment in the 2008 recession in the UK, followed by small firms and then large firms as presented in Crawford et al. (2013). To address this question I amend the model, however, in a way that keeps it static. I have attempted to answer this question in an infinite horizon setting; however, it became clear that the question I want to address cannot be answered.[23]

I now amend the model, such that each entrepreneur j maximizes $u(c) + v(i)$, where $u(c) = \ln c$ and $v(i) = \ln i$. The entrepreneur gets utility from consuming $u(c)$, and I interpret $v(i)$ as the utility she gets from bequeathing to a next generation. In other words, $v(i)$ is a valuation of a bequest. Instead of bequeathing capital to the next generation,

[23]This is not possible in an infinite horizon version, specially that I am assuming that firms only differ in the initial level of capital they start with. Dynamics in an infinite horizon model are studied by shocking the model in a steady state. This will not allow studying the behaviour of middle-sized firms. All firms will converge to a single steady state level of capital, regardless of the initial condition they start from. In other words, as firms converge to their steady state, a single representative firm will arise that will either be above or below $\hat{k}$ in the steady state. The capital distribution at the start of the economy as given by Equation (2), will collapse to a spike in the steady state. Therefore, the comparisons between firms' responses (small, medium and large) which is the question I am seeking to answer, would not be possible.

the entrepreneur bequests the consumption good. The next generation of entrepreneurs converts i to capital and then utilise it in production. Accordingly, I further interpret i as investment.[24] I follow Galor and Zeira (1993) in the way I amended the model, to allow for bequests to a next generation.

5.1 Nash Bargaining

Similar to before, the lender and borrower negotiate and agree the credit contract. Credit contracts are assumed to be written for one period only. The division of the match surplus is agreed through a Nash Bargaining solution. The solution will be as given in Equations (4) and (5) previously.

5.2 The Entrepreneur's Problem

Now the entrepreneur maximises $u(c) + v(i)$. She will maximise her profits, which are given in terms of the consumption good. Given the structure of her utility, she will divide those profits equally between consumption and investment.[25]

5.2.1 The Audit Case

The entrepreneur maximises her utility by choosing c and i. Since her resources are divided equally between consumption and bequeathing, she maximises her share from the production surplus. This is done by choosing her optimal demand for x:

[24] Unlike Fisher (2006) where the investment specific shocks are persistent, I assume that those shocks take the form of white noise. An alternative set up where capital is bequeathed, clouds my analysis. When capital is bequeathed, I cannot isolate the adverse effect a positive shock to q would have on firms' investment. A positive shock to q, means that the amount in terms of the final consumption good that entrepreneurs can dedicate for investment declines. This is because of a decrease in the entrepreneur's total resources in terms of the final consumption good. However, bequeathing capital means that the capital which will be passed to a future generation, is purchased in the same period of the shock occurrence. Therefore, since capital became cheaper in the period of the shock, the amount of capital bequeathed can be more than what the entrepreneur would have in the absence of the shock. The positive effect on the bequeathed capital coming from the positive shock to q can partly, fully or more than fully offset the adverse effect on capital accumulation coming from the reduction in firms' resources. I want to evaluate the change in investment (the amount spent on capital accumulation) in terms of the final consumption good, and not in terms of the capital good itself.

[25] The entrepreneur's utility function is a monotonic transformation of a Cobb-Douglas utility function with equal weights for both of its arguments.

$$\max_{c,i,x} U = \ln c + \ln i \tag{25}$$

subject to

$$c + i \leq P\left(zk^{\alpha}x^{1-\alpha} - \gamma\left(zk^{\alpha}x^{1-\alpha} - x - \frac{k}{q}\right) - x + \frac{(1-\delta)k}{q}\right) + (1-P)\frac{k}{q} \tag{26}$$

I substitute for c in Equation (25), using the budget constraint (1.26). I then take the first-order conditions with respect to x and i:[26]

$$\frac{\mathrm{d}U}{\mathrm{d}x} = 0 \Rightarrow x_A^* = ((1-\alpha)z)^{\frac{1}{\alpha}}\, k \tag{27}$$

$$\frac{\mathrm{d}U}{\mathrm{d}i} = 0 \Rightarrow i = c \tag{28}$$

Using the budget constraint (26) to substitute for c in the optimality condition obtained in (28):

$$i = \frac{1}{2}\left(P\left(zk^{\alpha}x^{1-\alpha} - \gamma\left(zk^{\alpha}x^{1-\alpha} - x - \frac{k}{q}\right) - x + \frac{(1-\delta)k}{q}\right) + (1-P)\frac{k}{q}\right) \tag{29}$$

Finally, using (27) to substitute for x in (29), gives the optimal investment decision under audit, i_A^*:

$$i_A^* = \frac{1}{2}\left((1-\gamma)P(z)^{\frac{1}{\alpha}}\left((1-\alpha)^{\frac{1-\alpha}{\alpha}} - (1-\alpha)^{\frac{1}{\alpha}}\right) + \frac{1 + P\gamma - P\delta}{q}\right)k \tag{30}$$

5.2.2 The No Audit Case

The unaudited entrepreneur does not know the realisation of her idiosyncratic shock. Nevertheless, the lender still lends but uses the entrepreneur's undepreciated capital as a collateral to ensure the repayment of x, in case the realisation of the idiosyncratic shock is equal to 0. The entrepreneur takes this uncertainty into account in her optimisation problem:

[26] The function is concave; thus, the second order condition is satisfied.

$$\max_{c,i,x} U = \ln c + \ln i \tag{31}$$

subject to

$$c + i \leq P\left(zk^{\alpha}x^{1-\alpha} - \gamma\left(zk^{\alpha}x^{1-\alpha} - x - \frac{k}{q}\right) - x + \frac{(1-\delta)k}{q}\right) + (1-P)\left(-x + \frac{(1-\delta)k}{q}\right) \tag{32}$$

I substitute for c in Equation (31), using the budget constraint (32). I then take the first-order conditions with respect to x and i:

$$\frac{\mathrm{d}U}{\mathrm{d}x} = 0 \Rightarrow x_A^* = \left(\frac{(1-\alpha)P(1-\gamma)}{1-P\gamma}\right)^{\frac{1}{\alpha}} k \tag{33}$$

$$\frac{\mathrm{d}U}{\mathrm{d}i} = 0 \Rightarrow i = c \tag{34}$$

Using the budget constraint (32) to substitute for in the optimality condition obtained in (34):

$$i = \frac{1}{2}\left(P\left(zk^{\alpha}x^{1-\alpha} - \gamma\left(zk^{\alpha}x^{1-\alpha} - x - \frac{k}{q}\right) - x + \frac{(1-\delta)k}{q}\right)\right) + (1-P)\left(-x + \frac{(1-\delta)k}{q}\right) \tag{35}$$

Finally, using (33) to substitute for x in (35), gives the optimal investment decision under no audit and non-binding borrowing constraint, i_{NA1}^*:

$$i_{NA1}^* = \frac{1}{2}\left((1-\gamma)Pz\left(\frac{(1-\alpha)Pz(1-\gamma)}{1-P\gamma}\right)^{\frac{1-\alpha}{\alpha}} - (1-P\gamma)\left(\frac{(1-\alpha)Pz(1-\gamma)}{1-P\gamma}\right)^{\frac{1}{\alpha}} + \frac{1+P\gamma-\delta}{q}\right) \tag{36}$$

When $q > \bar{q}$, the entrepreneur's borrowing constraint in binding, and I use the maximum borrowing amount $x = (1/q)(1-\delta)k$ to substitute for x in equation (35):

$$i_{NA2}^* = \frac{1}{2}\left((1-\gamma)Pz\left(\frac{1-\delta}{q}\right)^{1-\alpha} + \frac{P\gamma(2-\delta)}{q}\right)k \tag{37}$$

which the optimal investment decision under no audit and binding borrowing con-

straint.

5.3 The Household's Problem

The household's preference remained unchanged. Additionally, the entrepreneur's optimal demand for x is as given before in Equations (7) and (9). Accordingly, the solution to the household's problem is similar to what has been presented in Section 4.3, and the audit threshold $\hat{k}$ is the same as given in Section 4.4.

5.4 Conditions on the Model's Parameters

In order for the household and the entrepreneur to engage in lending and borrowing instead of choosing to stay in autarky, the production technology $f(k, x)$ must be sufficiently productive compared to the technologies $g(E)$ and $h(k)$. The household's and the entrepreneur's resource constraints remained the same, and therefore the conditions given in Equations (21) to (24) still apply.

6 Results and Discussion

As explained earlier, the lenders' optimisation will generate an endogenous audit threshold for the level of capital, $\hat{k}$. It implies discriminating credit treatment for the firms that fall below the threshold level, compared to the ones that fall above it. Since $\hat{k}$ is a function of exogenous economic parameters, when economic conditions change, so does $\hat{k}$. The direction in which $\hat{k}$ shifts will determine whether some firms will start getting better credit treatment or worse.

In the results presented in this section, the distribution of the entrepreneurs' net worth, which is their endowments of capital is divided into three intervals that are equal in length. The three intervals represent the three groups of firms; small, medium and large. Moreover, I let $\hat{k}$ be at the centre of the capital distribution before the occurrence of any shock. Accordingly, with $\hat{k}$ being at the centre of the distribution, half of the medium firms will fall on the left and the other half on the right of the auditing threshold $\hat{k}$. In other words, in the absence of shocks, half of the medium firms are treated as large by lenders and thus being audited, while the other half are treated as small firms and forced to provide collaterals to obtain credit. This proposition is supported by a market research. The UK Competition and Market Authority (CMA (2015)) has carried out an investigation into SMEs' banking relationships, where one-to-one interviews were conducted. A significant number of mid-sized firms within the sample reported being able to renegotiate banking fees, especially when a dedicated relationship manager was

assigned to them. Despite firms across the sample attempting to negotiate banking terms, it was medium sized businesses in general and high turnover mid-sized firms in specific, that were able to develop a more peer-to-peer relationships.

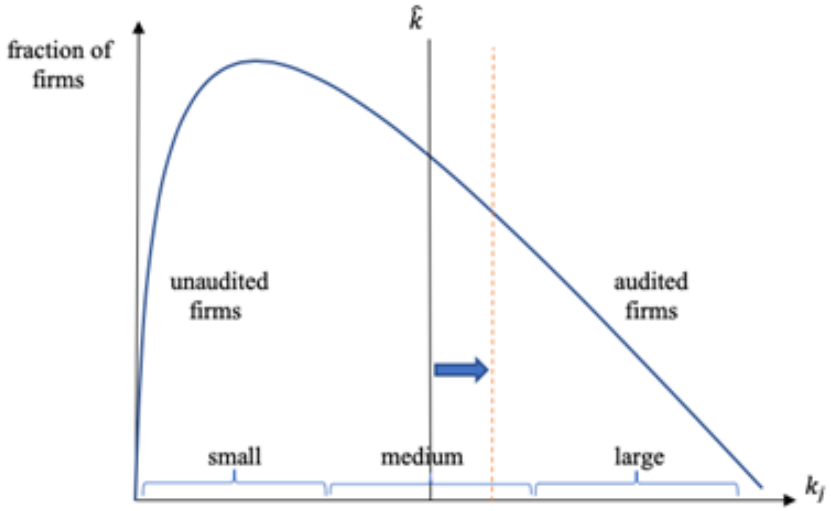

Figure 4: Adverse economic shocks shifting $\hat{k}$ to the right, imply worse credit treatment to some previously audited firms.

I hypothesize that during adverse economic conditions which cause a reduction in credit extended to all firms, this threshold level of firm size shifts to the right as shown in Figure 4. Consequently, the firms which get affected the most (the ones that suffer the largest percentage reduction in their ability to obtain credit) following a shock, are the firms which were earlier on the right of this threshold but are now on the left of it. In other words, it is the firms that were earlier treated as large firms; being audited and borrowing without the need for a collateral, but now are treated as small firms; not being audited and required to place a collateral to borrow. Therefore, although all firms face a reduced credit in a recession, some of the medium sized firms suffer a double hit. They face the general credit reduction like all firms, but they also suffer from a change in credit treatment due to the shifting audit threshold level of the banks.

Accordingly, the idea is to make a comparison in the average responses for entrepreneurs' investment between: *(i)* audited entrepreneurs, who remain audited (i.e. large firms); *(ii)* unaudited entrepreneurs, who remain unaudited (i.e. small firms); and *(iii)* medium-sized firms, which some of them were audited but become unaudited following the shocks. Since the model is static, the results are generated by comparing the model outcomes with shocks, relative to the outcomes in the absence of shocks.

6.1 The UK in the Great Recession

The 2008 recession in both the UK and US started with a large drop in asset prices, that later spread to a wide recession affecting the whole of their economies. Rather than attempting to quantitatively match the reported results in Figure 1.1, I try to qualitatively explain those variations in firms' investment responses. The 2008 large drop in asset prices has led to a decline in the value of capital. This in turn has resulted in tougher credit conditions to small firms in particular, as they rely more on property collaterals to obtain loans (Chadha et al. (2017); Haldane (2017)). Moreover, both Bahaj et al. (2016) and Pinter (2019) have found empirical evidence that the decline in house prices has caused a reduction in SMEs' loans and investment in the UK. Hence, I proxy for it by a positive shock to q.

Additionally, I proxy for the economy-wide recession that followed by a negative shock to z. Given the theoretical framework presented, each of the two shocks will generate different responses between small, medium and large firms. Each shock causes those different responses based on a certain channel. In terms of the empirical regularities given in the UK, the model predicts that two channels must have been acting at the same time. However, just having the two channels working at the same time (two shocks), will not be sufficient in explaining how the empirical regularity came into place. What also matters is the relative size of the shocks to each other.

Instead of considering the combined effect of the two shocks only, I first examine each shock in isolation. This is to allow understanding the underlying channel that is associated with each shock, z and q. Moreover, in the numerical exercises presented, I consider a situation in which the economy moves from non-binding borrowing constraints to binding borrowing constraints. This is motivated by the credit crunch that characterised the 2008 recession. Net bank lending to the non-financial private sector, stood at 12 per cent of the UK's GDP in 2007. However, it dropped to less than 1 per cent in 2009 (Martin and Rowthorn (2012)).

6.1.1 Neutral Productivity Shock, z

I examine the response of firms' investment to a negative neutral productivity shock, while holding all other parameters unchanged. More precisely, I consider a 2 per cent reduction in the value of 2. Table 2 provides the economy's parameters values, both in the absence of a shock and when the shock occurs. It also provides the values of $\bar{q}$ and $\hat{k}$ that are implied by the given parameters' values.

As can be seen in Figure 5, when there is only a negative shock to , ceteris paribus, an audited firm's investment responds by more compared to the investment of an unaudited firm. This is formally given by Equations (38) and (39). The derivatives in those two

	P	α	γ	δ	A	z	q	$\bar{q}$	$\hat{k}$
Pre-shock	0.8	0.3	0.5	0.1	30	2	1.124	1.13271	184.041
Post-shock	0.8	0.3	0.5	0.1	30	1.96	1.124	1.21162	196.861

Table 2: The economy's parameters values; in the absence of a shock and when a 2 per cent negative shock to occurs.

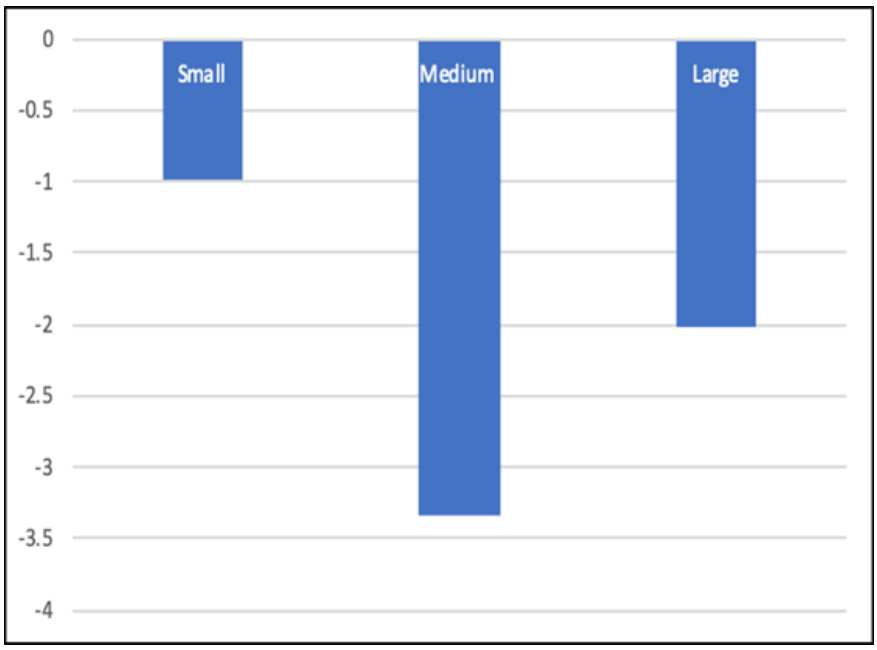

Figure 5: The percentage impact on firms' investment due to a 2 per cent negative shock to z.

equations are positive, which imply that in the event of a negative shock to z, audited firms' investment will drop by a higher percentage relative to unaudited firms' investment. In other words, the inequality signs will change direction, with the left-hand side being more negative than the right-hand side.

$$\frac{\partial i_A^*/\partial z}{i_A^*} > \frac{\partial i_{NA1}^*/\partial z}{i_{NA1}^*} \tag{38}$$

$$\frac{\partial i_A^*/\partial z}{i_A^*} > \frac{\partial i_{NA2}^*/\partial z}{i_{NA2}^*} \tag{39}$$

To understand the reason behind this result, I first consider the percentage effect of a change in z on the optimal borrowing demands, x_A^* and x_{NA1}^*, and find them to be equal:

$$\frac{\partial x_A^*/\partial z}{x_A^*} = \frac{\partial x_{NA1}^*/\partial z}{x_{NA1}^*} \tag{40}$$

I let y be the firm's output (i.e. $y = zk^{\alpha}x^{1-\alpha}$), and hence Equation (40) will also imply the following:

$$\frac{\partial y_A^*/\partial z}{y_A^*} = \frac{\partial y_{NA1}^*/\partial z}{y_{NA1}^*} \tag{41}$$

This would lead us to think that the percentage investment responses should be the same between audited and unaudited entrepreneurs, in contrast to Equation (38). However, this conclusion does not take into account the uncertainty that the unaudited entrepreneur faces when deciding her optimal demand for x. With probability $1 - P$, the unaudited entrepreneur production will be unsuccessful, and she will be forced to pay back the amount she borrowed using her capital resources which have already depreciated at a rate δ.

Considering the impact that a shock to z would have on an entrepreneur's expected resources would lead to an accurate conclusion. I let R be the total expected resources of the entrepreneur as given by the right-hand side of the budget constraints in Equations (26) and (32). I substitute for x in R using x_A^* and x_{NA1}^*, thus obtaining both the audited and unaudited entrepreneur's total expected resources in equilibrium, R_A^* and R_{NA1}^* respectively.

$$\frac{\partial R_A^*/\partial z}{R_A^*} > \frac{\partial R_{NA1}^*/\partial z}{R_{NA1}^*} \tag{42}$$

When considering the case between an audited entrepreneur and an unaudited entrepreneur with a binding borrowing constraint, then the channel of transmission is clearer, since:

$$\frac{\partial x_{NA2}^*/\partial z}{x_{NA2}^*} = 0 \tag{43}$$

The impact of the shock on the audited entrepreneur will come from both a direct and an indirect channel; a reduction in debt (indirect) and a reduction in output (direct). For the unaudited entrepreneur with a binding borrowing constraint, the impact of a shock to z will only come through the direct channel, which is a reduction in output.

The above discussion explains the differences in investment response between small and large firms. However, to explain the largest decline in investment that is exhibited by medium firms, I need to look at the effect of a shock to z on the capital threshold for auditing. Medium firms fall on both sides of $\hat{k}$, and a negative shock to z will cause $\hat{k}$ to shift to the right. Therefore, some of the medium firms which were audited before the shock, will now fall under the threshold level $\hat{k}$. This will result in those firms facing a discriminating credit treatment relative to before. It follows that for those firms, a negative shock to z will cause a double hit. The first is in the form of a contraction in debt that both audited and unaudited firms suffer alike, as explained before. Additionally, the change in the credit conditions that those firms face will present a further contraction. Since

$x_A^* > x_{NA1}^*$, it follows that those medium firms (the ones which post-shock fell below $\hat{k}$) will have a much larger drop in their investment. Consequently, the average investment of all medium firms declines by a higher percentage relative to small and large firms.

6.1.2 Investment Specific Technology Shock, q

I now examine the response of firms' investment to a positive IST shock, while holding all other parameters unchanged. More precisely, I consider a 10 per cent increase in the value of q. Table 3 provides the economy's parameters values, both in the absence of a shock and when the shock occurs.

	P	α	γ	δ	A	z	q	$\bar{q}$	$\hat{k}$
Pre-shock	0.8	0.3	0.5	0.1	30	2	1.124	1.13271	184.041
Post-shock	0.8	0.3	0.5	0.1	30	2	1.2364	1.13271	169.63

Table 3: The economy's parameters values; in the absence of a shock and when a 10 per cent positive shock to q occurs.

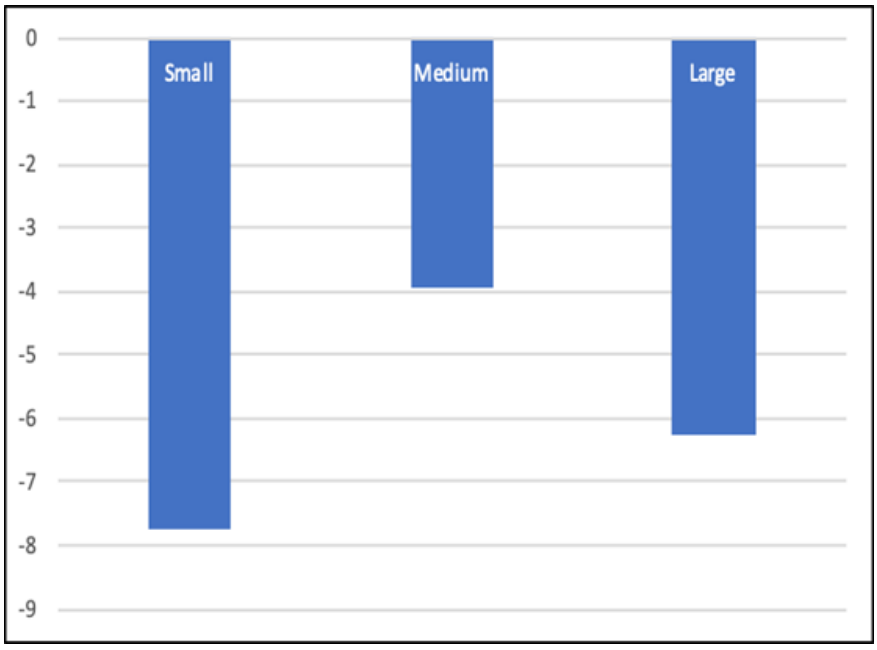

Figure 6: The percentage impact on firms' investment due to a 10 per cent positive shock to q.

As can be seen in Figure 6, when there is only a positive shock to q, *ceteris paribus*, an unaudited firm's investment responds by more than the investment of an audited firm. This is formally given by Equations (44) and (45). The derivatives in those two equations are negative, which imply that in the event of a positive shock to q, audited firms' investment will drop by a smaller percentage relative to unaudited firms' investment.

$$\frac{\partial i_A^*/\partial q}{i_A^*} > \frac{\partial i_{NA1}^*/\partial q}{i_{NA1}^*} \tag{44}$$

$$\frac{\partial i_A^*/\partial q}{i_A^*} > \frac{\partial i_{NA2}^*/\partial q}{i_{NA2}^*} \tag{45}$$

A positive shock to q, means a decrease in the value of an entrepreneur's net worth in terms of the final consumption good. Although this decline is equally shared between audited and unaudited entrepreneurs, unaudited entrepreneurs face the uncertainty where with probability $1-P$ their production will be unsuccessful. In such event, they are forced to pay back the amount they borrowed using their capital resources which have already depreciated at a rate δ. Therefore, an increase in q would have a relatively greater negative impact on the unaudited entrepreneurs expected total resources in equilibrium. This in turn will translate to a greater expected percentage reduction in the resources allocated for investment. This is formally given in Equation (46), where the derivatives are negative.

$$\frac{\partial R_A^*/\partial q}{R_A^*} > \frac{\partial R_{NA1}^*/\partial q}{R_{NA1}^*} \tag{46}$$

The effect of a positive shock to on investment will be even stronger in the case where the unaudited entrepreneur borrowing constraint is binding. This is because an increase in will also lower the maximum amount that the unaudited entrepreneur can borrow, $x = (1/q)(1-\delta)k$. This in turn will result in larger percentage decline her expected resources.

The above discussion explains the differences in investment response between small and large firms. However, as can be seen in Figure 6, medium firms exhibit the smallest percentage drop in investment. To explain this, I need to look at the effect of a shock to q on the capital threshold for auditing, since medium firms fall on both sides of $\hat{k}$. A positive shock to q will cause $\hat{k}$ to shift to the left. Consequently, some medium firms which were unaudited before the shock, will now fall above the threshold level $\hat{k}$, resulting in them facing a better credit treatment relative to before. It follows that for those firms, a positive shock to q will form an improvement. The positive effect of the improvement in credit treatment on those firms is larger than the negative effect of an increase in q on their expected resources, R. Accordingly, those firms (the ones which post-shock fell above $\hat{k}$) will experience an increase in investment following the shock. This increase for those firms will in turn make the average investment of all medium firms decline by a lower percentage relative to small and large firms.

6.1.3 Shocks to both z and q

As described earlier, each shock, z and q, will give different responses between small, medium and large firms through two different channels of transmission. In terms of the empirical regularity given in the UK, the theoretical framework predicts that it must have been the case that there were two channels acting at the same time. However, having the two channels working at the same time (two shocks), will not be sufficient on its own in explaining how the empirical regularity came into place. What also matters is the relative size of the shocks to each other. In light of the of the earlier discussion, the model predicts that the effect of the neutral productivity shock z on $\hat{k}$, should have been stronger than the effect of the investment specific technology shock q. In this case, medium firms will respond by more compared to other firms on average. In the same time, when comparing small and large firms' investment responses, the effect of the channel of q must have been stronger than the channel of z. This is for small firms' investment to respond by more compared to large firms' investment. I now consider a 10 per cent increase in the value of q combined with a 2 per cent reduction in the value of z, as shown in Table 4:

	P	α	γ	δ	A	z	q	$\bar{q}$	$\hat{k}$
Pre-shock	0.8	0.3	0.5	0.1	30	2	1.124	1.13271	184.041
Post-shock	0.8	0.3	0.5	0.1	30	1.96	1.2364	1.21162	193.055

Table 4: The economy's parameters values; in the absence of a shock and when a 10 per cent positive shock to combined with a 2 per cent negative shock to occur.

In the results shown in the Figure 1.7, the economy has moved from $z/q = 1.77936$ to $z/q = 1.58525$ following the shocks to z and q. The choice of relative size of the two shocks to each other, makes the two underlying channels interact to generate larger percentage decline in investment for small firms, relative to large firms. Furthermore, the relative size of the shocks is also crucial for determining the effect on the average investment of medium firms. A negative shock to z would cause $\hat{k}$ to shift to the right, while a positive shock to q would cause $\hat{k}$ to shift in the opposite direction. The model predicts that the effect of the shock to z must have dominated the effect of shock to q on $\hat{k}$, such that $\hat{k}$ has shifted to the right. This will result in a double hit to some medium firms, and therefore a greater percentage decline in medium firms' investment on average.

To further clarify the above argument and how the two different channels work, I now consider two other alternatives. I run the two alternatives of: *(i)* a 2 per cent negative shock to z combined with a 5 per cent positive shock to q, and *(ii)* a 0.5 per cent negative shock to z combined with a 10 per cent positive shock to q. Those two exercises confirm that the relativity of the two shocks to each other matters. The two shocks shift $\hat{k}$ in opposite directions, and the generated results will be determined by which of the two effects dominates the other.

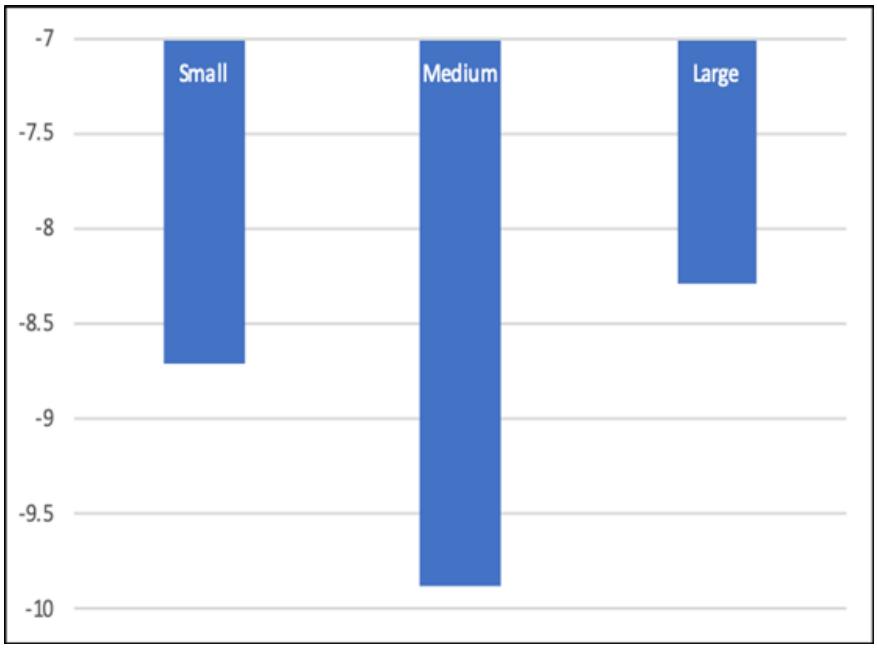

Figure 7: The percentage impact on firms' investment due to a 2 per cent negative shock to z, combined with a 10 per cent positive shock to q.

	P	α	γ	δ	A	z	q	$\bar{q}$	$\hat{k}$
Pre-shock	0.8	0.3	0.5	0.1	30	2	1.124	1.13271	184.041
Post-shock	0.8	0.3	0.5	0.1	30	1.96	1.1802	1.21162	196.861

Table 5: The economy's parameters values; in the absence of a shock and when a 5 per cent positive shock to q combined with a 2 per cent negative shock to z occur.

As shown in Table 5, following the two shocks the economy remains in a situation of non-binding borrowing constraints for unaudited entrepreneurs. The combined effect of the two shocks have caused $\hat{k}$ to shift to the right, resulting in a double hit to some medium firms as shown in Figure 8. Nevertheless, the negative shock to z was substantially large relative to the positive shock to q, such that large firms' investment dropped by a larger percentage compared to small firms. In other words, when comparing large firms and small firms' investment, the effect of the channel caused by a negative shock to z, has dominated the effect of the channel caused by a positive shock to q.

On the other hand, when I consider the other alternative of a 0.5 per cent negative shock to z combined with a 10 per cent positive shock to q, then the economy moves to a situation where the borrowing constraints of unaudited entrepreneurs become binding:

In this example, the combined impact of the shocks will make small firms investment decline by a greater percentage relative to large firms. This is consistent with the empirical facts. However, when considering the effect on $\hat{k}$, the negative shock to z was substantially small relative to the positive shock to q, such that the auditing threshold has shifted to the left. This has improved the credit treatment for some medium firms, which caused

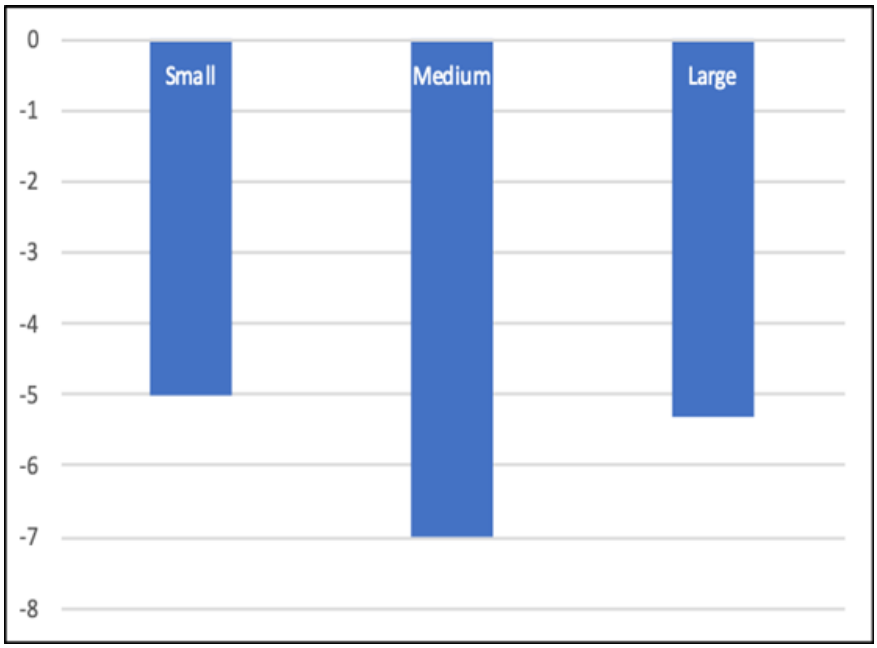

Figure 8: The percentage impact on firms' investment due to a 2 per cent negative shock to z, combined with a 5 per cent positive shock to q.

	P	α	γ	δ	A	z	q	$\bar{q}$	$\hat{k}$
Pre-shock	0.8	0.3	0.5	0.1	30	2	1.124	1.13271	184.041
Post-shock	0.8	0.3	0.5	0.1	30	1.99	1.2364	1.1518	175.091

Table 6: The economy's parameters values; in the absence of a shock and when a 5 per cent positive shock to q combined with a 2 per cent negative shock to z occur.

their investment to increase. Accordingly, the total effect on the average investment of all medium firms, was to decline by a smaller percentage relative to small and large firms.

6.2 The US in the Great Recession

Kudlyak and Sanchez (2017) have used US data covering the period between 1958Q4 and 2014Q2 to study the behaviour of small and large firms. Their results challenge the predictions of the financial frictions theoretical literature, where adverse shocks are propagated through the most financially vulnerable firms. They found that large firms turnover and short-term debt have contracted proportionately more than small firms during the recent recession.

When I examined their results further, I conclude that their results are due to the way in which they have defined small and larger firms. The data they used is aggregated into eight asset classes, with those classes remaining constant through the entire sample. However, they define small firms to be the ones below the twenty-fifth percentile of firms' turnover distribution in each period. Therefore, as the distribution of firms' sales

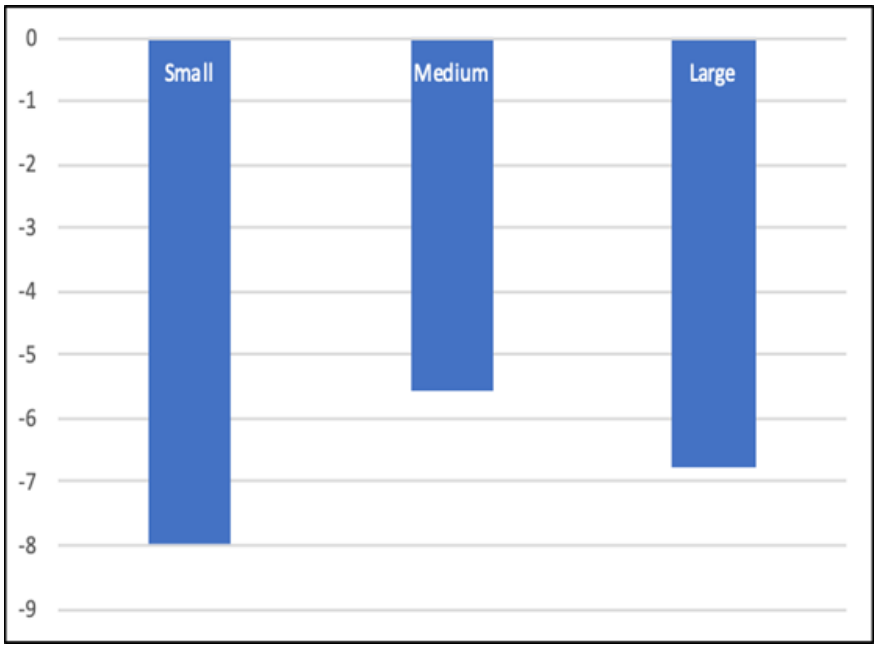

Figure 9: The percentage impact on firms' investment due to a 0.5 per cent negative shock to z, combined with a 10 per cent positive shock to q.

continues shifting to the right with time, the asset classes in which data is aggregated into remains constant. For example, in the early years of their sample, firms with total assets of $5 million and under were small firms, while in the end of their sample (precisely by the start of the 2008 recession and onward) firms with total assets of $1000 million and under were considered to be small firms. I find this very implausible; nevertheless, I show that my model can generate such results if I follow their definition of small firms. The implication of their definition of small firms on my model, is that medium firms will be grouped with large firms, where medium firms exhibit the largest average percentage contraction in debt and output, following adverse shocks.

I now repeat the same exercises that I conducted before; however, I perform two changes before obtaining the numerical results. The first is reducing the bargaining power of the lenders. The banking sector in the US is less concentrated than in the UK, hence the change. The second change is re-dividing the entrepreneurs' net worth distribution, such that now small firms are the ones at the bottom twenty-fifth percentile, while large firms are the ones occupying the top twenty-fifth percentile. I now consider a 10 per cent increase in the value of q combined with a 2 per cent reduction in the value of z, as shown in Table 7:

The economy starts from a situation where the unaudited entrepreneurs' borrowing constraints are binding and remain binding following the shocks. Figures 10 and 11, present the results for small versus large firms, where large firms exhibit larger percentage decreases in their debt and output.

	P	α	γ	δ	A	z	q	$\bar{q}$	$\hat{k}$
Pre-shock	0.8	0.3	0.5	0.1	30	2	1.124	0.867454	264.91
Post-shock	0.8	0.3	0.5	0.1	30	1.99	1.2364	0.927882	275.793

Table 7: The economy's parameters values; in the absence of a shock and when a 10 per cent positive shock to q combined with a 2 per cent negative shock to z occur.

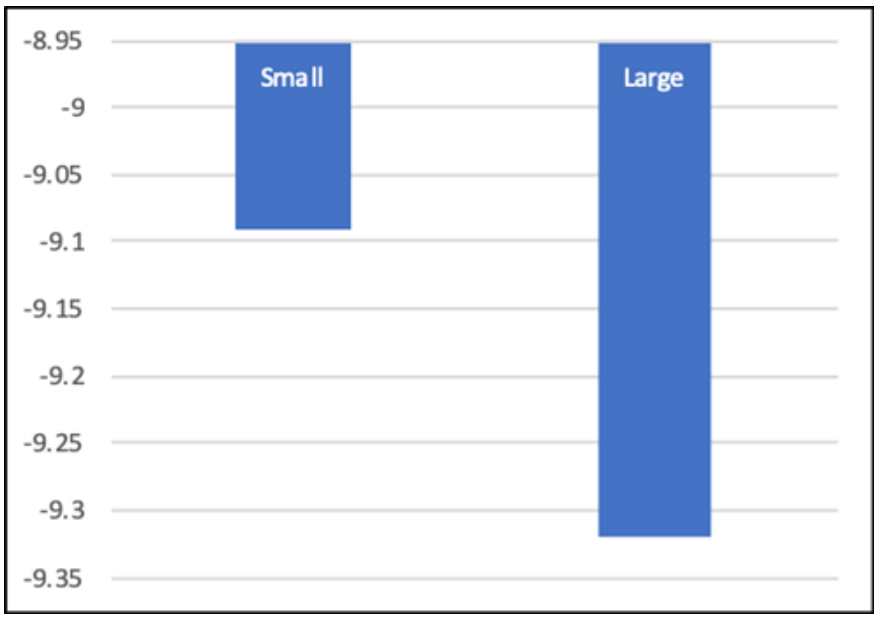

Figure 10: The percentage impact on firms' debt due to a 2 per cent negative shock to z, combined with a 10 per cent positive shock to q.

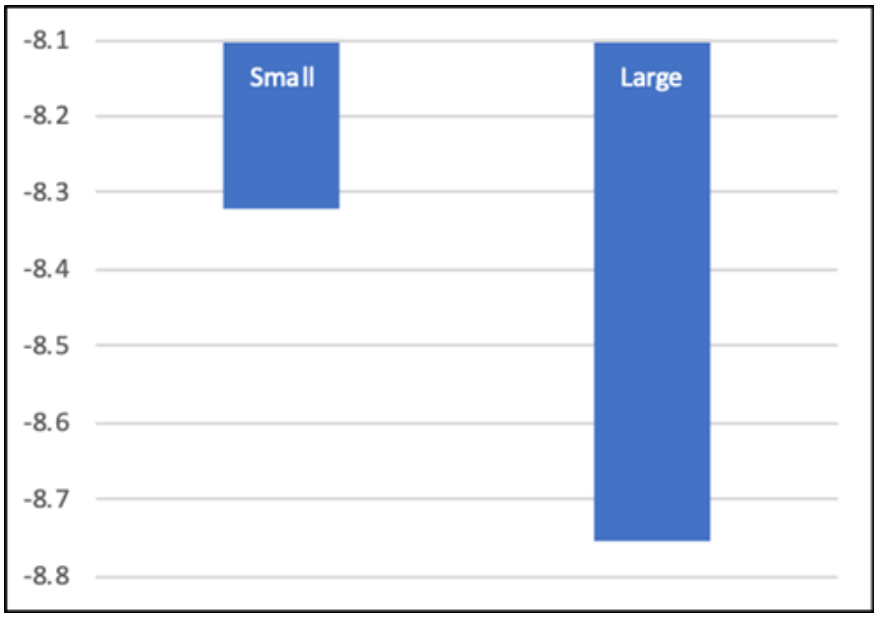

Figure 11: The percentage impact on firms' output due to a 2 per cent negative shock to z, combined with a 10 per cent positive shock to q.

However, in Figures 12 and 13 I breakdown the results to small, medium and large firms. As can be seen the larger decreases exhibited by large firms were due to medium firms being grouped with them. Following the previous definition of small firms would not change the qualitative results. However, it is worth noting that following the definition of Kudlyak and Sanchez (2017) would make the response of the "large" firms even more severe, since I am adding with them more small unaudited firms than I would have under the previous definition.

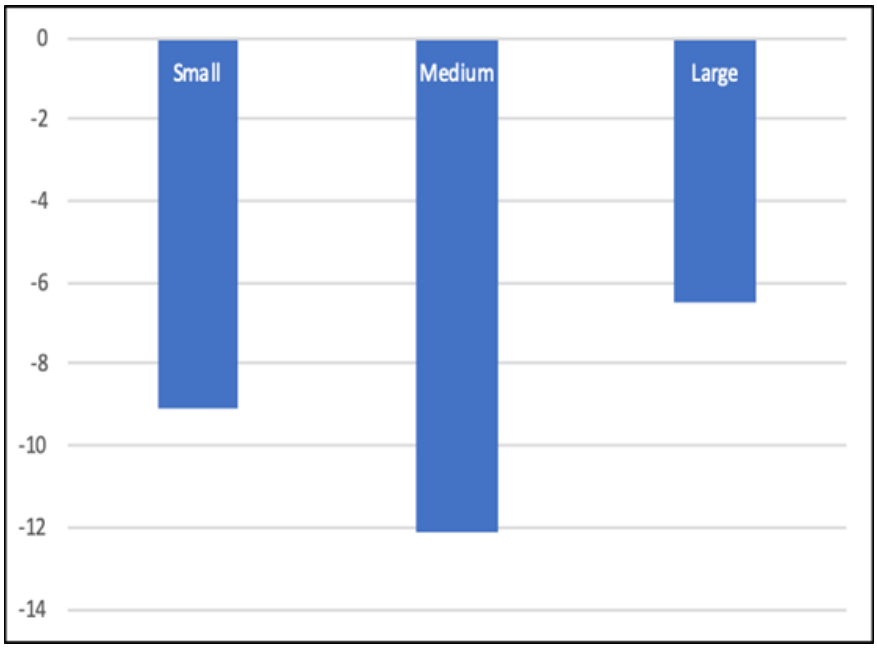

Figure 12: A breakdown of the impact on firms' output that was presented in Figure 10.

7 Conclusion

The model developed in this paper has addressed the two main research questions. The first is the reason behind the discriminating credit treatments offered by banks to firms based on their size. The mechanism that explains this credit discrimination, can also explain the differences in firms' investment responses to economic shocks based on their size, as was reported Crawford et al. (2013). In addition, contrary to the proposition of Kudlyak and Sanchez (2017) that their results challenge the predictions of the financial frictions theoretical literature, my model shows that such empirical results can be generated with a mechanism that propagates adverse shocks through the most financially vulnerable firms.

To the best of my knowledge, all the theoretical models generate discriminating credit treatments and contracts, by imposing exogenous heterogeneity between small and large firms. This exogenous heterogeneity can be in one or more dimensions where it is expressed as types, instead of the firms' initial condition which is their size. The premises on

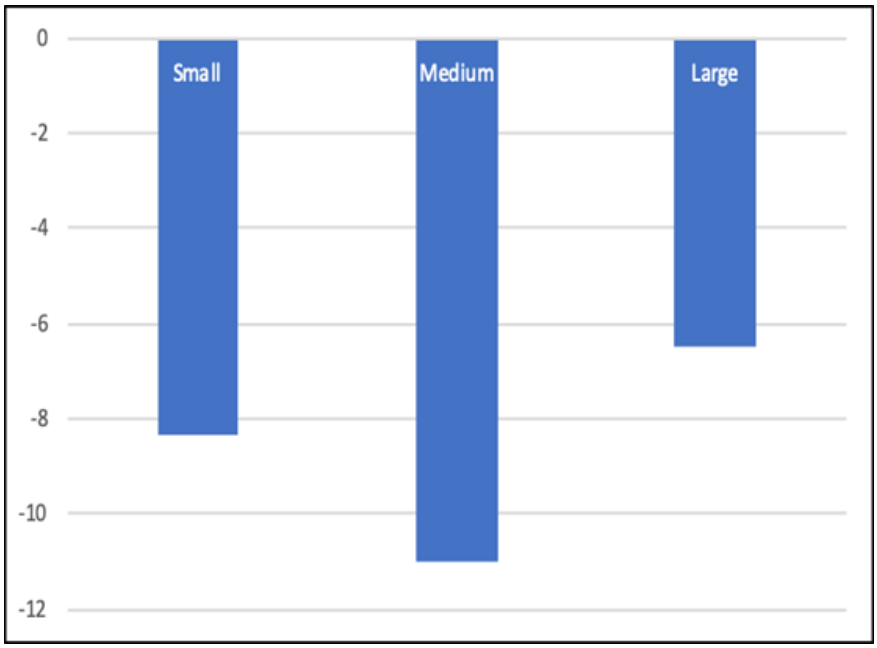

Figure 13: A breakdown of the impact on firms' output that was presented in Figure 11.

which those models are built are different from the premises that I start from. Therefore, those models will not generate the empirical regularity which I am trying to explain. Some models abstract from imposing different characteristics between firms that lead to credit discrimination, and simply assume the heterogenous credit treatment exogenously. For instance, in Bernanke et al. (1999) the credit treatments to firms are assumed to be different, and on the bases of this assumption they generate their results. On the other hand, my model implies that because firms are of different sizes, the differing credit treatments are arising endogenously due to the fixed audit cost that must be incurred by banks.

In the other financial frictions' models, ceteris paribus, the amount of credit that firms need depend on their net worth. In other words, the difference between what they own and what they need to have. When considering models that apply the net worth channel more specifically, the amount of credit that firms can get depends on the value of the firms' assets. Accordingly, when there is an adverse shock to the value of firms' assets, shocks will propagate differently for firms that differ in their net worth, and hence their access to credit. The equivalent to this result in my model is when there is a positive shock to q, where the numerical results generated are compared to the results of the other models. However, in my model, when there is a negative shock to z and in the absence of any shock to q, my results would differ. Large firms which enjoy a better access to credit, will have a greater response than small firms. As stated earlier, I am not taking many other heterogeneities into account. For instance, large firms enjoy higher success rates and greater bargaining power. Another criticism is that negative shocks to z, cause reduction in credit through a reduction in the demand for credit. A reduction in credit that is due to

a reduction in supply, only occurs when there is a positive investment specific technology shock and the unaudited entrepreneurs' borrowing constraints are binding.

Due to the importance of medium sized firms as discussed early in the paper, and since they are the most vulnerable to adverse shocks as it was established by my model; policies that aim at reducing the informational asymmetries between lenders and borrowers should be beneficial. The establishment of systematic approaches to the assessment of businesses by independent bodies, and which can be accepted by banks should reduce the financing difficulties faced by mid-sized firms. This would involve outsourcing part of the banks auditing processes to independent third parties, where firms can contribute towards their auditing costs. A similar suggestion to reduce informational asymmetries was proposed by Binks and Ennew (1997). Finally, medium sized firms can be a candidate for obtaining finance outside of the traditional banking system, if equity financing can be improved. Mason and Harrison (2015) have studied equity financing for British SMEs during the financial crisis. They found that the matching between firms and investors followed a less systematic approach and was mostly a coincidence-based process.

Future research, that builds on the theoretical framework presented in this paper, should look at the effect of including a full banking sector in the model. With this inclusion, financial intermediation and credit screening will be the role played by the banks, which receive deposits from the households and grant loans to the entrepreneurs. The inclusion of a banking sector would not in itself be sufficient to analyse financial fragility, however. Other essential elements to be added include: *(i)* the explicit introduction of a monetary sector in the model in order for liquidity to have a role; and *(ii)* agents' intertemporal decision-making when the model is extended to multiple periods (Peiris et al. (2018)).

One possible way for adopting the aforementioned changes is to follow the framework developed by Baliga and Polak (2004) in order to compare German and Anglo-Saxon financial systems, which differ in the degree that firms' credit is monitored. The Anglo-Saxon system is more geared towards traded bonds – pre-screening borrowers at the start, with little monitoring thereafter. The German system follows a more hands-on approach, with a very close relationship banking through non-traded bank loans. In their model either the lender or the entrepreneur makes the choice between monitored and non-monitored loans, depending on which agent being allocated the full bargaining power. The choice between the two types of loans is determined through a set of conditions that arise endogenously in the model. It is worth mentioning that their model follows a two periods setup. This would be especially beneficial since extending the model presented in this paper to an infinite horizon will cause the distribution of aggregate capital stock given in Equation (2) to collapse to a spike in the steady state – preventing the comparisons between firms' responses (small, medium and large), which is the question I am addressing in this paper.

Appendix A

A.1 General Equilibrium Model

I have attempted to address the research questions in a general equilibrium setup; however, an equilibrium does not always exist over the entire capital space. I start by a description of the model's environment, and then follow it by a discussion of the equilibrium.

I consider a two-period economy, $t = 1, 2$. There are two agents in this economy; a household and an entrepreneur. Both agents get utility from consumption, but consumption only occurs in $t = 2$. Each agent has access to a storage technology that gives one-to-one between periods. The household receives an endowment E of the consumption good in $t = 1$. The endowment is assumed to be sufficiently large, such that the resource constraint in lending to the entrepreneur will not bind. The entrepreneur owns a firm with a production technology $f(k, l) = zk^{\alpha}l^{1-\alpha}$. The output of this technology is the consumption good, and it is transferable one-to-one to the capital good. The entrepreneur starts in $t = 1$ with an endowment of capital k, and hires labour which is supplied by the household. Moreover, capital depreciates only when engaging in production. Precisely, a fraction $\delta \in (0, 1)$ of the entrepreneur's capital depreciates when she engages in production. The production starts in $t = 1$ with output being realised in $t = 2$. I assumed that labour costs are paid upfront in $t = 1$. To pay the upfront labour cost, the entrepreneur borrows from the household in . I let b be the amount that the entrepreneur borrows to pay the upfront labour cost.

The output realisation in $t = 2$ is subject to an idiosyncratic shock. With prior probability P, output will be strictly positive (the ex-post realisation of the idiosyncratic shock is 1), and with probability $1 - P$ output will be zero (the ex-post realisation of the idiosyncratic shock is 0). In $t = 1$, the entrepreneur is either audited or not, by the household which she is borrowing from. Audit is a choice made by the household and reveals the realisation of the idiosyncratic shock, that is otherwise unobservable without engaging in production. If the entrepreneur is audited and found to be good, then she borrows, hires labour and produces. If she is found to be bad, then she would not borrow or hire labour, and simply stores her capital endowment and consumes it in $t = 2$. On the other hand, if the entrepreneur is not audited, then she still borrows, hires labour and engages in production. The entrepreneur's capital is assumed to be perfectly observable by the household.

In $t = 1$, the household has a choice between two options. He can either store his whole endowment and consumes it in $t = 2$, or engage in lending and supplying labour. If the household chooses to lend, then he has a further two options. He can either use part of the endowment to pay the fixed audit cost, $A > 0$, and based on the audit outcome he lends or not, or he can choose to lend without auditing. The household stores the remainder

of his endowment for consumption in $t = 2$. Since consumption occurs only in $t = 2$, then the opportunity cost from lending is zero, thus $r = 0$. If audit is conducted, then the realisation of the idiosyncratic shock becomes a common knowledge. Therefore, if the realisation is found to be bad, the entrepreneur will not hire labour or borrow, although workers would like to work. The realisation of output from production in $t = 2$ is assumed to be perfectly observable to the household.

Finally, the entrepreneur who is unaudited and gets a bad realisation of her idiosyncratic shock, chooses to default. Specifically, if the output realisation of the unaudited entrepreneur is zero in $t = 2$, she defaults on her debt, and cannot be forced to pay back. Accordingly, the household's incentive from auditing is to avoid the possibility of the entrepreneur defaulting on the debt if her production is unsuccessful. At low levels of capital, the loss in the household's expected utility from paying the audit cost, will be greater than the loss in his expected utility from the entrepreneur defaulting on the debt, if the entrepreneur's production is unsuccessful. As the entrepreneur's capital grows larger, the loss in the household's expected utility from paying the audit cost, will be smaller than the loss in his expected utility from the entrepreneur defaulting on the debt, if the entrepreneur's firm project turns to be bad.

A.1.1 The Entrepreneur's Problem

The entrepreneur maximizes her utility by choosing consumption in the second period c', the amount of the consumption good that she wants to borrow b, and the amount of labour she wants to hire l. Moreover, since an entrepreneur can be subjected to two different credit treatments, depending on whether she is being audited or not, there will be two versions of the optimisation problem.

The Audit Case In $t = 1$, with a prior probability $1 - P$, the entrepreneur's production will be found unsuccessful through audit, and hence she will not borrow to hire labour. Accordingly, her problem is given by:

$$\max_{c',b,l} E[\beta c'] \tag{47}$$

subject to

$$\begin{aligned} c' &= zk^{\alpha}l^{1-\alpha} - b + (1-\delta)k \quad \text{with probability } P \\ c' &= k \quad \text{with probability } 1 - P \\ b &= wl \end{aligned} \tag{48}$$

where c' indicates that consumption takes place in the second period, w is the wage

rate, and $\beta \in (0,1)$ is the implied discount factor. I substitute for c' in (47) using the entrepreneur's lifetime budget constraint, which is given in (48):

$$\max_{l} P\beta \left[zk^{\alpha}l^{1-\alpha} - wl + (1-\delta)k\right] + (1-P)\beta k \tag{49}$$

Taking the first order condition and solving for the optimal labour demand under audit, gives:[27]

$$l = \left(\frac{z(1-\alpha)}{w}\right)^{\frac{1}{\alpha}} k \tag{50}$$

The No Audit Case Under the no audit case, the entrepreneur will not be audited; however, she still borrows and hires labour. If the entrepreneur's realisation of the idiosyncratic shock is unfavourable, then she defaults and cannot be forced to pay back. Accordingly, her problem is given by:

$$\max_{c',b,l} E[\beta c'] \tag{51}$$

subject to

$$\begin{aligned} c' &= zk^{\alpha}l^{1-\alpha} - b + (1-\delta)k \quad \text{with probability } P \\ c' &= (1-\delta)k \quad \text{with probability } 1-P \\ b &= wl \end{aligned} \tag{52}$$

Substituting for c' in (51) using the entrepreneur's lifetime budget constraint, which is given in (52):

$$\max_{l} P\beta \left[zk^{\alpha}l^{1-\alpha} - wl + (1-\delta)k\right] + (1-P)\beta(1-\delta)k \tag{53}$$

Taking the first order condition and solving for the optimal labour demand under audit, gives:[28]

$$l = \left(\frac{z(1-\alpha)}{w}\right)^{\frac{1}{\alpha}} k \tag{54}$$

Since the entrepreneur can default on her debt if her production is unsuccessful, then her labour demand will be the same under audit and no audit.

[27] The function is concave; thus, the second order condition is satisfied.
[28] The function is concave; thus, the second order condition is satisfied.

A.1.2 The Household's Problem

The household maximizes his utility by choosing consumption in the second period c' and the amount of labour he wants to supply l. Moreover, since the household can either audit the entrepreneur or not, there will be two versions of the optimisation problem.

The Audit Case

$$\max_{c',l} E\left[-\frac{l^2}{2}+\beta c'\right] \tag{55}$$

subject to

$$c' = zc' = E - A + wl - b + b \quad \text{with probability } P$$

$$c' = E - A \quad \text{with probability } 1-P \tag{56}$$

In the good state of the world, with probability P, the amount of debt b is subtracted indicating lending, then added back to indicate debt repayment. I substitute for c' in (55) using the household's lifetime budget constraint, which is given in (56):

$$\max_{l} P\left[-\frac{l^2}{2}+\beta\left(E-A+wl\right)\right]+(1-P)\beta(E-A) \tag{57}$$

Taking the first order condition and solving for the optimal labour supply under audit, gives:[29]

$$l = \beta w \tag{58}$$

The No Audit Case

$$\max_{c',l} E\left[-\frac{l^2}{2}+\beta c'\right] \tag{59}$$

subject to

$$c' = E + wl - b + b \quad \text{with probability } P$$

$$c' = E + wl - b \quad \text{with probability } 1-P \tag{60}$$

Debt repayment only occurs in the good state of the world, since the entrepreneur cannot be forced to pay back. I substitute for c' in (59) using the household's lifetime budget constraint, which is given in (60):

$$\max_{l} P\left[-\frac{l^2}{2}+\beta(E+wl)\right]+(1-P)\left[-\frac{l^2}{2}+\beta E\right] \tag{61}$$

[29]The function is concave; thus, the second order condition is satisfied.

Taking the first order condition and solving for the optimal labour supply under audit, gives:[30]

$$l = P\beta w \tag{62}$$

Since the entrepreneur can default on her debt if her production is unsuccessful, the household's labour supply is less than what it would be under the audit case. In other words, in the bad state of the world, the household will be supplying labour with no return.

A.1.3 The Equilibrium

The equilibrium is characterised by the labour market clearing. Since I have two cases depending on whether audit takes place or not, two equilibrium levels of labour and wages will arise.

The Audit Case I equate the labour supply and demand as given in equations (50) and (58), and solve for the equilibrium wage under audit, w_A^*:

$$w_A^* = (z(1-\alpha))^{\frac{1}{1+\alpha}} \left(\frac{k}{\beta}\right)^{\frac{\alpha}{1+\alpha}} \tag{63}$$

I use (63) to substitute for in the labour supply given in equation (58) and obtain:

$$l_A^* = [z\beta(1-\alpha)]^{\frac{1}{1+\alpha}} (k)^{\frac{\alpha}{1+\alpha}} \tag{64}$$

which is the equilibrium level of labour under audit.

The No Audit Case I equate the labour supply and demand as given in equations (54) and (62), and solve for the equilibrium wage under no audit, w_{NA}^*:

$$w_{NA}^* = (z(1-\alpha))^{\frac{1}{1+\alpha}} \left(\frac{k}{P\beta}\right)^{\frac{\alpha}{1+\alpha}} \tag{65}$$

I use (65) to substitute for in the labour supply given in equation (62) and obtain:

$$l_{NA}^* = [zP\beta(1-\alpha)]^{\frac{1}{1+\alpha}} (k)^{\frac{\alpha}{1+\alpha}} \tag{66}$$

which is the equilibrium level of labour under no audit.

[30] he function is concave; thus, the second order condition is satisfied.

A.1.4 Obtaining the Audit Threshold, $\widehat{k}$

This is a general equilibrium set up, where agents form their decisions based on the prices they receive in equilibrium. Accordingly, I first obtain the threshold for auditing in terms of the wage rate. I obtain the household's utility under the audit and no audit cases, by substituting for l in equations (57) and (61) using equations (64) and (66), respectively.

$$EU_A^H = \frac{1}{2}P\beta^2 w^2 + \beta E - \beta A \tag{67}$$

$$EU_{NA}^H = \frac{1}{2}P^2\beta^2 w^2 + \beta E \tag{68}$$

The Audit Case The household prefers the audit equilibrium iff $EU_A^H \geq EU_{NA}^H$. I solve this inequality for w and obtain:

$$w \geq \left(\frac{2A}{\beta(P-P^2)}\right)^{\frac{1}{2}} = \hat{w} \tag{69}$$

where $\hat{w}$ is the wage threshold for the household's auditing decision. To obtain the auditing threshold in terms of the entrepreneur's observable capital stock $\hat{k}_A$, I compare between the equilibrium wage under audit (63) and the wage threshold for auditing (69) and solve for k:

$$k \geq \left(\frac{2A}{P-P^2}\right)^{\frac{1+\alpha}{2\alpha}} (\beta)^{\frac{-1+\alpha}{2\alpha}} (z(1-\alpha))^{-\frac{1}{\alpha}} = \hat{k}_A \tag{70}$$

The No Audit Case Similarly, the household prefers the no audit equilibrium iff $EU_{NA}^H \geq EU_A^H$. I solve this inequality for w and obtain:

$$w < \left(\frac{2A}{\beta(P-P^2)}\right)^{\frac{1}{2}} = \hat{w} \tag{71}$$

To obtain the no auditing threshold in terms of entrepreneur's observable capital stock $\hat{k}_{NA}$, I compare between the equilibrium wage under audit (65) and the wage threshold for auditing (71) and solve for k:

$$k \geq \left(\frac{2A}{P-P^2}\right)^{\frac{1+\alpha}{2\alpha}} (\beta)^{\frac{-1+\alpha}{2\alpha}} P\,(z(1-\alpha))^{-\frac{1}{\alpha}} = \hat{k}_A \tag{72}$$

A.1.5 Numerical Exercise

The auditing and no auditing capital thresholds, $\hat{k}_A$ and $\hat{k}_{NA}$, as given in equations (70) and (72) are not the same. They differ by a factor P. For instance, if I assume that $A = 0.5$,

$P = 0.6$, $\alpha = 0.3$, $\beta = 0.95$ and $z = 1$, then $\hat{k}_A = 76.77$ and $\hat{k}_{NA} = 46.06$.

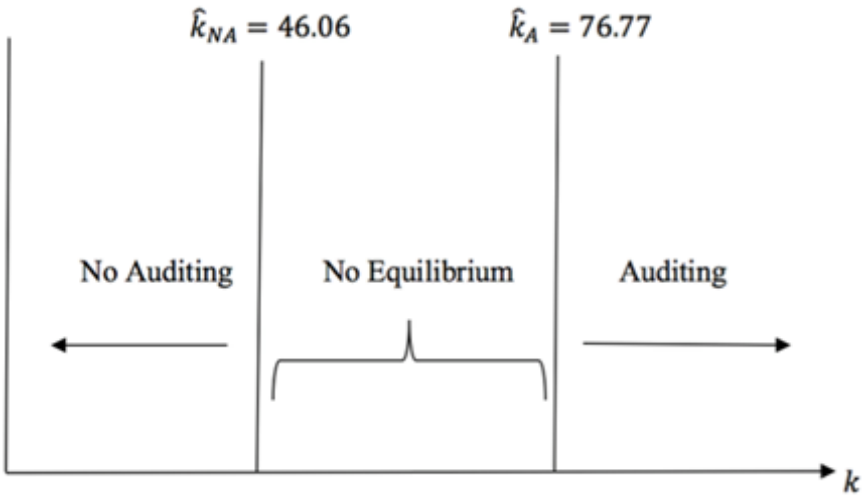

Figure 14: The auditing and no auditing capital thresholds.

Figure 14 illustrates the issue, where in the interval $(\hat{k}_{NA} = 46.06, \hat{k}_A = 76.77)$ no equilibrium exists, and the incentive compatibility breaks down. To further illustrate the issue, I conduct a numerical exercise where for different values of the capital stock k, I calculate the equilibrium wage rate in the audit and no audit cases. I assume the parameter values to be: $A = 0.5$, $P = 0.6$, $\alpha = 0.3$, $\beta = 0.95$, $\delta = 0.1$, $E = 800$ and $z = 1$, which give the wage threshold for auditing $\hat{w} = 2.09427$.

k	w_A^*	w_{NA}^*
40	1.80174	2.02716
$\hat{k}_{NA} = 46.06$	1.86139	2.09427
46.5	1.86544	2.09883
55	1.93913	2.18174
60	1.97846	2.22599
76.2	2.09066	2.35222
$\hat{k}_A = 76.77$	2.09427	2.35629
80	2.11427	2.37879

Table 8: The equilibrium wage rate in the audit and no audit cases, that correspond to different values of the entrepreneur's capital k.

Recall from equations (69) and (71) that auditing is preferred when $w \geq \hat{w}$ and no auditing is preferred when $w < \hat{w}$. As can be seen in Table 8, an equilibrium exists, and incentive compatibility is preserved in the intervals $[0, \hat{k}_{NA} = 46.06]$ and $[\hat{k}_A = 76.77, \infty)$. However, this does not apply in the interval $(\hat{k}_{NA} = 46.06, \hat{k}_A = 76.77)$. This is due to the functions w_A^* and w_{NA}^* not intersecting in the space of k and w, as shown in Figure 15.

Note that w_{NA}^* is always greater than w_A^*, since $P < 1$. Counterfactually, if the functions are hypothetically given as in Figure 16, then there will be a single capital threshold for

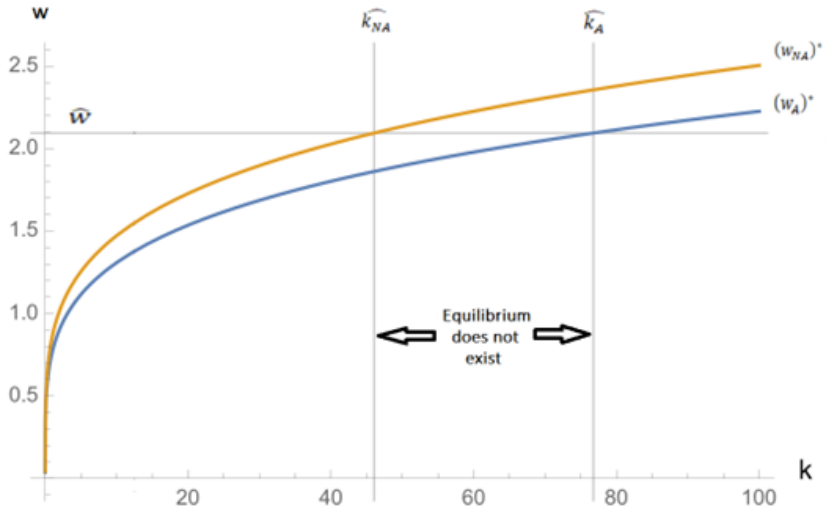

Figure 15: A plot of the functions w_A^* and w_{NA}^* , in the space of k and w.

the auditing decision and an equilibrium will exist over the entire capital space.

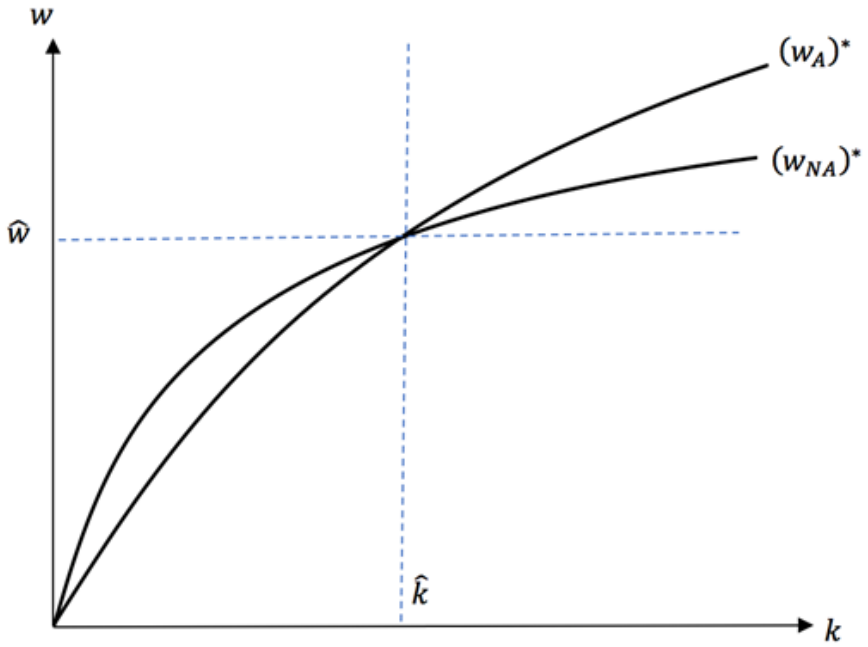

Figure 16: A hypothetical plot of the functions w_A^* and w_{NA}^* in the space ofof k and w, when there is a unique capital threshold for auditing.

I have amended the assumption on capital depreciation, such that capital always depreciates between $t = 1$ and $t = 2$. However, I arrive at the same conclusion, where an equilibrium does not always exist over the entire capital space. In addition, I have also tried to change the assumption on the enforceability of debt repayment, such that an unaudited entrepreneur is collaterally constrained in her borrowing. In this case, the wage threshold for the household's auditing decision is given by:

$$w \geq \left(\frac{2A}{\beta(P-1)}\right)^{\frac{1}{2}} = \hat{w} \tag{73}$$

The above expression implies that $\hat{w} < 0$, since $P \in (0,1)$. The same outcome is obtained when solving for the threshold level of no auditing (i.e. when $EU^H_{NA} \geq EU^H_A$). Since prices are positive, this suggests that for any wage rate that arise in equilibrium, the household will always choose to audit. Accordingly, the model fails in answering the first research question.

A.2 Choice of Parameters' Values

A.2.1 The Prior Probability of Entrepreneurs' Success, P

According to ONS (2012), the two years average survival rate for UK businesses just before the recession was around 80 per cent. I have chosen the two years average survival rate, since Crawford et al. (2013) results cover the period of 2008 and 2009. Moreover, according to the US Small Business Administration, SBA (2016), the one-year average survival rate for US businesses just before the recession started was around 80 per cent. I have chosen the one-year average survival rate, since Kudlyak and Sanchez (2017) results cover one year of recession only.

A.2.2 The Lenders' Bargaining Power, γ

Instead of justifying the precise choice for the value of γ that was reported in the results, I explain the reason behind choosing different values for γ between the UK and the US. In an investigation by the UK Competition and Markets authority (CMA (2016)), it was found that the biggest four banks in the UK hold between them around 85 per cent of the banking sector that deals with firms. SNL Financial which is a subsidiary of S&P, has complied data on the US banking concentration.[31] It found that the share of the industry's assets held by the biggest five American banks, have grown from around 10 per cent in 1990 to more than 44 per cent by 2014. Moreover, the percentage of total liabilities and assets held by the biggest five banks as a share of the industry's total is larger in the UK compared to the US (BIS (2001)). Accordingly, although the banking concentration in the US is high as well, it is not as severe as in the UK.

[31] Check: Vanderpool, C. (2014). "5 Banks Hold More Than 44% of US Industry's Assets", SNL Financial (S&P Global Market Intelligence), 2 December 2014, available at https://www.snl.com/InteractiveX/Article.aspx?cdid=A-30025507-14130.

A.3 Robustness Checks

In this section I perform robustness checks. I replicate the numerical exercises presented in the results section of the paper, under other possible scenarios. Precisely, I check the robustness of the results to the economy remaining in the same borrowing constraints regime, binding or non-binding, following the shocks. The results robustness is also examined to a change in the lenders' bargaining power. In all cases, the results remain intact.

A.3.1 Borrowing Constraints' Regimes

In what is shown below, results remain qualitatively the same. The initial values of z and q and their values after the shocks z' and q', were chosen such that if the economy has started in a situation of binding or non-binding borrowing constraints, it would remain in the same situation following the shocks. To make those economies comparable to the economy presented in the results earlier, the ratios z/q and z'/q' for the economies considered were maintained the same as in the economy presented in the results earlier.

Binding Borrowing Constraints As can be seen in Table 9, the economy starts from a situation of binding borrowing constraints and remains in that regime following the shocks. Additionally, Figure 17 demonstrates that the model's qualitative results stay intact.

	P	α	γ	δ	A	z	q	$\bar{q}$	$\hat{k}$
Pre-shock	0.8	0.3	0.5	0.1	30	3	1.6856	0.29319	20.364
Post-shock	0.8	0.3	0.5	0.1	30	2.94	1.85416	0.313614	21.6488

Table 9: The economy stays in a binding borrowing constraints regime; in the absence of a shock, and when a 10 per cent positive shock to q combined with a 2 per cent negative shock to z occur.

Non-Binding Borrowing Constraints As can be seen in Table 10, the economy starts from a situation of non-binding borrowing constraints and remains in that regime following the shocks. Additionally, Figure 18 demonstrates that the model's qualitative results continue to hold.

A.3.2 Lenders' Bargaining Power

I examine the robustness of my results to a change in the lenders' bargaining power. I replicate the numerical exercises for two alternative values of γ, 0.01 and 0.99.

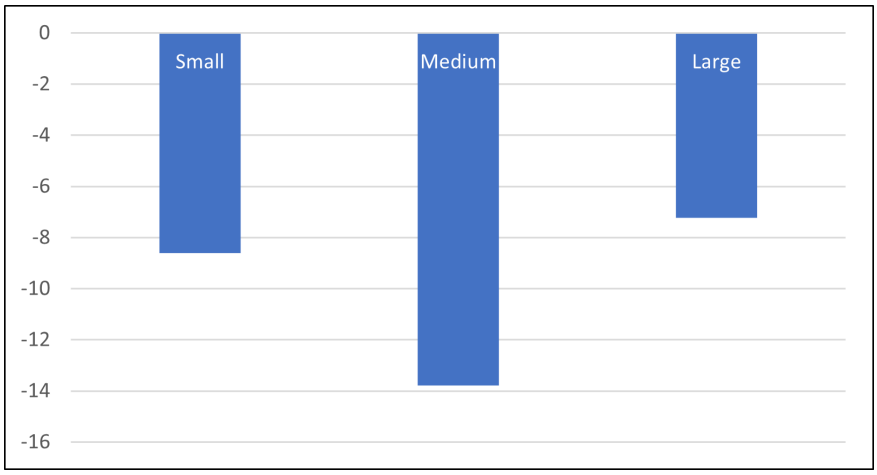

Figure 17: The results for a 2 per cent negative shock to z combined with a 10 per cent positive shock to q, while the economy remains in a binding borrowing constraints regime.

	P	α	γ	δ	A	z	q	$\bar{q}$	$\hat{k}$
Pre-shock	0.8	0.3	0.5	0.1	30	1	0.5612	11.417	1855.02
Post-shock	0.8	0.3	0.5	0.1	30	0.98	0.61732	12.2124	1984.24

Table 10: The economy stays in a non-binding borrowing constraints regime; in the absence of a shock, and when a 10 per cent positive shock to q combined with a 2 per cent negative shock to z occur.

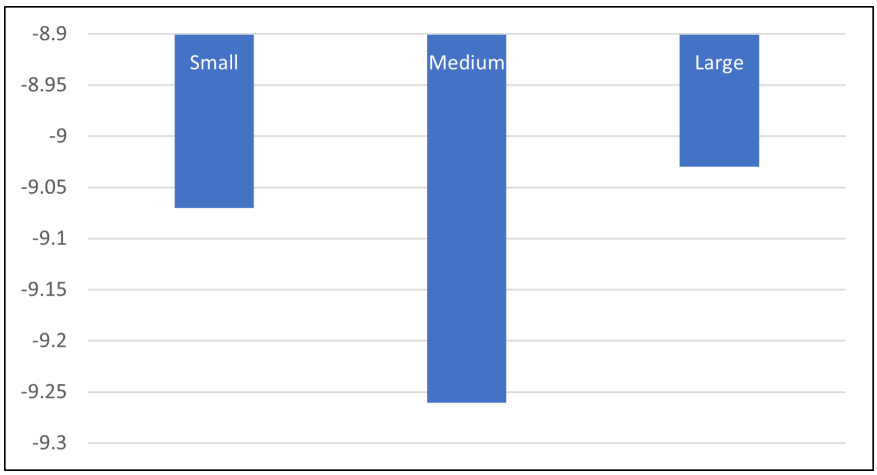

Figure 18: The results for a 2 per cent negative shock to z combined with a 10 per cent positive shock to q, while the economy remains in a non-binding borrowing constraints regime.

Setting $\gamma = 0.01$ As can be seen in Table 11, the economy starts from a situation of binding borrowing constraints and remains in that regime following the shocks. Additionally, Figure 1.19 demonstrates that the model's qualitative results remain unchanged.

	P	α	γ	δ	A	z	q	$\bar{q}$	$\hat{k}$
Pre-shock	0.8	0.3	0.5	0.1	30	2	1.124	0.621018	9271.87
Post-shock	0.8	0.3	0.5	0.1	30	1.96	1.2364	0.664279	9652.75

Table 11: A 10 per cent positive shock to q combined with a 2 per cent negative shock to z, while setting $\gamma = 0.01$.

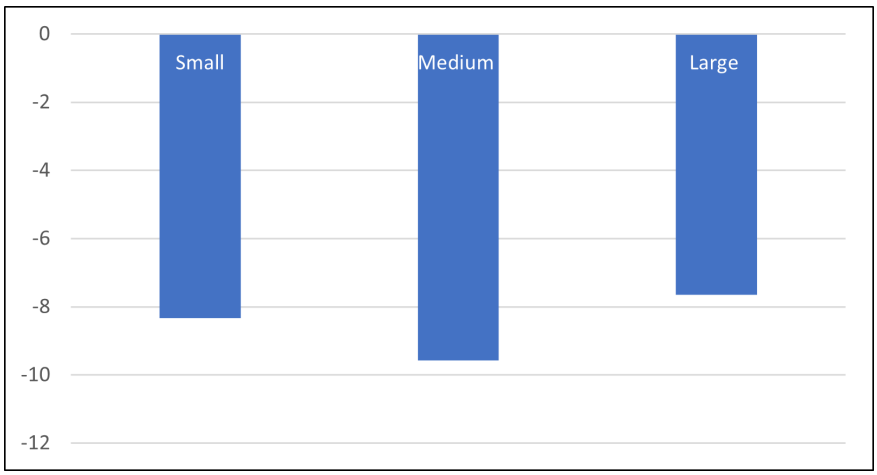

Figure 19: Investments' responses to a 10 per cent positive shock to q combined with a 2 per cent negative shock to z, while setting $\gamma = 0.01$.

Setting $\gamma = 0.99$ As can be seen in Table 12, the economy starts from a situation of non-binding borrowing constraints and remains in that regime following the shocks. Additionally, Figure 20 demonstrates that the model's qualitative results remain unchanged.

	P	α	γ	δ	A	z	q	$\bar{q}$	$\hat{k}$
Pre-shock	0.8	0.3	0.5	0.1	30	2	1.124	15266.1	28.8392
Post-shock	0.8	0.3	0.5	0.1	30	1.96	1.2364	16329.5	30.8482

Table 12: A 10 per cent positive shock to q combined with a 2 per cent negative shock to z, while setting $\gamma = 0.99$.

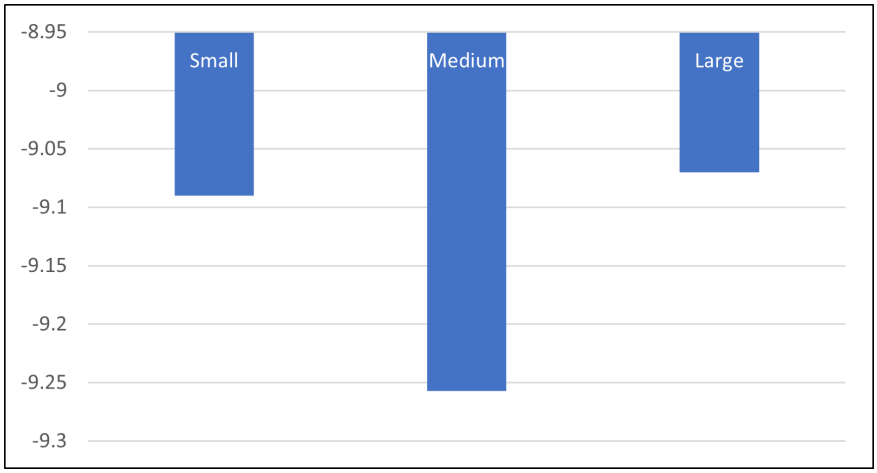

Figure 20: Investments' responses to a 10 per cent positive shock to q combined with a 2 per cent negative shock to z, while setting $\gamma = 0.99$.

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