Relative Valuation in Precious Metals Markets

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Abstract

Price ratios convey information about the relative valuation of traded assets. We estimate valuation pressure, which captures the degree of misalignment between gold and silver prices. While valuation pressure correlates positively with gold and silver variances, VIX, global equity market volatility, and economic policy uncertainty, it provides unique information for pricing uncertainty in the gold and silver markets. We formalize three channels through which changes in the gold-silver ratio affect asset price volatility and test them using intraday futures data for gold and silver from 2009 to 2025. First, valuation pressure itself has both short- and long-run effects on future volatility in both markets, where a 1-standard-deviation increase in valuation pressure leads to approximately 3% and 4% increases in the variance of gold and silver, respectively. Second, re-balancing pressure, reflecting valuation-induced portfolio adjustments, reduces subsequent volatility in the silver market. Third, attention pressure, linking relative valuation changes to heightened investor attention, leads to modest volatility increases in the silver market. These effects are stronger for silver and are most pronounced when gold is relatively expensive. Incorporating valuation pressure extracted from the gold-silver ratio tends to improve out-of-sample volatility forecasts of the best benchmark model. The gold-silver ratio thus captures pricing dynamics relevant for risk management and portfolio allocation.

Keywords: Relative Valuation; Gold; Silver; Gold-Silver Ratio; Realized Volatility; Local Projections; Machine Learning.

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1. Introduction

Gold is one of the prominent commodities in global financial markets despite limited industrial use. It is widely recognized as a safe haven asset (Baur and McDermott, 2010, Baur and Lucey, 2010, Boubaker et al., 2020)[1] and as an effective hedge against geopolitical risk (Baur and Smales, 2020). Investors' preference for gold, despite its higher volatility than low-risk assets such as Treasury bonds, likely reflects its historical role as a currency, its status as a store of value, and its perceived reliability in turmoil (Baur and McDermott, 2016). Gold's investment role continues to motivate new pricing models that capture its distinctive characteristics.[2] Silver, although sometimes also viewed as a safe haven (Li and Lucey, 2017), has received less attention in the literature. It has been found to exert feedback effects on equity markets (Baur and McDermott, 2016), yet it remains less popular and is often regarded as "gold's younger sibling", occupying a secondary position among precious metals. Unlike gold, silver has substantial industrial utility, in particular for solar energy, battery technologies, and medical devices.[3]

Despite differences in their economic roles, gold and silver prices are often jointly analyzed using the gold-silver ratio (GSR), defined as the number of ounces of silver required to buy one ounce of gold. The GSR is used as a valuation benchmark when analyzing one or both metals (e.g., Reuters, 2025a; Reuters, 2025b), a foreshadower of equity market movements (Bloomberg, 2020), and a sentiment indicator (Charles Schwab, 2025). A higher ratio may signal greater economic uncertainty, when silver's industrial demand weakens and safe-haven demand for gold rises; historically, it has also been interpreted as silver being relatively undervalued versus gold, while a lower ratio suggests the opposite. In this study, we propose three mechanisms, valuation misalignment, portfolio re-balancing, and investor attention, through which changes in the GSR can signal subsequent shifts in gold and silver price volatility.

A relevant stream of the literature examines short- and long-run linkages between gold and silver markets, often through cointegration. The rationale for treating gold and silver as a unified market stems from their use in the jewellery industry and their safe-haven properties, yet it is not straightforward because each metal has distinct, commercially important uses with few viable substitutes. Using monthly data from 1971 to 1990, Escribano and Granger

[1] A safe haven asset is typically defined as one that is either uncorrelated or negatively correlated with other assets during periods of financial stress.

[2] See Jermann [2025] for a recent study published in the Review of Financial Studies.

[3] Table A1 presents global demand and supply trends for gold and silver in recent years, highlighting notable differences between the two markets. Gold has experienced strong and sustained demand from central banks, reflecting its role as a reserve asset. In contrast, silver's relevance in photography has diminished, while its industrial demand has been steadily increasing. Notably, the silver market has been in a structural deficit since 2021.

[1998] find that cointegration may have occurred in some periods, especially during and after the 1979–1980 commodity price bubble. With a longer sample (1970–2011), Baur and Tran [2014] study the role of bubbles and financial crises and report cointegration, with gold prices driving the relationship mainly during bubble-like episodes and crises. Schweikert [2018] analyze monthly spot prices (August 1971–May 2017) and daily spot and futures prices (April 1980–May 2017), and although decades of comovement are documented, they cannot estimate a single constant cointegrating coefficient, concluding that the relationship is asymmetric and market-condition dependent. Other studies also fail to find a stable relationship (e.g., Ciner, 2001; Figuerola-Ferretti and Gonzalo, 2010), whereas Lucey and Tully [2015] report a weak but generally stable one. Overall, the direction and strength of price transmission vary across price regimes (Schweikert, 2018); gold can have significant positive or negative effects on silver, while the reverse link is weaker (Liu and Su, 2019; Mishra et al., 2019). This uncertainty is reflected in the high risk of gold-silver spread trading (Wahab and Cohn, 1994; Liu and Chou, 2003).

The dynamics and drivers of precious metals' prices have been studied extensively for decades.[4] For example, Batten et al. [2010] show that gold price volatility is strongly influenced by monetary variables, a link not found for silver.[5] Arouri et al. [2012] examine long memory and structural breaks in precious metals' volatility and conclude that long memory is the more important driver of future volatility. Related work considers volatility forecasting (e.g., Gupta and Pierdzioch, 2022; Gkillas et al., 2020; Khalifa et al., 2011; Raza et al., 2023).[6] Although this literature highlights exogenous factors (e.g., climate and geopolitical risks) as useful for improving volatility forecasts, it does not explicitly study forecast gains from the gold-silver interrelationship. A notable exception is the study of Lyócsa and Molnár [2016], who use univariate and multivariate heterogeneous autoregressive (HAR) models to forecast one-day-ahead volatilities of gold and silver. They find limited evidence that including one metal's volatility improves forecasts for the other; in a generalized HAR framework modeling both metals jointly, forecasts are comparable to those from combining univariate models.

To some extent, our study relates to the gold–platinum ratio literature. Given gold's dual role as a consumption good (mainly jewellery) and a financial asset that stores value during market distress, and platinum's similar consumption uses, Huang and Kilic [2019] argue that the gold–platinum price ratio should be relatively insulated from shocks to consumption and jewellery demand. It may therefore reflect variation in aggregate equity market (tail) risk. They

[4]For a comprehensive overview of the gold literature, see O'Connor et al. [2015]; for an in-depth discussion of the financial economics of silver, refer to Vigne et al. [2017].

[5]For further studies on macroeconomic and financial drivers in precious metals markets, refer to Mo et al. [2018] and Dinh et al. [2022].

[6]For related work on price predictability, see Auer [2016] and Pierdzioch et al. [2016].

find that the ratio robustly predicts future stock returns, even after controlling for dividend yields, price-to-earnings ratios, and term premia. Subsequent work extends this line of research to international and industry-specific stock returns (Pham and Rudolf, 2021; Han et al., 2020), expected returns on gold, platinum, and Bitcoin [Huynh et al., 2020], bond risk premia [Bouri et al., 2021], and stock market bubbles [Demirer et al., 2024].

Research on relative valuation remains relatively scarce. Fang [2020] examines whether gold price ratios, which represent the relative valuations of gold, predict aggregate stock returns and conclude that gold price ratios positively predict future stock returns in-sample, but fail to generate significant out-of-sample forecasting performance. Baur et al. [2020] show that the relative value of gold to commodities, equities, dividends, and bond yields varies substantially over time. Their analysis indicates that inflation expectations and uncertainty are the primary drivers, whereas macroeconomic fundamentals play a minor role. The only study that integrates the GSR into a risk forecasting framework is Salisu et al. [2022], who use GSR in levels as a proxy for global risk and find that it improves forecasts of stock market tail risks at a monthly frequency.

Our work has several contributions. First, we introduce three mechanisms through which changes in a price ratio may be transmitted to subsequent short- and long-term changes in volatility in theoretical terms. The first one, rooted in the cointegration literature, interprets deviations in the ratio as valuation shocks, with their squared magnitude defining valuation pressure, a measure of temporary misalignment that should induce corrective price adjustments. The second mechanism draws on portfolio theory. When a ratio deviates unexpectedly, it can imply non-optimal portfolio weights and trigger re-balancing behavior. We capture this effect through re-balancing pressure, defined as the interaction of valuation pressure with unexpected volume fluctuations. Finally, guided by an attention-based view of asset prices, we conceptualize attention pressure as the joint effect of valuation pressure and unexpected increases in investor attention.

Our second contribution is defining an empirical setup to test whether these transmission mechanisms help forecast realized volatility. Using machine learning techniques and sixteen years of intraday gold and silver futures data, we extract the unpredictable components of changes in the GSR, trading volume, and investor attention, and combine them with local projection models that incorporate a wide range of market risk controls. Our findings reveal that valuation pressure robustly predicts higher future variance for both metals at horizons from one day to one quarter, with stronger effects for silver and primarily arising when gold appreciates relative to silver. In contrast, re-balancing pressure systematically dampens volatility, consistent with the idea that portfolio adjustments counteract momentum driven price movements. Investor attention appears to play a relatively minor role overall. Significant effects for re-balancing and attention pressures are identified mainly for silver. These dynamics are particularly pronounced when gold is relatively expensive.

Third, motivated by these in-sample results, we evaluate whether relative valuation can be harnessed to improve out-of-sample volatility forecasts. We find that valuation pressure provides incremental informational content beyond liquidity measures and implied volatility. When combined with these predictors, valuation pressure further enhances forecasting performance. Moreover, we show that when high-frequency data are unavailable and range-based variance measures are used instead to estimate price variation, the forecasting accuracy improvements are even more pronounced. Together, our results demonstrate that the GSR contains rich information about market risk that is not fully captured by traditional volatility measures. This emphasizes the broader potential of relative valuation ratios as tools for understanding and forecasting risk in financial markets.

The paper is structured as follows. In the following Section 2 we discuss the three transmission mechanisms. In Section 3 we provide information about the high-frequency data and the control variables used in our study. Section 4 describes the methodology in three parts. First, we specify how are expected values and hence shocks estimated, next, we define specifications that test transmission mechanisms in the local projection framework and finally, we describe the methodology for the out-of-sample study. Section 5 presents and discusses our main findings. The final section concludes and suggests directions for future research.

2. Relative valuation and price behavior

Gold and silver share several features as precious metals. Both are traded globally, denominated primarily in U.S. dollars, and are considered as potential hedges against inflation, currency depreciation, and financial stress. However, gold has limited industrial use and is valued primarily as a monetary and investment asset, reflecting its historical role as a store of value, its use in official reserves (strong central bank demand), and its tendency to attract demand during periods of elevated uncertainty and risk aversion. Silver, by contrast, combines precious metal characteristics with substantial industrial use. Its demand is increasingly linked to electronics, solar photovoltaics, vehicle electrification, grid infrastructure, and other cyclically sensitive technologies. Consistent with this distinction, prior research shows that gold and silver are exposed differently to macroeconomic and monetary forces (Batten et al., 2010), and that their long-run relationship is unstable and regime dependent. A narrative interpretation of an increase in the GSR is that it reflects heightened investor risk aversion and stronger safe-haven demand for gold relative to expectations of economic, particularly industrial, growth that support silver demand.

However, our main argument is based on the unexpected relative valuation shock that potentially pushes gold and silver prices out of their long-run equilibrium. A sufficiently large build-up of such shocks might eventually lead to volatility effects through price adjustments towards the long-run equilibrium, price adjustments induced through portfolio re-balancing and through increased investor attention associated with such valuation shocks. Our valuation misalignment is different from the Huang and Kilic [2019] who use the level of the gold-to-platinum ratio directly as a macro-finance state variable.

2.1. Valuation shock and valuation pressure

The literature suggests an existence of a regime dependent, or time-varying relationship between gold and silver prices, thus gold and silver should be cointegrated at least intermittently. The existence of such a long-run equilibrium relationship, despite short-term deviations, also implies that unexpected deviations in the GSR should provide price(trading)-relevant information and may thus also improve our understanding of market risk on these two markets, specifically the behavior of price fluctuations. As the spread deviates substantially from its long-run equilibrium, it is likely to revert, with potential implications for the price volatility of one or both metals.

To clarify how the GSR can be interpreted through a potentially time-varying cointegrating relationship, suppose that gold and silver prices satisfy

$$G_t = \alpha_t + \beta_t S_t + u_t, \tag{1}$$

where $G_t > 0$, $S_t > 0$, and u_t is a mean-zero stationary process with finite variance, $u_t \sim I(0)$. Consistent with evidence that the relationship between gold and silver changes over time, we allow the intercept and slope to vary. We assume that $\alpha_t \sim I(0)$ is persistent and stationary, while β_t is a slow-moving, potentially trend-stationary process (e.g. $\beta_t = \delta_0 + \delta_1 t + \nu_t, \nu_t = \gamma\nu_{t-1} + \eta_t, |\gamma| < 1$). Defining $GSR_t = G_t/S_t$ we have

$$GSR_t - \beta_t = \frac{\alpha_t + u_t}{S_t}. \tag{2}$$

This representation provides a link between the cointegrating relationship and the GSR. The ratio consists of the slow-moving relative-valuation benchmark β_t, a premium or discount α_t/S_t, and the transitory deviation u_t/S_t, expressed relative to the silver price. To characterize the long-run interpretation of this representation, let the silver price follow a geometric Brownian motion sampled at discrete times,

$$S_t = S_0 e^{\mu t + \sigma W_t}, \qquad S_0 > 0, \quad \mu > 0, \tag{3}$$

where μ denotes the long-run log-price drift and W_t is a standard Brownian motion evaluated at t. In discrete time,

$$W_t = \sum_{k=1}^{t} \varepsilon_k, \qquad \varepsilon_k \overset{i.i.d.}{\sim} N(0,1).$$

Under this parameterization,

$$\frac{1}{t} \ln\left(\frac{1}{S_t}\right) = -\frac{\ln S_0}{t} - \mu - \sigma \frac{W_t}{t} \xrightarrow{a.s.} -\mu, \tag{4}$$

because $W_t/t \xrightarrow{a.s.} 0$, hence

$$S_t^{-1} = e^{-(\mu + o(1))t}$$

almost surely. Since α_t and u_t are stationary processes with finite moments, they grow more slowly than S_t, and therefore $\frac{\alpha_t}{S_t} \xrightarrow{a.s.} 0$, $\frac{u_t}{S_t} \xrightarrow{a.s.} 0$, consequently $GSR_t - \beta_t \xrightarrow{a.s.} 0$. Conditional on the assumed cointegrating relationship and positive long-run growth in the silver price, the GSR therefore tracks the slow-moving relative-valuation benchmark β_t in the long run.

At shorter horizons, all three components of the ratio can vary. Under the assumption that u_t is mean-zero and mean-independent of S_t and $\mathcal{I}_{t-1}$,

$$\mathbb{E}\left[\frac{u_t}{S_t} \,\middle|\, \mathcal{I}_{t-1}\right] = 0.$$

The innovation in the GSR can then be written as

$$\begin{aligned} GSR_t^{\perp} &\equiv GSR_t - \mathbb{E}\left[GSR_t \mid \mathcal{I}_{t-1}\right] \\ &= \{\beta_t - \mathbb{E}\left[\beta_t \mid \mathcal{I}_{t-1}\right]\} + \left\{\frac{\alpha_t}{S_t} - \mathbb{E}\left[\frac{\alpha_t}{S_t} \,\middle|\, \mathcal{I}_{t-1}\right]\right\} + \frac{u_t}{S_t}. \end{aligned} \tag{5}$$

We refer to $GSR_t^{\perp}$ as the *valuation shock*. It is a reduced-form innovation that contains unexpected movements in β_t and α_t/S_t, as well as the scaled transitory deviation u_t/S_t. When the persistent components of the relationship are well captured by the conditional expectation, so that their one-step-ahead innovations are relatively small, $GSR_t^{\perp}$ is expected to be dominated by u_t/S_t. Under this interpretation, the valuation shock provides an empirical proxy for an unexpected deviation between gold and silver prices, without requiring it to equal the cointegrating residual exactly.

The decomposition into expected and unexpected components follows the identification strategies of Baker and Wurgler [2006] and Constantinides et al. [2025]. Our implementation is predictive because $\mathbb{E}[GSR_t \mid \mathcal{I}_{t-1}]$ is estimated using only information available at $t-1$. Information arriving during period t is therefore included in the forecast innovation.

The relationship between the valuation shock and future volatility is an empirical question. Following the cointegration literature, we intuitively associate a large instantaneous misalignment between prices with a need for adjustment in one or both prices, especially if there is a build-up of such shocks over time. This suggests that the effect of a shock at time t can have delayed and persistent impacts on the underlying prices. The relationship between the valuation shock and volatility might depend on the sign of the shock, i.e. whether gold or silver is becoming relatively more expensive. We therefore also use a magnitude-based measure, which is based on the square of the shock, which we refer to as the *valuation pressure*. For interpretation purposes, it is defined as the demeaned log of the squared shock,

$$VP_t = \ln\left(GSR_t^{\perp}\right)^2 - \overline{\ln\left(GSR_t^{\perp}\right)^2}. \tag{6}$$

2.2. Volume shocks and re-balancing pressure

Changes in the prices of risky assets in a portfolio will almost surely lead to changes in portfolio weights, as such price changes are rarely of the same proportion i.e. $GSR_{t+1} \neq GSR_t$. If the new weights deviate from the investor's 'optimal' weights, it might create re-balancing pressure that should be accompanied by increased trading activity and, hence, price volatility.

To illustrate what might impact the re-balancing pressure more formally, assume a two-asset portfolio of a risk-averse investor with returns given as $R^G_{t+1} = G_{t+1}/G_t$ and $R^S_{t+1} = S_{t+1}/S_t$ for gold and silver, respectively. The change in the valuation ratio is $q_{t+1} = R^G_{t+1}/R^S_{t+1} = GSR_{t+1}/GSR_t$. The next-period portfolio weights for gold ($w^G_{t+1} \in [0,1]$) and silver ($w^S_{t+1} = 1 - w^G_{t+1}$) can be expressed as a change in the valuation ratio,

$$w^G_{t+1} = \frac{w^G_t q_{t+1}}{w^G_t q_{t+1} + (1 - w^G_t)}, \qquad w^S_{t+1} = 1 - w^G_{t+1}. \tag{7}$$

The optimum weights at time t for a risk-averse investor will be denoted as $\boldsymbol{w}^*_t$ and given a risk-aversion parameter $\gamma > 0$, a column vector of gross returns $\boldsymbol{\mu}_t$ and covariance matrix of returns $\boldsymbol{\Sigma}_t$, they are determined as a solution to: $\boldsymbol{w}^\star_t = \arg\max_{\boldsymbol{w}\in\mathbb{R}^2:\,\mathbf{1}^\top\mathbf{w}=1} \left\{\boldsymbol{\mu}_t^\top \boldsymbol{w} - \frac{\gamma}{2}\boldsymbol{w}^\top\boldsymbol{\Sigma}_t\boldsymbol{w}\right\}$, where $\mathbf{1} = (1,1)^\top$. For each of the two assets, the re-balancing pressure can be understood to be associated with a difference between the current and the optimum weights, i.e. $w^G_{t+1} - w^{G,*}_{t+1}$ which can be decomposed into

$$w^G_{t+1} - w^{G,*}_{t+1} = (w^G_{t+1} - w^G_t) - (w^{G,*}_{t+1} - w^{G,*}_t) + (w^G_t - w^{G,*}_t). \tag{8}$$

The decomposition shows that the re-balancing pressure is driven by the: i) mechanical drift $(w_{t+1}^G - w_t^G)$ of the weights, which is a function of the change of the valuation ratio q_{t+1}, ii) revisions in the belief $(w_{t+1}^{G,*} - w_t^{G,*})$ about the optimum mean-variance weights and iii) as the previous weight might not have been at the optimum, the previous tracking error $(w_t^G - w_t^{*,G})$. This decomposition also shows (we can substitute Eq. 7 into Eq. 8), that the shock to the valuation ratio might increase the re-balancing pressure, especially if that has not led to changes in the belief about the expected returns, variances and co-variances. Furthermore, the pressure increases if there is already a build-up of the tracking error as not all valuation shocks will lead to trades due to trading constraints like financing constraints, trading costs, position limits, mandate constraints etc.

In an empirical setup, we do not observe the re-balancing pressure directly. As the decomposition shows, it is related to changes in the valuation ratio, and in order to get closer to the optimum weights $\boldsymbol{w}_{t+1}^*$, one needs to trade $(w_{t+1}^{G,*} - w_{t+1}^G)W_{t+1}/G_{t+1}$ units, where W_{t+1} is the value of the portfolio before the trade. Therefore it should be related to the valuation pressure and volume as well. Let V_t denote the volume of trades for a given asset on day t and the expectation $\mathbb{E}[V_t | \mathcal{I}_{t-1}]$. The unexpected *volume shock* (excess volume) is given by

$$V_t^{\perp} = V_t - \mathbb{E}[V_t | \mathcal{I}_{t-1}] \,. \tag{9}$$

We define the *re-balancing pressure* as the interaction between the magnitude of the valuation shock and the volume shock $V_t^{\perp}$,

$$RP_t = V_t^{\perp} \times \left(GSR_t^{\perp}\right)^2 . \tag{10}$$

Higher re-balancing pressure is given if the magnitude of an unexpected valuation shock (valuation pressure) coincides with the volume shock. To illustrate a potential impact of the re-balancing pressure on the volatility, consider a positive valuation shock, arising from a relative gold price increase, which given Eq. (8) increases the weight of the gold and if that is not immediately followed by changes in beliefs, the re-balancing means selling gold, thus acting against the price increase that led to the re-balancing pressure in the first place. If re-balancing acts against the direction of the valuation shock, we might observe a negative relationship between re-balancing pressure and volatility.

2.3. Attention shock and attention pressure

As gold and silver prices are likely often observed by the same market participants, changes to the price valuation ratios might also be informative as they trigger behavioral biases.

Market participants often compare gold and silver prices side by side, which can create reference dependence. The GSR may display stickiness or mean reversion (anchoring), and/or round-number thresholds can trigger disproportionate trading activity. The relative value between gold and silver is often referred to by the media (e.g., Reuters, 2025b,a, Bloomberg, 2020), we also argue for a potential mechanism, where gold and silver volatilities are driven by increased attention that the relative value of the two commodities receives from the media.

The literature offers several explanations for the existence of the relationship between investors' attention and price changes. The limited attention explanation of Barber and Odean [2008] argues that investors trade mostly attention grabbing stocks. The rational inattention theory of Sims [2003, 2006] argues that economic agents have selective and costly attention which leads agents to process not only a limited amount of information, but also a specific type of information. Prices might therefore reflect which investors have paid attention to what information and when. Tetlock [2007] shows how a sentiment extracted from a column article in Wall Street Journal had the potential to lead price and volume changes on the stock market. Recently, multiple studies have shown that sentiment from various media, like newspapers or micro-blogging sites, lead future variation of asset prices (e.g. Audrino et al., 2020, Halousková and Lyócsa, 2025).

Let A_t denote the attention to gold and silver and $\mathbb{E}[A_t|\mathcal{I}_{t-1}]$ the expectation. The unexpected *attention shock* is given by

$$A_t^{\perp} = A_t - \mathbb{E}[A_t|\mathcal{I}_{t-1}]\,. \tag{11}$$

Similarly as above, we define the *attention pressure* as the interaction between the magnitude of the valuation shock and the attention shock $A_t^{\perp}$,

$$AP_t = A_t^{\perp} \times \left(GSR_t^{\perp}\right)^2. \tag{12}$$

If there is an increased magnitude of the valuation shock (valuation pressure), the increased attention shock might amplify or dampen the effect of the valuation shock on the volatility.

3. Data

3.1. Intraday data: Gold and Silver

To estimate and forecast the volatility of gold and silver, we use intraday transaction data from Globex for CME gold and silver futures contracts spanning the period from 2 September 2009 to 18 June 2025. The data, sourced from LSEG Tick History, are sampled at one-minute frequency. Continuous futures series are constructed by selecting the contract with the highest trading volume for each day. Gold and silver futures are traded at Globex from Sunday, 6:00 pm. to Friday 5:00 pm ET, with a daily 60-minute break beginning at 5:00 pm. We observe intraday prices $P_{t,n}$ that are evenly spaced in calendar time, every minute, with $t = 1, 2, ...$ denoting the trading day, $n = 1, 2, ...N$ the intraday observation, with $P_{t,0}$ corresponding to the first observation at 6:00 pm. and $P_{t,N}$ to the closing observation for that day, defined to be at 5:00 pm. A price of $P_{t,n}$ corresponds to a price of a last trade within a given sampling period. The daily close-to-close return is given by $R_t = 100 \times (\ln P_{t,N} - \ln P_{t-1,N})$.

3.2. Realized measures: price variation and liquidity

The 1-minute price resolution allows us to use sub-sampling and averaging that increases the efficiency of the realized variance estimates (Zhang et al., 2005) also following approaches standard in the literature (e.g. Patton and Sheppard, 2015, Li and Tang, 2025). We use $K = 5$ different 5-minute price grids that should result in a more accurate estimates. The estimate of the average price variation over H trading days, that includes non-trading period variation is given by

$$RV_{t,H} = \frac{10^4}{H} \sum_{h=1}^{H} \left[\underbrace{\left(\ln \frac{P_{t+h+1,0}}{P_{t+h,N}} \right)^2}_{\text{non-trading period variation}} + \underbrace{\frac{1}{K} \sum_{\delta=1}^{K} \sum_{j=1}^{\lfloor (N-\delta+1)/K \rfloor} \left(\ln \frac{P_{t+h,\delta+Kj-1}}{P_{t+h,\delta+K(j-1)-1}} \right)^2}_{\text{trading period variation}} \right]. \tag{13}$$

Here $\lfloor . \rfloor$ denotes the closest lower integer. The non-trading-period variation is the squared return from the closing price on day $t+h$ to the opening price on day $t+h+1$. Our estimator is a special case of the whole-day variance estimator of Hansen and Lunde [2005], where the latter uses a weighted sum of the non-trading and trading period variation. The weights are based on the historical co-variation between the non-trading and trading period variation. For Globex data, the calculation of the co-variation between the trading and non-trading period is not straightforward as the non-trading period lasts for just 60 minutes from Sunday to Friday, but is over a day from Friday to Sunday, hence we prefer our simplified approach. The average realized variance over H days is our variable of interest in the sub-sequent analysis.

Motivated by empirical evidence that liquidity measures have predictive power for future volatility (e.g. Valenzuela et al., 2015), we add one liquidity measure to our set of control variables, the quoted spread, which has recently been shown to outperform alternative liquidity benchmarks in forecasting volatility of U.S. equities (Stašek and Lyócsa, 2024). In order to estimate volume-weighted quoted spread, we set $M_\delta = \lfloor (N - \delta)/K \rfloor$ and define the $H_{t,\delta} = \max\{P_{t,\delta+Kj} : j = 0, 1, ..., M_\delta\}$ and $L_{t,\delta} = \min\{P_{t,\delta+Kj} : j = 0, 1, ..., M_\delta\}$ as the highest and lowest prices in a given grid δ of a trading day t. We use the volume of trades $V_{t,\delta+Kj}$ that corresponds to a given 1-minute period of a given sampling grid δ to calculate weights $w_{t,\delta+Kj} = V_{t,\delta+Kj}/\sum_{j=0}^{M_\delta} V_{t,\delta+Kj}$. With $P_{t,\delta+Kj,\text{ask}}$ and $P_{t,\delta+Kj,\text{bid}}$ denoting the last ask and bid quotes for intraday observation $\delta + Kj$ on day t, the scaled averaged volume-weighted quoted spread is given as

$$QS_t = K^{-1} \sum_{\delta=1}^{K} \left[100 \times \frac{2}{H_{t,\delta} + L_{t,\delta}} \times \sum_{j=0}^{M_\delta} w_{t,\delta+Kj} \left(P_{t,\delta+Kj,\text{ask}} - P_{t,\delta+Kj,\text{bid}} \right) \right]. \quad (14)$$

3.3. Attention indices

To proxy for investor's interest towards the gold and silver markets, we retrieve daily search volume indices from Google trends, that correspond to a specific daily keyword/phrase (in English) and geographically to the whole world. Our sample of attention measures starts on 1 January 2009, but the maximum number of consecutive days that can be accessed in a single file is limited by Google to around 270 days. Within each such batch of data (sub-sample) the search volume index is scaled and rounded to integers that range from 0–100. Consecutive batches of data cannot be knitted together as they represent different scales (see Bleher and Dimpfl, 2022). Instead, we retrieve batches of data that overlap with three months of data. The overlapping period is used to estimate the scaling constant needed to rescale and merge individual batches into a single time series, following the approach of Lyócsa and Plíhal [2022].

We work with three attention indices. The first is a proxy for the interest towards the gold market. It is created as an average (across each day) of knitted search volume indices corresponding to the phrases: 'gold', 'gold market', 'gold price', 'gold price today', 'gold spot price'. The second is a proxy for the interest towards the silver market and is created using the phrases: 'silver', 'silver market', 'silver price', 'silver price today', 'silver spot price'. The third is our key attention measure of interest, where the proxy is the interest towards the gold and silver prices together, and uses three keywords: 'gold silver', 'gold silver price' and 'gold silver market'.

Such indices are subject to sampling variation error, which comes from the fact that the retrieved search volume indices are based on a sample of user queries. Therefore, the same requests of data might result in different time series, a problem recognized in the literature (Eichenauer et al., 2022, Cebrián and Domenech, 2024). A standard solution to mitigate the sampling error is to use an average across multiple suitable keywords and samples. Therefore, apart from using multiple keywords, we repeat the whole procedure described above five more times, as instead of starting data retrieval on 1 January 2009, we start on 1 February, March, April, May and June 2009, resulting in six time-series that are aggregated into the attention index.

The resulting attention indices are denoted as A_t^G and A_t^S for attention to gold and to silver, respectively, and A_t for the attention index that proxies attention to gold and silver, which is used in Eq. (11). In our specifications, the indices of attention to gold (A_t^G) only and to silver only (A_t^S) are used as control variables that help to identify the joint attention to gold and silver.

3.4. Daily control variables

In this section, we describe the set of control variables which are used at two instances. First, they are used to predict valuation ratios in order to estimate the unexpected part (i.e. shocks) of changes in relative values. Second, these are used in local projection equations to control for predictable changes in the volatility of gold and silver prices. Apart from the liquidity measure, we include the equity market implied volatility VIX[7], which is commonly referred to as the 'fear gauge', a widely recognized in the literature as a predictor of asset price volatility [Bekaert and Hoerova, 2014]. Given the VIX's established forecasting performance, a natural benchmark is whether a proposed model offers incremental information beyond what is already captured by the VIX, which is why we include it also in out-of-sample models. We also include the CBOE's gold implied volatility index GVZ, which corresponds to the expected 30-day volatility of returns of the underlying SPDR Gold Shares ETF.[8] As data for the CBOE's silver implied volatility index were not available,[9] specifications modeling silver's volatility use the GVZ instead.

In addition, we use a set of global financial market risk measures. First, the recently introduced COVOL of Engle and Campos-Martins [2023], based on a generalization of multivariate volatility models to incorporate volatility shocks that are common to all assets and asset classes. For this study, we use the daily global equity $COVOL_t$ times series that represents the common shock (innovations) across country level equity indices around the world (e.g. US, UK, France, South Korea, Kuwait, Germany).[10] $COVOL_t$ has been shown to act as a meaningful proxy of financial risk in a variety of contexts (e.g., Jabeur et al., 2025; Lang et al., 2024; Pham et al., 2025). We add a proxy for systemic risk in the U.S. financial sector, the SRISK, which is defined as the expected capital shortfall of a (financial) firm in an event of a broad market index declining by more than 40% in a six-month period.[11] We calculate the average firm-level

[7]Source: CBOE.
[8]Source: CBOE.
[9]The CBOE Silver ETF Volatility Index was decommissioned in 2022.
[10]See V-Lab/Covol website for more details and the data source.
[11]For more details, refer to Brownlees and Engle [2017].

SRISK by averaging the $SRISK_t$ across up to 900 U.S. financial institutions.[12] For our sample period, most of the time the $SRISK_t$ is actually negative, meaning that on average, the financial sector was well-capitalized for such an adverse market period. Increased liquidity risk on the equity market might increase flows into market of precious metals, potentially impacting their volatility as well. To control for such conditions, we also use Composite Liquidity Index, ILQ_t, which is a market-cap-weighted measure of market-wide liquidity for U.S. equities based on the Amihud [2002]'s price impact measure.[13]

We also make use of four uncertainty indices. First, the daily U.S. specific economic policy uncertainty index (EPU) of Baker et al. [2016], widely adopted to control for relevant economic policy innovations. The daily EPU_t is based on the news coverage aiming to identify articles related to economic policy uncertainty. Second, we use an equity market specific news uncertainty index, the US Equity Market Uncertainty Index (EMU_t), that is based on the (scaled) number of articles that mention at least one keyword from three sets: i) 'uncertainty' or 'uncertain', ii) 'economic' or 'economy' and iii) 'equity market', 'equity price', 'stock market', or 'stock price'.[14] Third, we use a text-based indicator of daily economic news in the U.S. designed by Shapiro et al. [2022].[15] The news sentiment index (NSI_t) is known to be correlated with lower frequency consumer sentiment and is frequently employed in the empirical finance literature (e.g., Jawadi et al., 2024; Osman et al., 2024; Yousaf et al., 2024). Fourth, we also control for possible adverse geopolitical events and associated risks, that might drive higher demand for precious metals, by using the Geopolitical Risk Index (GPR_t) developed by Caldara and Iacoviello [2022]. The GPR Index is constructed through an automated text analysis of the electronic archives of ten major newspapers, based on the frequency of articles referencing adverse geopolitical events, with keywords related to war, peace, military, terrorist, or nuclear risks (the GPR Threats sub-index) topics or to specific actions related to wars and/or terrorist acts (the GPR Acts sub-index).[16]

4. Methodology

4.1. Identification of valuation shocks

Our identification strategy for shocks is based on the decomposition of the valuation ratio into its expected and the unexpected component. This involves estimating the expected values

[12] The exact number of firms varies over the given sample period. Note that we take the average across all shortfalls irrespective of their sign.
[13] Data retrieved from V-Lab website.
[14] Both indices are retrieved from Policy Uncertainty website.
[15] Data retrieved from FRBSF website.
[16] Source: Matteo Iacoviello website.

of the gold-silver ratio, $\mathbb{E}[GSR_t|\mathcal{I}_{t-1}]$, volume, $\mathbb{E}[V_t|\mathcal{I}_{t-1}]$, and investor's attention, $\mathbb{E}[A_t|\mathcal{I}_{t-1}]$. This approach is motivated by the possibility that changes in the GSR_t might potentially be confounded with other market risks factors, which are now represented by the predicted values (e.g. Constantinides et al., 2025). The unexpected part, forecast error, allows us to study changes in the relative valuation ratio, that cannot be explained (confounded) by other lagged market risk factors. The expected values need to be estimated with an appropriate function $f(.)$ and a set of predictors $\boldsymbol{Z}$ observed in time $t-1$, i.e. the information set I_{t-1}.

To approximate the values of the function $f_t(.)$, we use an average of two day-ahead forecasts of GSR_t from an Elastic Net ($\hat{F}_t^{GSR,EN}(\boldsymbol{Z}|I_{t-1})$ or simply $\hat{F}_t^{GSR,EN}$) model and a Random Forest ($\hat{F}_t^{GSR,RF}$) model, that we will denote as $\hat{F}_t^{GSR}$. To estimate $\mathbb{E}[V_t|\,\mathcal{I}_{t-1}]$ and $\mathbb{E}[A_t|\,\mathcal{I}_{t-1}]$ we follow the same approach. As a set of predictors we include: GSR_t, ΔGSR_t, ΔGSR_t^2, RV_t^G, QS_t^G, R_t^G, RV_t^S, QS_t^S, R_t^S, VIX_t, GVZ_t, $COVOL_t$, $SRISK_t$, ILQ_t, EPU_t, EMU_t, NSI_t, GPR_t, V_t^G, V_t^S, A_t, A_t^G, and A_t^S, with the superscript denoting either gold or silver. We use one-day lags of these predictors, their average value over the past 5 trading days, their average value over the past 22 trading days, in the spirit of Corsi [2009]'s HAR model, while we also add a linear time-trend and day-of-the-week dummies, a structure akin to the volatility models that are used in the literature.

In order to estimate the hyper-parameters of the Elastic net and the Random forest models, we use a rolling-window approach, with a sliding window of 504 observations and a validation sample of 252 observations. The first forecast $\hat{F}_t(\boldsymbol{Z}|I_{t-1})$ and thus $GSR_t^{\perp}$ that is available for our analysis is the 757^{th} observation, corresponding to 20 August 2012, and is based on the model estimated using hyper-parameters that led to lowest mean-square forecast error of the valuation ratio over the past 252 observations. These optimum hyper-parameters are re-estimated after each trading day.

Within the elastic net modeling framework, we use two hyper-parameters, the regularization λ and the $\alpha \in \{0, 0.2, 0.4, 0.6, 0.8, 1\}$ that puts either all weight for the LASSO (Tibshirani, 1996) penalization ($\alpha = 1$), Ridge penalization (Hoerl and Kennard [1970b,a] ($\alpha = 0$) or between those two (Zou and Hastie, 2005). For the regularization parameter λ, we use 100 different values between λ_{max} and $\lambda_{min} = 10^{-4} \times \lambda_{max}$, where the λ_{max} is based on the maximum λ such that all coefficients (except the intercept) shrink to 0 using a sample of the first 504 observations across all three α values. The resulting 100 λ values between λ_{min} and λ_{max} are spaced equally on the log-scale.

In the random forest (Breiman, 2001) framework, we work with two hyper-parameters and two additional are being set ad-hoc. Specifically, each tree is estimated to the maximum depth possible by minimizing the square error, where the tree depth is only limited by the minimum node size of 5. These two hyper-parameters are set to allow creation of deep trees that are likely to overfit the data, which is turn is mitigated by two additional hyper-parameters that are tuned: i) the number of trees where we consider $250, 500, 1000$ and the ii) the number of variables that are randomly selected for each split, which is set to $3, 5, 7$. The hyper-parameters are re-estimated after each trading-day, following the same approach as described in the previous paragraph. The final estimate of the $\mathbb{E}[GSR_t|\mathcal{I}_{t-1}]$ is given by the average of the forecasts $\hat{F}_t^{EN}(\boldsymbol{Z}|I_{t-1})$ and $\hat{F}_t^{RF}(\boldsymbol{Z}|I_{t-1})$, which is an approach that lowers the impact of the model choice uncertainty.[17]

4.2. Local projections

We use the local projections (LP) framework of Jorda [2005] to study whether changes in the gold-silver ratio are associated with the future price variation of gold and silver, $RV_{t,H}$. The LP framework allows us to use predictive regressions, that are often employed in the volatility modeling literature, in order to estimate impulse-responses of volatility to shocks. The predictive regressions are estimated for each forecasting horizon H, and for gold or silver individually. We use three sets of right-hand side variables. The first captures the volatility persistence as in the heterogeneous autoregressive volatility model of Corsi [2009], where the column vector $\boldsymbol{RM}_t$ stacks a constant, $\ln(RV_t^D)$, $\ln(RV_t^W)$ and $\ln(RV_t^M)$. This is denoted as the HAR model. The second specification adds valuation shocks, denoted as the HAR-GSR model. The third specification adds a column vector $\boldsymbol{C}_t$ that includes a set of control variables, the HAR-GSR-C. This third, most general, specification that leads to our main results is given by

$$\begin{aligned}\ln RV_{t,H} = {} & \boldsymbol{\beta}_H^{\mathrm{T}}\boldsymbol{RM}_t + \boldsymbol{\delta}_H^{\mathrm{T}}\boldsymbol{C}_t + \\ & \underbrace{\gamma_{1,H}GSR_t^{\perp} + \gamma_{2,H}VP_t}_{\text{valuation}} + \underbrace{\gamma_{3,H}V_t^{\perp} + \gamma_{4,H}RP_t}_{\text{portfolio}} + \underbrace{\gamma_{5,H}A_t^{\perp} + \gamma_{6,H}AP_t}_{\text{attention}} + \epsilon_{t,H},\end{aligned} \tag{15}$$

with $\epsilon_{t,H}$ the usual error term and with the parameters stacked in vectors $\boldsymbol{\beta}_H$ and $\boldsymbol{\delta}_H$. The Eq. (15) is based on logs if possible, which is explicitly reported in Tables 3 and 4. We estimate the model in Eq. (15) via OLS for $H = 1, 2, ..., 66$ and the resulting estimates of $\hat{\gamma}$ coefficients are the basis for the impulse response functions (IRF). In the IRF plots, we report standardized effects that correspond to the percentage change in variance $RV_{t,H}$ given a one-standard deviation increase in the regressor, i.e. $100[\%]\left(e^{\hat{\gamma}_{k,H}\times\sigma_k} - 1\right)$. The LP framework is applied for gold and silver volatility models separately and the standardization allows a direct

[17] Refer to Bates and Granger [1969], Granger and Ramanathan [1984], Timmermann [2006] for seminal studies in the area of forecast combinations.

comparison of the effects for the gold and silver volatility models.

The response of gold or silver variance to a relative appreciation of gold, $PS_t = I\left(GSR_t^{\perp} > 0\right)$, might be different from depreciation of gold, $NS_t = I\left(GSR_t^{\perp} \leq 0\right)$, where $I(.)$ is an indicator function. The resulting specification is given by

$$\begin{aligned}\ln RV_{t,H} = \boldsymbol{\beta}_H^{\mathrm{T}} \boldsymbol{RM}_t + \boldsymbol{\delta}_H^{\mathrm{T}} \boldsymbol{C}_t + \\ \underbrace{PS_t \times \left(\gamma_{1,H} GSR_t^{\perp} + \gamma_{2,H} V_t^{\perp} + \gamma_{3,H} A_t^{\perp}\right)}_{\text{positive valuation shock}} + \\ \underbrace{NS_t \times \left(\eta_{1,H} GSR_t^{\perp} + \eta_{2,H} V_t^{\perp} + \eta_{3,H} A_t^{\perp}\right)}_{\text{negative valuation shock}} + \epsilon_{t,H}.\end{aligned} \tag{16}$$

Allowing for different slopes in the shock led to high-correlation of shocks with the valuation, portfolio and attention pressure variables, which are thus omitted in this specification.

While the previous specification traces short-term effects of relatively more or less expensive gold, we might compare the volatility response for longer periods, where gold was relatively more expensive as silver. We identify such periods when the expected gold-silver ratio, $\hat{F}_t^{GSR}$ was above the median $\tilde{\hat{F}}_t^{GSR}$, so the period of relatively more expensive gold is denoted as $GE_t = I\left(\hat{F}_t^{GSR} > \tilde{\hat{F}}_t^{GSR}\right)$ and a relatively more expensive silver period as $SE_t = I\left(\hat{F}_t^{GSR} \leq \tilde{\hat{F}}_t^{GSR}\right)$. Within our sample, silver tends to be relatively more expensive in the periods that reside mostly in the first-half of the sample, although not exclusively. The resulting specification is

$$\begin{aligned}\ln RV_{t,H} = \underbrace{GE_t \times \left(\gamma_{1,H} GSR_t^{\perp} + \gamma_{2,H} VP_t + \gamma_{3,H} V_t^{\perp} + \gamma_{4,H} RP_t + \gamma_{5,H} A_t^{\perp} + \gamma_{6,H} AP_t\right)}_{\text{Gold relatively more expensive}} + \\ \underbrace{SE_t \times \left(\eta_{1,H} GSR_t^{\perp} + \eta_{2,H} VP_t + \eta_{3,H} V_t^{\perp} + \eta_{4,H} RP_t + \eta_{5,H} A_t^{\perp} + \eta_{6,H} AP_t\right)}_{\text{Silver relatively more expensive}} + \\ \boldsymbol{\beta}_H^{\mathrm{T}} \boldsymbol{RM}_t + \boldsymbol{\delta}_H^{\mathrm{T}} \boldsymbol{C}_t + \epsilon_{t,H}.\end{aligned} \tag{17}$$

4.3. Out-of-sample forecasting models and evaluation

The in-sample analysis is complemented by a forecasting study in which we investigate whether incorporating the valuation shock and its square (unscaled valuation pressure) can improve volatility forecasts. In the main results, we report results for ten forecasting models: the vanilla HAR model of Corsi [2009] and nine augmented HAR models that allow us to estimate the importance of the valuation, re-balancing, and attention pressures for forecasting purposes. The benchmark HAR model that predicts variances is given by:

$$RV_{t,H} = \beta_{0,H} + \beta_{1,H}RV_t^D + \beta_{2,H}RV_t^W + \beta_{3,H}RV_t^M + \epsilon_{t,H}. \tag{18}$$

Next, we use six models that are augmented with one extra variable,

$$RV_{t,H} = \beta_{0,H} + \beta_{1,H}RV_t^D + \beta_{2,H}RV_t^W + \beta_{3,H}RV_t^M + \beta_{4,H}X_t + \epsilon_{t,H}, \tag{19}$$

corresponding to $\left(GSR_t^{\perp}\right)^2$, $V_t^{\perp} \times \left(GSR_t^{\perp}\right)^2$ or $A_{t-1}^{\perp} \times \left(GSR_t^{\perp}\right)^2$, which are the unscaled (no logs) and not demeaned valuation, re-balancing and attention pressures, respectively, and to QS_t, VIX_t, or GVZ_t, where the quoted spread (e.g. Stašek and Lyócsa 2024) and implied volatility indices (e.g. Bekaert and Hoerova 2014) were selected because they are known to improve volatility forecasts for other assets. Note that attention is observed only at the end of day t. Therefore, in contrast to the in-sample study, we use $t-1$ for the out-of-sample part. As GVZ_t proved to be the most relevant variable, while VIX_t was less useful for gold and silver volatility forecasts, we also estimated the following specification

$$RV_{t,H} = \beta_{0,H} + \beta_{1,H}RV_t^D + \beta_{2,H}RV_t^W + \beta_{3,H}RV_t^M + \beta_{4,H}GVZ_t + \beta_{5,H}X_t + \epsilon_{t,H}, \tag{20}$$

where the extra variable X_t corresponds to either $\left(GSR_t^{\perp}\right)^2$, $V_t^{\perp} \times \left(GSR_t^{\perp}\right)^2$, or $A_{t-1}^{\perp} \times \left(GSR_t^{\perp}\right)^2$ in order to assess whether any of the valuation pressures can further improve volatility forecasts.

The resulting models allow us to identify: i) the relative importance of the valuation, re-balancing, and attention pressures for forecasting purposes, and ii) whether any of the variables can lead to forecast improvements beyond those provided by gold's implied volatility index. We estimate all HAR models using a WLS estimator with weights given by $1/RV_{t,H}$, following one of the recommended estimators from Clements and Preve [2021], as this mitigates the impact of extremes on the estimated parameters. Models are estimated using a rolling window of 756 observations (approx. 3 years) for forecasting horizons $H = 1, 2, ..., 66$.

To evaluate the accuracy of forecasts[18] from model m, $F_{t,H}^m$, we use two loss functions that are known to lead to a consistent ranking of forecasting models even in the presence of a noisy volatility proxy, as shown by Patton [2011]: the mean squared error (MSE) and the

[18] As volatility model can produce implausible forecasts, we follow the existing volatility literature (e.g. Patton and Sheppard [2015], Bollerslev et al. [2016]) and use a sanity check. In our case, if the forecast at time t lies outside of the range off $RV_{r,H}, r = 1, 2, ..., t-H$, we winsorize the forecast to the closest observed extreme.Bollerslev et al. [2016] winsorized with estimation-window mean variance instead.

QLIKE,

$$MSE_H^m = (T - T_F + 1)^{-1} \sum_{t=T_F}^{T} \left(F_{t,H}^m - RV_{t,H}\right)^2 \tag{21}$$

and

$$QLIKE_H^m = (T - T_F + 1)^{-1} \sum_{t=T_F}^{T} \left(\frac{RV_{t,H}}{F_{t,H}^m} - \ln \frac{RV_{t,H}}{F_{t,H}^m} - 1\right), \tag{22}$$

respectively, where T_F corresponds to $2 \times 756 + 1$, as the first 756 observations are used to estimate the first shocks (e.g. $GSR_t^{\perp}$), and the following 756 observations are used to estimate the first volatility models, so the first forecast is for the 1513^{th} observation (10 October 2015). In the figures, we report forecasting improvements in percentage terms and whether a given model belongs to the set of superior models as indicated by the Diebold and Mariano [1995] test applied within the stepwise framework of Romano and Wolf [2005]. We use moving-block bootstrap with a block length of h and 999 replications.

5. Results

5.1. Data characteristics

Characteristics of the key variables of interest and the controls can be found in Table 1. Both gold and silver exhibit positive average and median daily returns mirroring the overall increase in the nominal value of both precious metals (see also Figure 1). The dynamics is different, with the daily realized variance of silver more than three times higher than that of gold. Stylized facts commonly observed in financial market volatility such as positive skewness, pronounced excess kurtosis, and high persistence are confirmed, with significant autocorrelation (see column EL in Table 1) and considerable autocorrelations of 0.05 (gold) and 0.09 (silver) even at the 66^{th} lag. The quoted spread is on average lower for gold. This characteristic of the market is in line with the ideas of Copeland and Galai [1983], Glosten and Milgrom [1985] that spreads represent the price of a straddle option strategy as market makers post quotes (and hold inventory). Thus, we should observe lower quoted spread on the market with lower volatility, which is the case between gold and silver futures markets.

Figure 1 illustrates the historical dynamics of prices, returns, and daily variances. Both metals experienced sharp price increases in 2011. Gold rose steadily from approximately \$800 in 2009 to a peak of \$1,900 in 2011, driven by concerns over inflation, global financial instability, and expansionary monetary policy. In contrast, speculative activity in the comparatively smaller silver market amplified price movements, propelling silver to nearly \$50 per ounce in April 2011, its highest level since 1980. These developments are mirrored in the spikes observed in daily variance measures, with silver exhibiting notably higher variance during this period.

Following the onset of the pandemic in 2020, both price series trended upward, with gold appreciating more sharply due to its role as a safe haven asset. Silver followed a similar trajectory, albeit at a slower pace, gradually catching up over time.

Insert Table 1 around here

Insert Figure 1 around here

Compared to volatility time series, characteristics of the relative valuation ratios are not that well understood. We observe that over our sample period, the GSR has an average value of 73, with substantial variation ranging from 31.72 to 124.36 (Table 1), which is driven by a long-term trend. This is visible not only from the Figure 2, but also from autocorrelations which show considerable persistence[19], thus relative value between gold and silver is stable in the short-run, consistent with the findings of Baur et al. [2020]. The upward trend in the GSR indicates that within our sample, gold has become relatively more expensive compared to silver.

Given the higher level of persistence of the GSR, the predicted (expected) values (see Figure 2) track the GSR closely. The valuation shock equivalent to the forecast errors to the gold-silver ratio (denoted as $GSR^{\perp}$), along with the squared shock, $(GSR^{\perp})^2$, are characterized by pronounced kurtosis, reflecting the presence of extreme values and heavy tails in the distribution (see Table 1).[20] The valuation shock, $GSR^{\perp}$, and its square, $(GSR^{\perp})^2$, visualized in Figure 2 show that particularly during periods of higher market turbulence (the pandemic period and the trade tensions associated with the U.S. tariff policies in 2025) the deviation between the observed GSR from its predicted values increases. These episodes are marked by pronounced volatility captured in the magnitude of the estimated shocks. Such periods might lead to valuation, re-balancing and/or attention related shocks that can lead market variances. If so, the fact that the valuation pressure and its magnitude display first-order autocorrelations of 0.55 and 0.35 and after twenty-two days still 0.17 and 0.08, respectively, suggests that these shocks may have an impact on gold and silver volatility over horizons extending beyond a few days.

Figure 3 plots shocks to trading volume for each metal, along with shocks to the investor attention to gold and silver series. Volume shocks for both gold and silver exhibit broadly similar dynamics and, aside from a small number of outliers, display a relatively stable and regular

[19] We reject the the null of a unit-root test of Elliott et al. [1996] for all time series with constant or, in the case of GSR, a trend (see the note below Table 1).

[20] Note that the sum of the average predicted gold-silver ratio and the corresponding shocks ($GSR^{\perp}$) would not equal the average observed GSR over the full sample period. This discrepancy arises because in Table 1, the GSR values are computed over the entire sample period, spanning from 2 September 2009 to 18 June 2025, whereas the estimation of shocks requires a rolling window approach. Specifically, the first three years of data are used to initialize the rolling prediction procedure, which necessarily excludes this initial segment from the shock estimation.

pattern over time. In contrast, attention shocks are notably more episodic. Pronounced spikes are observed around the onset and immediate aftermath of the pandemic, as well as in early 2025. These periods coincide with increased investor focus on precious metals, consistent with their role as safe haven assets. From Table 1, we also observe that the first-order autocorrelations of the shocks are much lower than that of the level series, suggesting that the forecasting models were able to capture considerable part of the predictable components of the volume and attention. In case of attention, the first-order autocorrelation went down from 0.95 to 0.41, while for both gold and silver trading volume it falls from the 0.54-0.58 range to below 0.10 for excess volume.

Insert Figure 2 around here

Insert Figure 3 around here

The upper diagonal of Table 2 reports the correlations with significances highlighted below the diagonal. Daily returns and variances of gold and silver futures exhibit a strong positive correlation of 0.77 for returns and 0.69 for variances, implying close co-movement of these precious metal markets. Similarly, the liquidity measure (quoted spread) and various control variables are significantly correlated with variances, suggesting their potential relevance in a volatility modeling framework. Most correlations are positive, which is consistent with expectations, except for the News Sentiment Index NSI. This negative association is intuitive: higher sentiment reflects greater optimism, which corresponds to a lower perceived level of risk. Among the control variables, the geopolitical risk proxy displays the weakest correlations with the other variables, in line with previous findings of (Lyócsa and Todorova, 2024), which is also expected given the exogenous natures of such risks. Within the set of control variables, the strongest correlations are observed between NSI and $SRISK$ (0.89), between EPU and EMU (0.63), and between VIX and NSI (-0.60). While these figures indicate that certain risk measures share overlapping informational content, the magnitudes of the correlations are not sufficiently high to raise concerns about multicollinearity in the in-sample models.

Interestingly, while the GSR is correlated with several market risk measures, with the strongest links to market illiquidity ILQ and to systemic risk $SRISK$, the valuation shock $GSR^{\perp}$ is less correlated with market risk measures (D1-D8 in Table 2), but appears more correlated with gold and silver price characteristics. The magnitude of the shocks, i.e. the squared shocks $(GSR^{\perp})^2$ exhibit mostly positive and significant association with controls and also with the contemporaneous realized variance levels of gold and silver. It shows that the shocks to the GSR are related to global and metal specific risk factors. The correlation of GSR with the VIX and GVZ was only -0.22 and 0.15 but $(GSR^{\perp})^2$ with VIX and GVZ is 0.30 and

0.35 suggesting that valuation-deviations are at least to some extent related to expected market uncertainty. Whether this reflects only common underlying drivers of precious metal markets or a more unique relationship arising from relative pricing dynamics will be investigated in the subsequent analysis.

Insert Table 2 around here

5.2. In-sample analysis

Tables 3 and 4 report the numerical results of HAR models for gold and silver volatility, for selected forecast horizons (daily $h = 1$, weekly $h = 5$, monthly $h = 22$ and quarterly $h = 66$). To assess the relevance of GSR for volatility modeling, we estimate three model specifications: (i) a baseline HAR model including only own historical volatility components, (ii) an extended model augmented with GSR components considering the three potential transmission channels as outlined in Section 2, and (iii) a full model that also incorporates a set of control variables. The third specification is also used to estimate impulse responses within the local projection framework. The sample period goes from 19 August 2012 to 18 June 2025, where the starting date corresponds to the 757^{th} observation in the sample (see Section 4.1) as it is the first out-of-sample shock, which ensures no look-ahead bias in the in-sample models.

The results for the baseline model confirm the pervasive importance of the assets' own historical volatility components. For day-ahead volatility, the most relevant predictor is the previous week's volatility. As the predictive horizon lengthens, the influence of recent volatility weakens, while longer-horizon historical volatilities gain explanatory power for both gold and silver. Enhancing the baseline model with GSR components hints to our first empirical results, that information provided by valuation ratio, the GSR, might be more relevant for silver than for gold. The explanatory power of the baseline model increases by 0.6% ($h = 5$) to 1.2% ($h = 22, 66$) percentage points in R^2_{Adj} for gold, while for silver it is from 1.1% ($h = 5, 22$) to 1.7% ($h = 66$) for silver. The model fit comparison shows, that while shocks to the valuation might be significant drivers of future volatility, market risk measures are even more important. In the following, we interpret results from the full model with all control variables.

Adding market risk proxies and further control variables improves the fit of models for both metals and across all forecasting horizons. Among the wider set of market risk factors, only few are significant and across multiple horizons for both gold and silver only implied volatility for gold market stands out. For short-term $h = 1, 5, 22$ predictions, a 1% increase in implied volatility on the gold market led to a 0.70% ($h = 66$) to 1.83% ($h = 5$) response in gold's variance and 0.63% ($h = 66$) to 1.29% ($h = 5$) response in silver's variance. Surprisingly, VIX had a positive and significant coefficient only for silver and only for selected horizons. The metal-specific attention was also included in the model in order to better isolate the effect of the attention towards both gold and silver (at the same time) and it proved to be significant for gold and silver in the short-term. We also find asymmetric volatility spillover effects, as interestingly, higher volatility in gold's metal tends to be associated with lower future silver's volatility (but not vice versa), despite the positive contemporaneous correlation between the realized volatilities of gold and silver (Table 2). The magnitude of the effect gradually declines as the forecasting horizon increases.

5.2.1. Valuation channel

Panels B1-B3 of Tables 3 and 4 report the estimated coefficients for the valuation, re-balancing, and attention channels. The valuation shock is insignificant at all horizons for both gold and silver; thus, the direction of the shock is not indicative of future price variation itself. However, the valuation pressure, which represents the magnitude of the shock to the GSR, is significant and positive at all horizons. For gold's variance, the coefficients imply that doubling the magnitude of the valuation shock leads to a 1.31% ($h = 66$) to 1.99% ($h = 1$) increase in gold's variance.[21] For silver, valuation pressure has an even larger effect, ranging from 1.95% ($h = 66$) to 2.95% ($h = 1$). The effects are generally larger in the short term, as one would expect.

In Figure 4, we display the impulse responses from the HAR-GSR-C model, based on the local projection analysis outlined in Section 4.2. The x-axis corresponds to the forecast horizon, while the y-axis corresponds to a one-standard-deviation response of variance in percentage changes. The effect sizes are thus comparable across various responses. The first two rows show volatility responses to valuation shocks and valuation pressure. We observe that valuation pressure is positive and statistically significant across all forecasting horizons and is also stronger for silver than for gold. The long-term impact suggests that the valuation deviation will eventually lead to higher uncertainty, even after controlling for metal- and market-specific risk factors under the HAR-GSR-C model.

5.2.2. Re-balancing channel

Panel B2 of Tables 3 and 4 reports the coefficients related to the re-balancing channel. The individual volume shock ($V_t^{\perp}$), where a positive value corresponds to an unexpected increase in daily trading volume, is the control, while a positive re-balancing pressure variable (RP_t)

[21] Given that VP_t is in the log of the squared shock (demeaning does not change the following calculations), doubling the magnitude of the shock $GSR_t^{\perp}$ (irrespective of its sign) increases VP_t by $\ln\left[4\left(GSR_t^{\perp}\right)^2\right] - \ln\left[\left(GSR_t^{\perp}\right)^2\right] = 2\ln 2 \approx 1.386$. Multiplying by the coefficient of 0.0094 gives 0.01303 and, since the dependent variable is the log of the variance, the implied effect is $100 \times \left(e^{0.01303} - 1\right) \approx 1.31\%$.

itself corresponds to a positive volume shock coinciding with a positive valuation pressure. As expected, the volume shock coefficient is positive for both gold and silver, but significant only for $h = 1$ for gold, with a one-standard-deviation increase in excess volume leading to a 4.77% ($h = 1$) increase in gold variance. For silver, excess volume shows a stronger effect, ranging from a 3.12% ($h = 5$) to a 6.67% ($h = 66$) increase in silver variance. The re-balancing pressure exhibits negative coefficients across all forecast horizons, with stronger effects in the short term, but under the HAR-GSR-C model the effect for gold is significant only in the short term. The size of the re-balancing effect is weaker than that of the valuation pressure, as one standard deviation change in the re-balancing pressure decreases gold volatility by 1.07% for $h = 1$, while for silver the significant effects (at least at the 10% significance level) range from -0.88% ($h = 22$) to -2.56% ($h = 1$). The documented negative coefficients are in line with the re-balancing interpretation, as a larger positive (negative) shock increases (decreases) the portfolio weight of gold (silver), which should lead to a trade against price momentum and thus decrease price volatility. The impulse responses in Figure 4 show the standardized effects across the forecasting horizons. The effects peak in the short term but persist only for silver.

5.2.3. Attention channel

Finally, in Panel B3 of Tables 3 and 4, the coefficients show that an unexpected increase in attention to gold and silver tends to be followed by higher gold and silver variance. The attention-driven volatility effect is stronger for gold, likely because it is a metal that remains consistently in the public spotlight.[22] A one-standard-deviation increase in attention leads to an increase in gold variance ranging from 1.65% ($h = 1$) to 2.41% ($h = 22$), while for silver, these effects are significant only for $h = 5, 22$, at 1.57% and 1.85%, respectively. The impulse responses in Figure 4 confirm that gold volatility appears to be, at least to some extent, driven by excess attention, while in the case of silver, the effects are smaller and significant only in a subset of shorter horizons. With respect to attention pressure, while we observe the expected positive coefficient across forecasting horizons and both metals, the effects are mostly not statistically significant. These results suggest weak evidence for the attention channel.

To sum up, shocks to the GSR are relevant drivers of both metals' variances in the short and long run. However, valuation shocks, the re-balancing channel, and the attention channel have different roles. The larger the shock to the ratio, the larger the volatility in the subsequent periods, which is mitigated by re-balancing pressure. The mechanism through attention has not been confirmed, as only excess attention alone is significantly associated with future levels

[22]In a sample from August 2012 to June 2025, the worldwide monthly Google search volumes are on average about 4.9 times higher for 'gold price' than for 'silver price' keywords.

of volatility, but not if it coincides with valuation deviations represented by valuation pressure.

Insert Table 3 around here

Insert Table 4 around here

Insert Figure 4 around here

5.3. Are the effects of relative valuation shocks state-dependent?

To evaluate whether the effects stem from unexpectedly high gold prices, we disentangle the valuation shocks. First, we separate the shock to the GSR into its positive part, when gold is relatively more expensive as silver, and into its negative part, as shown in Eq. (16). Second, we separate the shocks into those appearing during a period, when the expected gold-silver ratio, $\hat{F}_t^{GSR}$ was above or below the median $\tilde{\hat{F}}_t^{GSR}$, as shown in Eq. (17). Both decompositions look at asymmetric effects, but in the first case we investigate a short-term asymmetric effect in the valuation shocks, while the second aims at capturing long-term, price-regime dependent asymmetric effects.

5.3.1. Short-term effect: the sign of the shocks

The corresponding results based on the model in Eq. (16), where we are able to observe whether the relative appreciation of gold (blue color – positive valuation shock) has a distinct effect from the relative appreciation of silver (red color – negative valuation shock). The corresponding IRFs are shown in Figure 5. The signed shocks are now significant for both metals and across almost all horizons.

The IRFs shown in Figure 5 show the percentage effect of a one-standard-deviation change in the shocks. We observe that when gold becomes relatively more expensive (positive shocks), both gold and silver are followed by higher volatility. On the other hand, as silver becomes relatively more expensive (negative shocks), volatility in both markets tends to decrease (the positive coefficient is multiplied by a negative shock). The positive effect is, however, stronger, at least over horizons ranging from several days up to a month and particularly for the silver market. These findings imply that relative valuation shocks are important drivers of volatility, but it is the appreciation of gold that leads to stronger market reactions in both markets.

In our main results, volume shocks were found to lead silver volatility in both the short and long term, and we observe a similar relationship in this setting. We also find that these results are driven by both positive and negative valuation shocks. The shock decomposition does not reveal any asymmetries with respect to volume shocks and thus the re-balancing channel. Applying positive and negative valuation signs to the attention shocks also leaves the main results largely unchanged. The results show a positive association with future volatility, regardless of whether the concurrent relative valuation shock is positive or negative, but it is not significant.

Insert Figure 5 around here

5.3.2. Long-term effect: pricing regimes

Next, we examine volatility responses depending on the price regime, i.e., when the predicted GSR lies above or below the median of the GSR forecasts. As such regimes are persistent, we might consider them long-term effects. The corresponding results are shown in Figure 6, with the model in Eq. (17) being the same as in the main results but with regime-specific coefficients. The associated valuation pressure is substantially stronger in absolute terms during periods of relatively expensive gold. By comparison, valuation effects when silver is relatively more expensive are dampened, albeit still statistically significant for silver, but not for gold, which exhibits fewer statistically significant volatility responses overall, with effects concentrated primarily at short-term horizons. These results are consistent with the short-term effects found in the previous subsection, where it was the appreciation of gold, and here periods of relatively more expensive gold, that drove our main results. Therefore, relative gold appreciation increases volatility in both markets, while relative silver appreciation tends to decrease volatility in the long run.

Trading volume shocks exhibit stronger effects when silver is relatively more expensive than gold, with a positive impact on future volatility. The re-balancing pressure, one of the potential transmission mechanisms, shows that the decrease in variance associated with re-balancing pressure is stronger during periods when gold is relatively more expensive. Once again, the underlying dynamics of the gold market seem to be the main force driving our main results.

Finally, looking at the attention channel, we first observe that gold volatility is more sensitive to attention shocks when silver is relatively more expensive. Attention pressure is significant only for silver, similarly to our main results, but is again driven by periods of relatively more expensive gold prices.

Overall, the results indicate that all three transmission channels – valuation, re-balancing, and attention pressure – are amplified when gold is relatively more expensive, either on a day-by-day basis (signed shocks) or over a potentially longer time period (GSR above the median). The relative mispricing, as indicated by shocks in the gold-silver ratio, is potentially triggering stronger volatility responses when gold is 'expensive'.

Insert Figure 6 around here

5.4. Out-of-sample analysis

In this section, we discuss whether relative GSR shocks contain incremental information that could be utilized in an out-of-sample setting. Numerical results for the one-day, one-week, one-month, and one-quarter forecast horizons are presented in Table 5. Figure 7 provides a comprehensive overview by plotting the *percentage* improvements (in %) in forecasting accuracy for both considered loss functions (MSE and QLIKE) across all horizons up to 66 days against the benchmark vanilla HAR model. Positive values indicate improvements relative to the benchmark, whereas negative values reflect the extent to which a given model underperforms compared to the plain HAR benchmark model. A dot on the curve indicates that the given model belongs to a set of superior models. The results can be discussed in two groups: models augmented by only one additional variable (6 models) and models containing the implied volatility of gold along with other variables (3 models).

5.4.1. HAR models augmented by a single variable

Across both assets, the HAR models augmented with $\left(GSR_t^{\perp}\right)^2$ (light-blue), $V_t^{\perp} \times \left(GSR_t^{\perp}\right)^2$ (light-orange), or $A_{t-1}^{\perp} \times \left(GSR_t^{\perp}\right)^2$ (green) yield limited forecast improvements across both assets. Only valuation pressure leads to around 2% improvements against the benchmark HAR model in MSE for gold and silver in the short term,. How good are these improvements compared to known drivers of volatility? Among other considered explanatory variables, including the quoted spread (brown), the VIX (dark grey), and the GVZ (dark blue), the implied volatility of gold emerges as the most useful single variable. The model incorporating the quoted spread provided no improvement, while VIX led to similar short-term forecast improvements. However, the implied volatility from the gold market, GVZ, led to forecast improvements for both gold and silver that ranged from 1% to almost 8%, depending on the forecast horizon and metal. Under QLIKE, the statistically significant forecast improvements are around 10% to 15% for gold and go above 20% for silver. These are non-trivial forecast improvements. The HAR+GVZ model ranks among the best-performing models overall and delivers remarkably consistent forecast gains across all horizons, loss functions, and both metals.

These are new findings alone, as gold implied volatility decisively outperforms the VIX, which is commonly regarded as the canonical 'fear gauge' of financial markets. While the strong performance of the GVZ is unsurprising given its direct focus on gold volatility, its results further demonstrate the relevance of market-specific implied volatility measures. Next, we also find that forecast improvements persist over substantially longer horizons for silver than for gold and are economically much larger for silver in terms of QLIKE. This is striking given that the dominant predictor is gold, rather than silver, implied volatility. Finally, only the magnitude of the shock leads to minor forecast improvements.

5.4.2. Models augmented by multiple variables

Next, we examine whether a combination of more variables can outperform the HAR model that uses only the GVZ variable. To preserve parsimony, the number of additional variables is restricted to no more than three. Motivated by the superior performance of the GVZ, we combine it with valuation pressure (dark orange), rebalancing pressure (purple), and attention pressure (light grey). Owing to the strong informational content of the GVZ, all three multivariate specifications perform similarly or better for both metals across most forecast horizons and loss functions. Notably, the models that combine gold implied volatility with valuation pressure (dark orange) emerge as the best-performing specifications among all models considered under the QLIKE loss, while under the MSE, even better performance is offered by combining HAR+GVZ and re-balancing pressure.

Compared to the HAR+GVZ model, these forecast improvements might not be trivial and are stronger for silver. For example, for $h = 5$ and gold volatility, HAR+GVZ achieves a forecast improvement of 6.38% (see Panel A of Table 5) and HAR+GVZ+Val. pressure achieves a forecast improvement of 8.32%. Under QLIKE, the forecast improvements are 14.02% for HAR+GVZ and 15.60% for HAR+GVZ+Val. pressure, with the later being significantly better as HAR+GVZ with the Diebold and Mariano [1995] test[23]. For silver, such forecast improvements are even stronger. These results thus show that the magnitude of the shock to the relative value of gold and silver does contain information that leads to improved volatility forecasts.

5.4.3. Out-of-sample results across all models

In Appendix D, we report out-of-sample results for $h = 1, 5, 22, 66$ day-ahead forecasts from additional models, all based on the baseline HAR specification augmented with additional variables: COVOL, EPU, US EMU, Illiquidity, NSI, SRISK, attention to a given metal, realized variance of the other metal, as well as combinations of the HAR+GVZ model that performed well with those variables. For both gold (Table D1) and silver (Table D2), we find that for one-day and five-day-ahead forecasts, regardless of the loss employed, the HAR and HAR-GVZ models augmented with valuation pressure are almost always among the top performers. For longer forecasting horizons, the model still performs well, showing that valuation deviations provide incremental information for managing risky positions in the gold and silver futures

[23]Pairwise results are available upon request.

markets.

5.5. Low-frequency volatility models: IRF and volatility forecasts

Previous analyses were based on volatility proxies derived from high-frequency data, which are known for their higher efficiency (e.g. Andersen et al., 2001, Molnár, 2012). However, such data are not always available due to higher costs. In such cases, a common approach is to use range-based volatility estimators, which rely on up to four daily price observations (open, high, low, and close) that are typically publicly accessible. Range-based estimators are widely recognized as a better alternative to the squared daily returns and represent an alternative for estimating daily variance in the absence of high-frequency data.

We repeat the main in-sample model analysis and out-of-sample study but now using range-based estimators. We expect, that given the lower efficiency of the range-based estimator, the in-sample effects should be smaller. However, we anticipated that valuation ratios would provide greater benefit in the out-of-sample study, as range-based models are not that efficient volatility estimators, which could make the supplementary information about pricing dynamics more relevant. The range-based analysis also serves as a robustness check of the main results reported above.

In our models, we use the range-based estimator of Garman and Klass [1980]. Let $h_t = 100 \times \left(\ln\left(\max_{\delta \in \{1,2,\ldots,K\}} H_{t,\delta}\right) - \ln P_{t,0}\right)$, $l_t = 100 \times \left(\ln\left(\min_{\delta \in \{1,2,\ldots,K\}} L_{t,\delta}\right) - \ln P_{t,0}\right)$ and $c_t = 100 \times (\ln P_{t,N} - \ln P_{t,0})$ denote scaled maximum, minimum and closing price (intraday-return). The range-based estimator for the variance on day t is given by

$$GK_t = 0.511(h_t - l_t)^2 - 0.019(c_t(h_t + l_t) - 2h_t l_t) - 0.383c_t^2, \tag{23}$$

and the multi-day estimator is

$$RB_{t,H} = H^{-1} \sum_{h=1}^{H} \left[\underbrace{\left[100 \times \ln \frac{P_{t+h+1,0}}{P_{t+h,N}}\right]^2}_{\text{non-trading period variation}} + \underbrace{GK_{t+h}}_{\text{trading period variation}} \right]. \tag{24}$$

We estimate the IRF using the same specifications as in Eq. (15) but instead of $RV_{t,H}$ we predict $RB_{t,H}$ and use lags of RB. Figure 8 presents the corresponding IRF.[24] The findings closely mirror those obtained when daily price variation was estimated using realized volatility (Figure 4). In particular, valuation shocks is insignificant for both metals, while valuation pressure is positive and significant, with stronger effects for silver, especially at short horizons.

[24]Numerical results for models predicting range-based volatility models are available upon request.

Volume shocks exhibit greater persistence and economic relevance for silver, whereas attention effects are more pronounced for gold only in shock form. Re-balancing pressure enters negatively for both metals, indicating a stabilizing role, but significant in the short-term and for silver only. Finally, the attention pressure is a factor is not significant. The results confirm that GSR-based shocks convey distinct information through valuation pressure transmission channel and to lesser extent through the rebalacing channel. Compared to the models that used realized variances, the effects of the valuation and re-balancing pressure are however smaller.

Insert Figure 8 around here

In the out-of-sample setting, we estimate HAR class models, but instead of realized variances, we use range-based estimates, which in this setup is also a proxy of the true variance. Numerical results are presented in Table 6 and can be directly compared against results from Table 5 for high-frequency models. Qualitatively, the results are similar in that we again find that individually only the magnitude of the valuation shock helps improving volatility forecasts (see Figure 9), especially in the short-run. The implied volatility from the gold market is the single most important predictor, similarly as before, but now, adding the magnitude of the valuation shock (orange) consistently out-performs that model and reaches up to 15% improvements under MSE for gold (up from 10%) and around 10% for silver. Forecast improvements under QLIKE are even larger, reaching above 30% for gold and silver alike. We also observe a pattern, where for gold, forecast improvements tend to decrease with the forecast horizon, after around 10 days, while for silver forecast improvements are more stable over time.

Insert Table 6 around here

Insert Figure 9 around here

5.6. Alternative volatility model specifications

We have re-run impulse response analysis using alternative volatility model specifications, where instead of the lagged volatilities we used intraday estimates of the continuous and jump components following Andersen et al. [2012] and corrections of Kolokolov and Renò [2024] for price staleness. Description of the estimates and the impulse responses can be found in Appendix B. In Appendix C, we also present impulse response functions calculated from a volatility model, where day-ahead volatility is predicted by positive and negative semi-variances following the work of Patton and Sheppard [2015]. In both instances, the estimated IRFs exhibit only minor differences relative to our main results, which implies that those results are robust to alternative approaches for capturing volatility persistence.

6. Conclusion

In this study, we argue that price ratios may provide relevant information about the relative value of traded assets and should therefore be indicative of future price movements; an idea that can be explained within both rational and behavioral asset-pricing models. Given a ratio between two prices, we propose three transmission mechanisms. The first is motivated by the co-integration literature, where deviations from the price ratio are interpreted as instantaneous misalignment between prices, i.e. the *valuation shock*. The square of this shock represents the magnitude of the misalignment and is referred to as the *valuation pressure*. The stronger the pressure, the greater the price adjustment that should occur in one or both assets in subsequent periods. The second mechanism is motivated by portfolio theory, where deviations from the price ratio correspond to a signal of non-optimal portfolio weights. We define *re-balancing pressure* as the interaction between the shock in trading volume, the *volume shock*, and valuation pressure, as this indicates that a shift in the ratio is also associated with unexpected trading volume. The third mechanism is motivated by the attention literature. A change in the price ratio may lead to an unexpected increase in investor interest in both assets, i.e. the *attention shock*. We define *attention pressure* as the interaction between such an attention shock and valuation pressure.

Using almost 16 years of intraday data on gold and silver futures prices, we test these mechanisms in an empirical framework by forecasting the future level of price volatility, measured by realized variance. Relative prices are represented by the gold-silver ratio. We use machine-learning models to estimate valuation, volume, and attention shocks, which represent the unpredictable components of changes in the GSR, trading volume, and investor attention. Using local projection models estimated with a broad set of market-risk controls, we provide evidence that valuation pressure leads to higher variance across all forecast horizons, from one day ahead to an increase in average variance over the subsequent quarter. The effect is stronger for silver and is driven mostly by gold appreciating relative to silver. For both metals, re-balancing pressure acts in the opposite direction by dampening variance and is significant mostly for silver. This result suggests that re-balancing may work against price momentum and thus lead to a decrease in variance. For example, if one asset becomes unexpectedly too expensive, its weight in the portfolio increases and, to reduce exposure to that asset, investors sell a fraction of it. Furthermore, we find evidence for the attention pressure in the main specifications only for silver, but the effect size is small. More detailed results show that the effects of the valuation, re-balancing, and attention pressures are more pronounced during periods when gold was relatively more expensive than silver (GSR above median).

Motivated by the in-sample results, we test whether valuation, re-balacing or attention pressure provide incremental information that could lead to more accurate volatility forecasts. We find that, on its own, only valuation pressure consistently improves the forecasting accuracy of the benchmark HAR model. The forecast improvements are minor compared with models that instead use a single market risk variable, such as the gold implied volatility index. However, adding valuation pressure to a model that includes the gold implied volatility index further increases forecast improvements. The magnitude of deviations in the gold-silver ratio is therefore useful in an out-of-sample volatility forecasting framework.

To account for cases when intraday data might not be available to market participants, we also re-estimate our models using low-frequency, range-based estimators of daily variance and find similar in-sample results, with a slightly weaker effect, while out-of-sample forecasting accuracy increases. The results show that if high-frequency data are not available, information from the price ratio may improve volatility forecasts even more.

Our analysis shows that the usefulness of the gold-silver ratio extends beyond its role as a descriptive market indicator, as it also provides predictive power, both in-sample and out-of-sample, for the future volatility of gold and silver. This suggests that the relative pricing dynamics between the two metals contains information about risk perceptions and market expectations that is not fully reflected in historical realized volatility. These results highlight the value of using relative valuation ratios to better understand and predict market volatility, and they offer useful insights for investors, analysts, and policymakers seeking to manage risk and make informed decisions in the precious metals market. Future research could extend this approach by examining additional relative valuation ratios and their forward-looking power in explaining the volatility of other financial assets.

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Tables and Figures

Figure 1: Historical evolution of gold and silver prices, daily returns and realized variances from 2 September 2009 to 18 June 2025

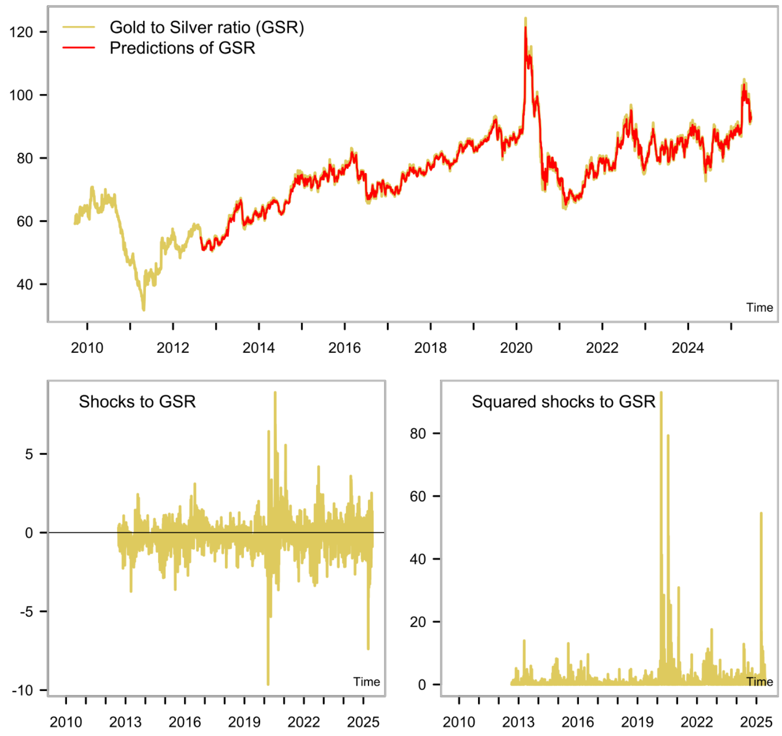

Figure 2: Gold-silver ratio series
Note: Sample period is from 2 September 2009 to 18 June 2025, where for shocks ($GSR^{\perp}$) and squared shocks to GSR $(GSR^{\perp})^2$, where the first 756 observations are being utilized to start the rolling window estimation of these values with the first (shock) observation recorded for 19 August 2012.

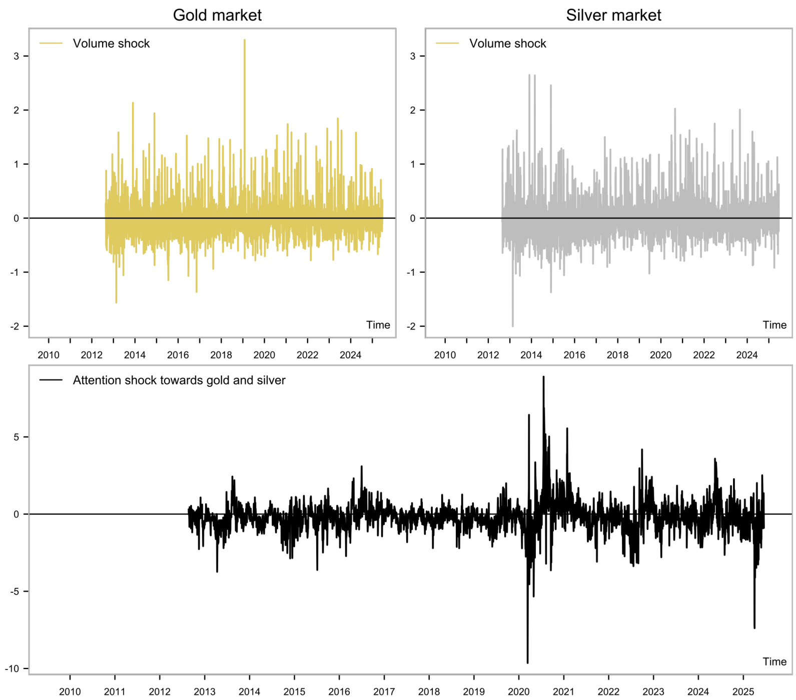

Figure 3: Shocks to trading volume and attention to gold and silver
Note: The first 756 daily observations are being utilized to start the rolling window estimation of shock series with the first (shock) observation recorded for 19 August 2012.

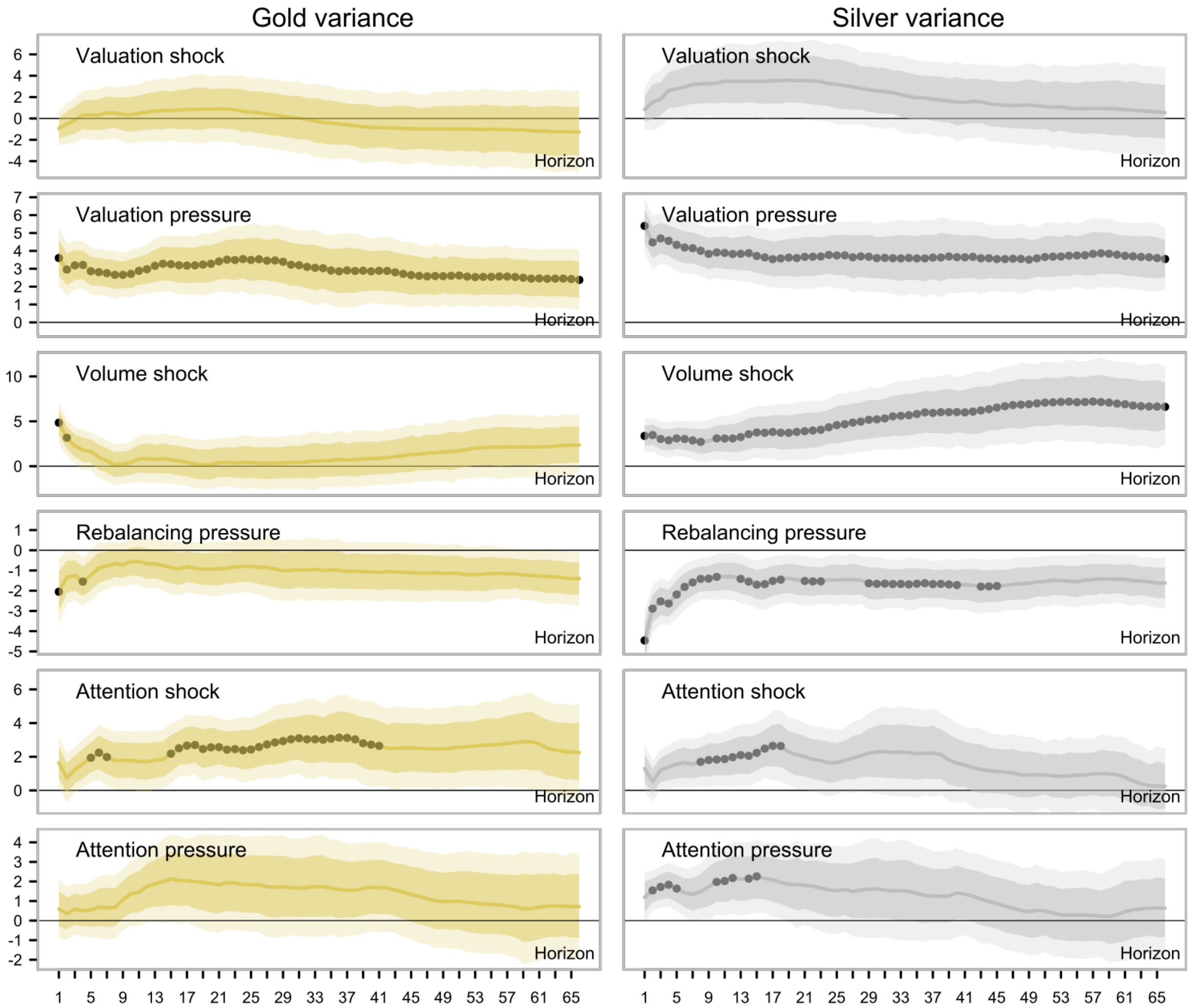

Figure 4: Local projection analysis
Note: The graphs show the coefficients for the corresponding terms and their respective 68% (darker) and 90% (lighter) confidence intervals for up to three months ahead gold and silver realized variance for model (15). The valuation pressure is a term considering the squared shocks in GSR (Eq. 6). The re-balancing pressure is the product of a shock in trading volume and the valuation pressure (Eq. 10). The attention pressure is the product of a shock in the attention to gold and silver, and the valuation pressure (Eq. 12). The coefficient estimates are obtained by running in-sample models spanning the period from 19 August 2012 to 18 June 2025. Coefficients statistically significant at the 5% level are marked with dots.

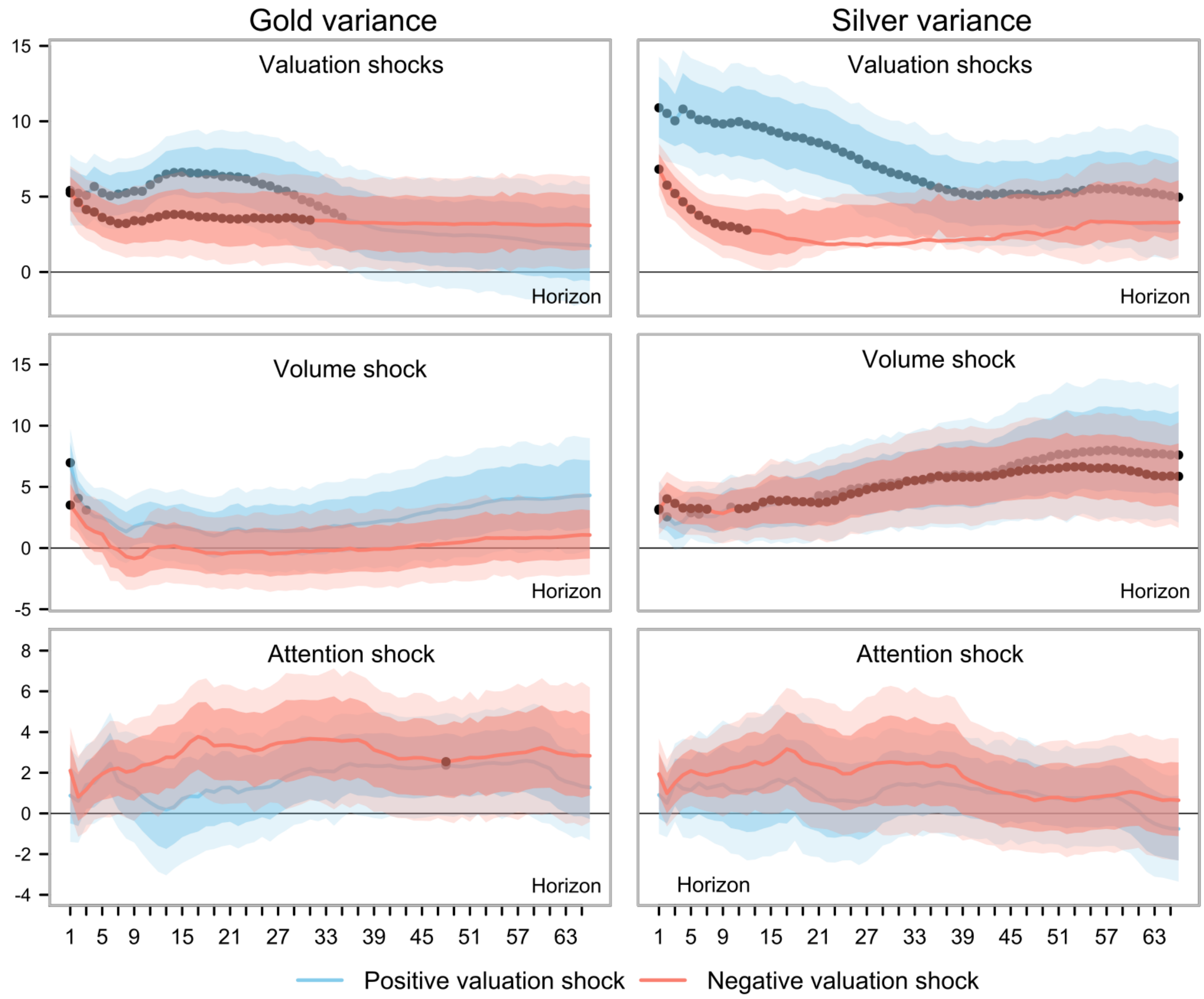

Figure 5: Local projection analysis for positive and negative GSR shocks

Note: The graphs show the coefficients for shocks in GSR, trading volume and attention shocks from model (16) and their respective 68% (darker) and 90% (lighter) confidence intervals for up to three months ahead gold and silver realized variance for the two individual cases when shocks are positive or negative, model (16).

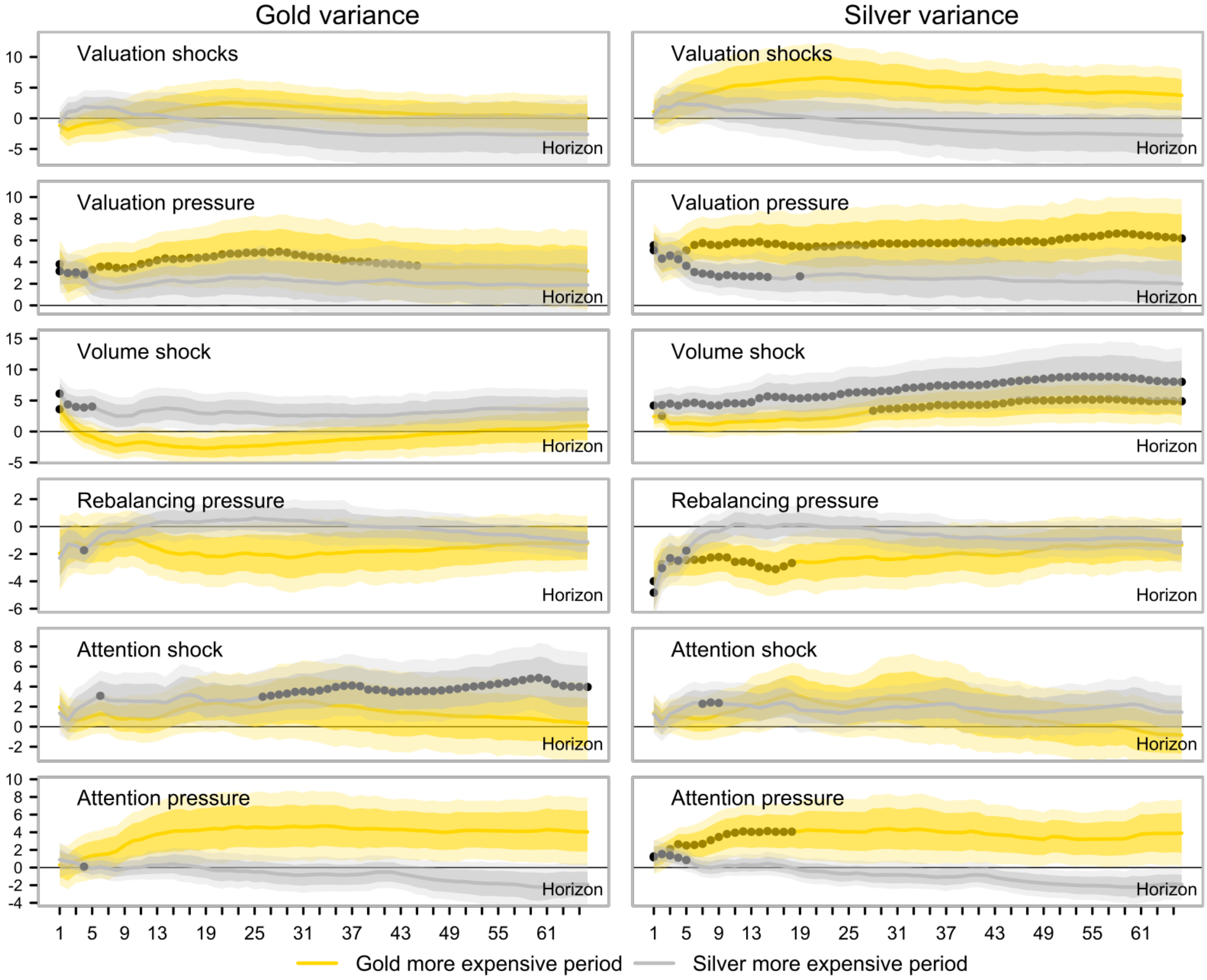

Figure 6: Local projection analysis when gold is relatively more/less expensive

Note: The graphs show the coefficients for the corresponding terms and their respective 68% (darker) and 90% (lighter) confidence intervals for up to three months ahead gold and silver realized variance for model (17) for the cases when the predicted GSR lies above (i.e. gold expensive relative to silver) or below the median (i.e. silver expensive relative to gold) of the GSR forecasts. The valuation pressure is a term considering the squared shocks in GSR (Eq. 6). The re-balancing pressure is the product of a shock in trading volume and the valuation pressure (Eq. 10). The attention pressure is the product of a shock in the attention to gold and silver, and the valuation pressure (Eq. 12). The coefficient estimates are obtained by running in-sample models spanning the period from 19 August 2012 to 18 June 2025.

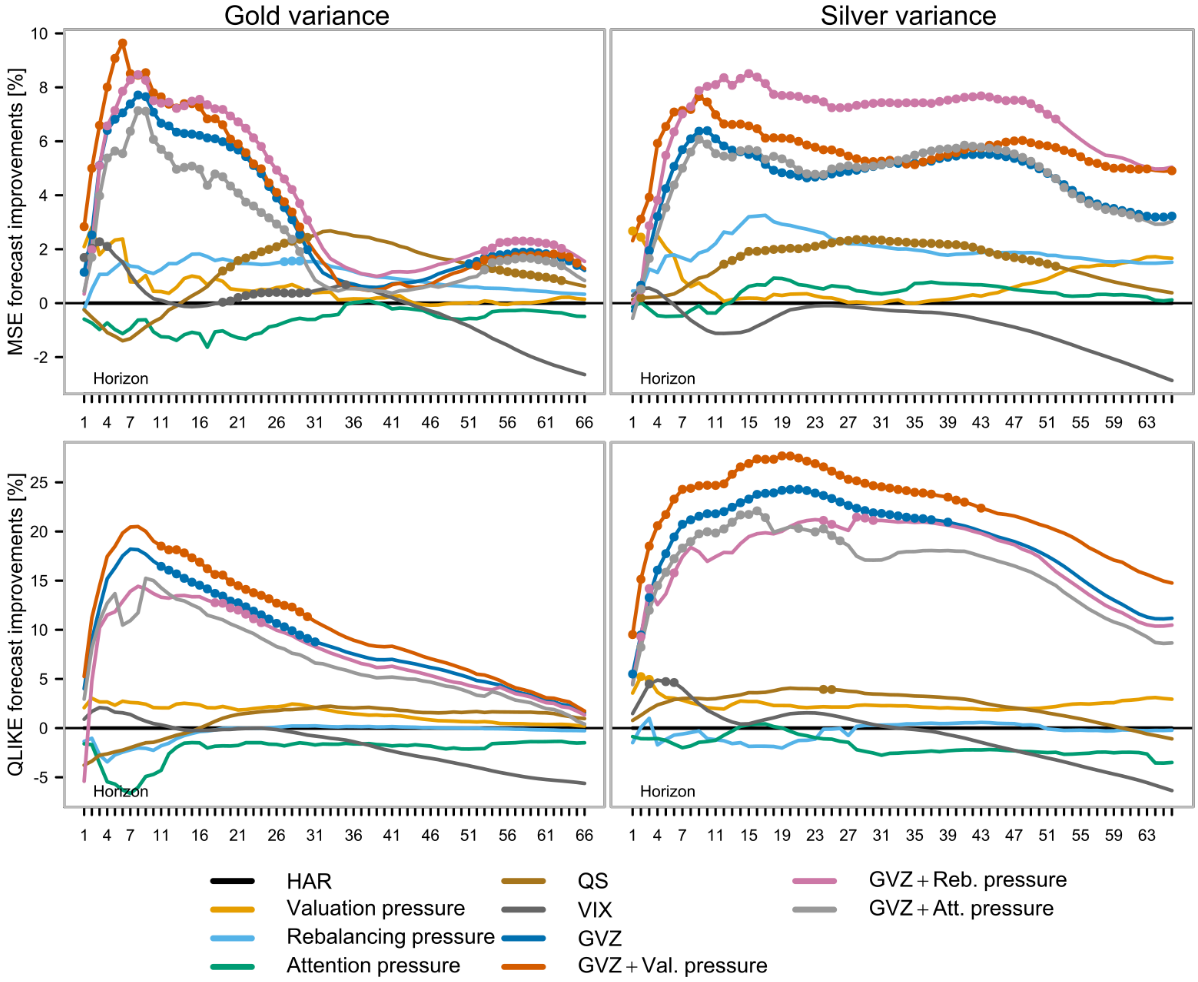

Figure 7: Out-of-sample forecasting performance

Note: This figure shows the % forecast improvements for the 10 competing models for MSE and QLIKE. HAR is the benchmark (plain) HAR model (horizontal line). The benchmark HAR model is expanded by one variable at a time, valuation pressure, rebalancing pressure, attention pressure, quoted spread, VIX and GVZ. Given the strong performance of the GVZ model, the HAR model with GVZ is expanded by valuation pressure (GVZ+Val. pressure), rebalancing pressure (GVZ+Reb. pressure) and attention pressure (GVZ+ Att. pressure). Dots indicate whether the given value belongs to the set of superior models, as indicated via the Romano and Wolf [2005] procedure with a 10% family-wise error rate, moving-block bootstrap with 999 samples and block length equal to the forecast horizon, while the test is the Diebold and Mariano [1995] test.

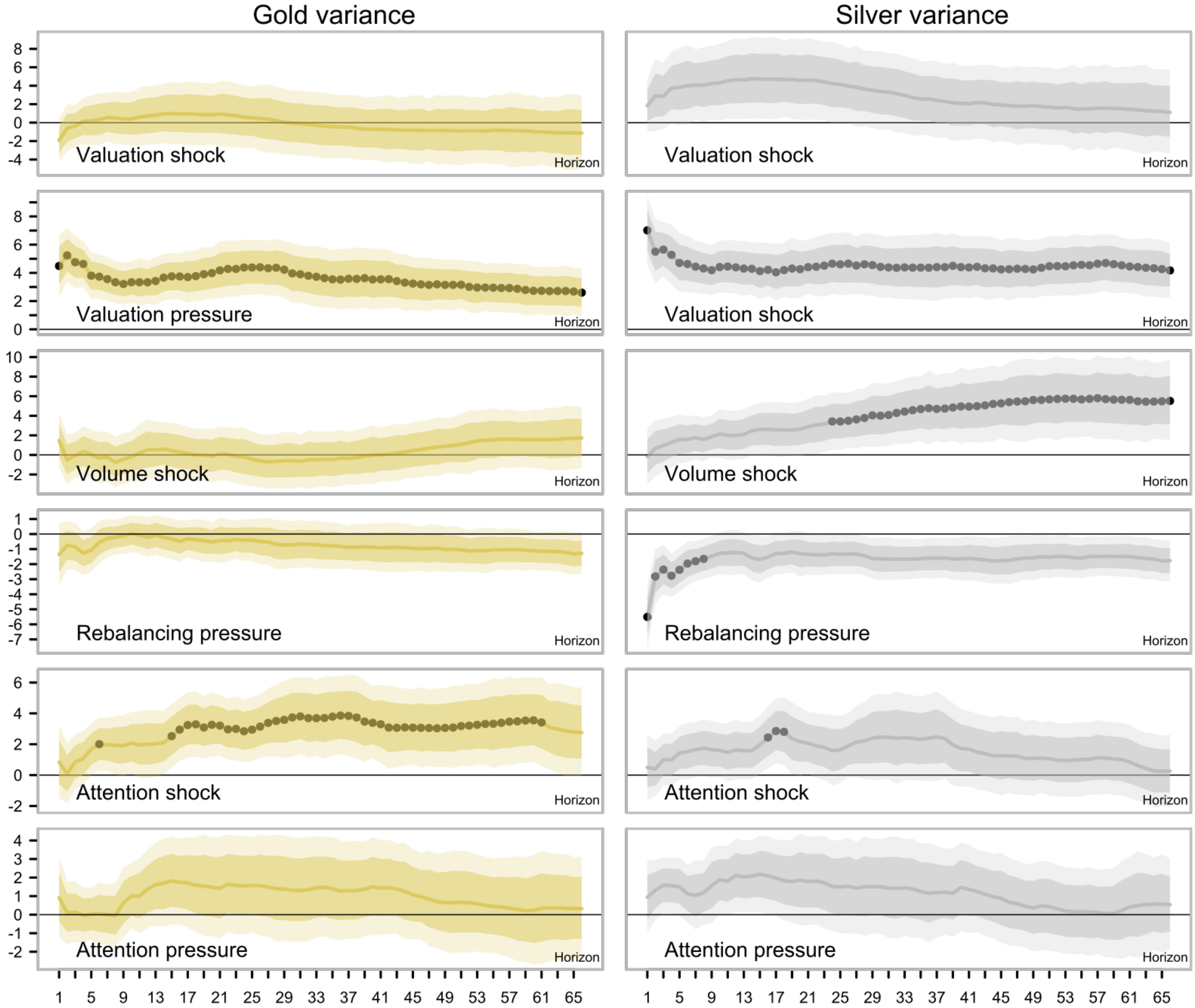

Figure 8: Local projection analysis for low-frequency, range-based models
Note: The graphs show the coefficients for the corresponding terms and their respective 68% (darker) and 90% (lighter) confidence intervals for up to three months ahead gold and silver realized variance for when volatility is estimated with daily ranges (i.e., no high-frequency data) for model (15). The valuation pressure is a term considering the squared shocks in GSR (Eq. 6). The re-balancing pressure is the product of a shock in trading volume and the valuation pressure (Eq. 10). The attention pressure is the product of a shock in the attention to gold and silver, and the valuation pressure (Eq. 12). The coefficient estimates are obtained by running in-sample models spanning the period from 19 August 2012 to 18 June 2025. The statistically significant coefficients are marked with thick dots.

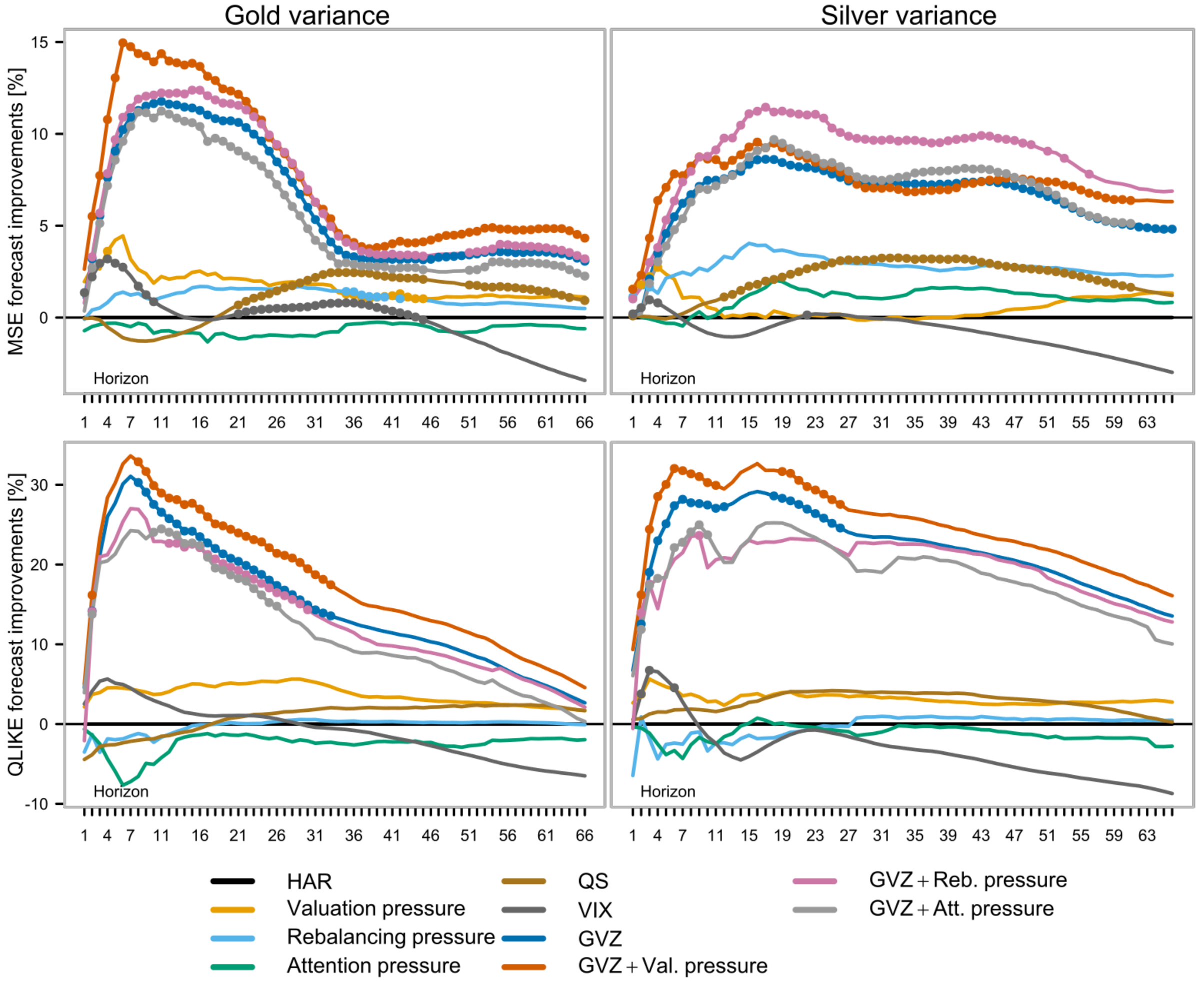

Figure 9: Out-of-sample forecasting performance for low-frequency data models

Note: This figure shows the % forecast improvements for the 10 competing models for MSE and QLIKE. HAR is the benchmark (plain) HAR model (horizontal line). The benchmark HAR model is expanded by one variable at a time, valuation pressure, rebalancing pressure, attention pressure, quoted spread, VIX and GVZ. Given the strong performance of the GVZ model, the HAR model with GVZ is expanded by valuation pressure (GVZ+Val. pressure), rebalancing pressure (GVZ+Reb. pressure) and attention pressure (GVZ+ Att. pressure). Dots indicate whether the given value belongs to the set of superior models, as indicated via the Romano and Wolf [2005] procedure with a 10% family-wise error rate, moving-block bootstrap with 999 samples and block length equal to the forecast horizon, while the test is the Diebold and Mariano [1995] test.

Table 1: Characteristics of key variables of interest

	Mean	SD	Min.	Q1	Median	Q3	Max.	Skew.	Kurt.	$\rho(1)$	$\rho(5)$	$\rho(22)$	$\rho(66)$	EL	ERS
Panel A: Gold market															
Daily return, R	0.03	0.98	-8.97	-0.48	0.04	0.55	6.03	-0.38	7.36	0.02	0.00	0.01	0.00		-15.41
Variance, RV	0.99	1.20	0.05	0.44	0.69	1.11	21.63	7.25	88.48	0.60	0.37	0.21	0.05	***	-31.74
Quoted spread, QS	0.01	0.00	0.00	0.01	0.01	0.01	0.03	1.49	7.75	0.85	0.77	0.65	0.50	***	-2.75
Volume, V	12.00	0.53	7.04	11.76	12.03	12.31	13.57	-1.66	12.13	0.54	0.37	0.29	0.24	***	-2.70
Excess volume, $V^{\perp}$	-0.02	0.31	-1.60	-0.19	-0.03	0.12	3.45	1.68	13.32	0.07	0.06	-0.01	-0.01	***	-53.58
Attention to Gold, A^G	56.43	14.01	22.15	46.78	56.95	66.22	121.03	0.08	3.07	0.95	0.89	0.81	0.54	***	-3.72
Panel B: Silver market															
Daily return, R	0.02	1.83	-15.17	-0.86	0.06	0.91	8.83	-0.64	9.24	0.02	0.00	0.02	-0.01		-9.99
Variance, RV	3.66	5.67	0.26	1.57	2.50	3.99	126.84	10.78	179.01	0.58	0.33	0.13	0.09	***	-32.52
Quoted spread, QS	0.03	0.01	0.00	0.03	0.03	0.04	0.09	1.78	10.55	0.86	0.75	0.60	0.43	***	-2.72
Volume, V	10.78	0.58	4.98	10.48	10.84	11.14	12.77	-1.38	9.81	0.58	0.42	0.37	0.32	***	-3.24
Excess volume, $V^{\perp}$	-0.02	0.34	-2.07	-0.21	-0.04	0.13	2.64	1.46	10.46	0.09	0.04	0.04	0.02	***	-52.48
Attention to Silver, A^S	56.63	12.90	18.19	50.30	57.51	65.37	105.35	-0.50	4.06	0.96	0.93	0.85	0.63	***	-2.19
Panel C: Gold and Silver															
GSR	72.96	13.79	31.72	64.27	74.74	83.32	124.36	-0.24	3.19	1.00	0.99	0.94	0.84	***	-3.24
$GSR^{\perp}$	-0.17	0.99	-10.66	-0.65	-0.18	0.26	8.66	0.28	14.72	0.55	0.36	0.17	-0.01	***	-19.68
$(GSR^{\perp})^2$	1.00	3.59	0.00	0.05	0.22	0.79	113.59	16.23	393.88	0.35	0.19	0.08	0.03	***	-38.05
Attention, A	50.99	13.07	22.06	43.11	52.00	59.30	99.50	-0.19	3.02	0.95	0.90	0.81	0.52	***	-2.38
Excess attention, $A^{\perp}$	0.11	3.90	-34.19	-1.36	0.45	2.11	27.20	-1.49	15.16	0.41	0.19	0.20	0.03	***	-9.77
Ivol gold, GVZ	17.33	4.67	8.88	14.34	16.76	19.68	48.98	1.14	5.82	0.98	0.90	0.70	0.40	***	-4.75
Panel D: Market Controls															
VIX	18.53	6.92	9.14	13.71	16.85	21.45	82.69	2.29	13.40	0.96	0.87	0.60	0.36	***	-5.84
$COVOL$	0.56	0.27	0.03	0.38	0.54	0.69	2.85	1.97	13.44	0.98	0.84	0.01	-0.01	***	-6.70
EPU	131.1	99.8	3.32	70.24	103.25	154.9	1,026	2.64	13.14	0.74	0.68	0.57	0.29	***	-6.11
EMU	72.45	114	4.80	14.05	35.85	82.57	1,922	5.21	47.35	0.58	0.46	0.34	0.10	***	-12.29
GPR	113.2	53	9.49	79.34	103.07	135.1	540.8	2.12	11.39	0.66	0.52	0.32	0.17	***	-5.54
ILQ	51.17	25.73	11.20	30.50	48.44	69.63	134.8	0.47	2.77	1.00	0.99	0.92	0.82	***	-4.03
NSI	-0.03	0.17	-0.67	-0.12	-0.01	0.09	0.33	-0.86	4.46	1.00	0.97	0.81	0.48	***	-3.76
$SRISK$	-922	1,226	-4,563	-1,718	-936	-50.3	1,203	-0.24	2.65	1.00	0.98	0.93	0.85	***	-4.44

Note: All observations correspond to daily data frequency measures. SD denotes standard deviation. Min and Max the minimum and maximum values. Q1-3 are the first, second and third quartiles. Skew is the skewness and Kurt denotes kurtosis. The $\rho(.)$ denotes the autocorrelation coefficient of the given order. EL denotes the p-value of the Escanciano and Lobato [2009] test of serial correlation, with *, **, *** denoting significant memory at the 10%, 5% and 1% level. ERS is the test statistics of the Elliott et al. [1996] test with the usual -1.96 left-handed critical value for a null of a unit-root, which is rejected for all time-series with constant or a trend (only for GSR). GSR is the gold-silver ration. VIX is the CBOE implied volatility index. GVZ is the CBOE gold implied volatility index. $COVOL$ is a measure of global common volatility shocks. $SRISK$ is the expected capital shortfall of US financial institutions. ILQ measures market wide liquidity in US equities. EPU is the US economic policy uncertainty index. EMU captures news-based uncertainty index. NSI is a daily sentiment indicator. GPR is the geopolitical risk index. Values are calculated for the period from 2 September 2009 to 18 June 2025, except for the GSR shock ($GSR^{\perp}$), GSR squared shocks $(GSR^{\perp})^2$, excess volume ($V^{\perp}$) and excess attention measures ($A^{\perp}$) where the first 756 observations are being utilized to start the rolling window estimation of these values with the first (shock) observation recorded for 19 August 2012.

Table 2: Correlations among variables under consideration

	Variables		A1	A2	A3	A4	A5	A6	A7	B1	B2	B3	B4	B5	B6	B7	C1	C2	C3	C4	C5	C6	D1	D2	D3	D4	D5	D6	D7	D8
Gold market	Daily return	A1	–	-0.11	-0.10	-0.04	-0.06	0.03	0.04	0.77	-0.10	-0.07	-0.03	0.00	0.04	0.02	0.01	0.21	-0.01	0.02	0.01	-0.01	0.01	0.01	0.06	0.06	0.01	-0.01	-0.02	-0.01
	Variance next day	A2	*	–	0.60	0.35	0.10	-0.06	-0.06	-0.15	0.69	0.44	0.32	0.13	-0.06	-0.07	-0.03	-0.12	0.32	-0.11	0.07	0.59	0.45	0.34	0.22	0.21	-0.02	0.22	-0.25	0.19
	Variance	A3	*	***	–	0.45	0.23	-0.24	-0.05	-0.15	0.48	0.69	0.42	0.01	-0.21	-0.07	-0.03	-0.13	0.39	-0.08	0.06	0.63	0.47	0.36	0.23	0.25	-0.02	0.22	-0.26	0.19
	Quoted spread	A4	*	***	***	–	-0.40	0.11	-0.20	-0.05	0.24	0.32	0.77	-0.38	-0.03	0.15	-0.39	-0.11	0.15	-0.09	0.03	0.61	0.34	0.35	-0.02	-0.05	-0.17	0.73	-0.17	0.66
	Volume	A5	***	***	***	***	–	-0.76	0.14	-0.07	0.06	0.15	-0.14	0.39	-0.45	-0.09	0.34	-0.02	0.12	0.12	0.05	-0.08	0.02	-0.05	0.04	0.12	0.08	-0.31	0.03	-0.32
	Volume shock	A6		***	***	***	***	–	0.01	0.04	-0.04	-0.17	0.03	-0.08	0.63	0.01	0.05	0.06	-0.05	-0.04	-0.05	-0.04	0.02	0.01	0.04	0.01	-0.02	-0.02	-0.05	-0.03
	Attention	A7	**			**			–	0.01	-0.02	-0.01	-0.07	0.16	0.03	0.56	0.08	-0.09	-0.03	0.68	-0.30	-0.29	-0.07	-0.08	0.00	0.06	0.03	-0.10	-0.08	-0.09
Silver market	Daily return	B1	***	***	***	*	***	*		–	-0.15	-0.14	-0.05	-0.01	0.02	0.03	-0.02	0.59	-0.06	0.00	0.00	-0.03	-0.04	-0.01	0.05	0.03	0.01	-0.01	-0.03	0.00
	Variance next day	B2	*	***	***	***				**	–	0.58	0.25	0.22	-0.08	-0.06	-0.10	0.04	0.49	-0.11	0.05	0.46	0.37	0.23	0.17	0.13	-0.06	0.15	-0.26	0.17
	Variance	B3	**	***	***	***	***	***		**	***	–	0.37	0.11	-0.20	-0.05	-0.10	0.02	0.58	-0.09	0.04	0.49	0.39	0.25	0.18	0.14	-0.07	0.16	-0.26	0.18
	Quoted spread	B4		***	***	***	**			*	***	***	–	-0.28	0.12	0.17	-0.10	-0.15	0.20	0.04	-0.02	0.49	0.41	0.30	-0.04	-0.03	-0.21	0.61	-0.22	0.54
	Volume	B5		***		***	***	***	***		***	***	***	–	-0.18	-0.07	0.31	0.12	0.17	0.09	-0.02	-0.19	-0.03	-0.07	0.07	0.10	0.09	-0.43	0.03	-0.38
	Volume shock	B6	*	**	***		***	***	*		***	**	***	***	–	-0.03	0.06	0.02	-0.09	-0.05	-0.05	-0.04	0.04	0.00	0.08	0.03	-0.01	-0.06	-0.04	-0.07
	Attention	B7							***	*						–	-0.05	-0.06	-0.02	0.71	-0.28	-0.14	-0.12	-0.02	-0.09	-0.06	-0.02	0.20	0.00	0.27
Gold and Silver	GSR_t	C1				***	***	**						***	***		–	-0.22	0.17	0.16	-0.02	-0.22	0.15	0.06	0.23	0.32	0.25	-0.67	-0.01	-0.71
	$GSR^{\perp}$	C2	***	*	*	**		***	**	***			***	**			***	–	-0.02	-0.13	0.03	0.00	-0.13	-0.09	0.01	-0.09	-0.03	-0.06	-0.07	0.03
	$(GSR^{\perp})^2$	C3		***	***		***	**			***	***	*	***	***		***		–	-0.05	0.02	0.30	0.35	0.17	0.21	0.19	-0.03	-0.04	-0.23	0.01
	Attention, A_t	C4		**				**	***		**	*			***	***		***		–	-0.28	-0.33	-0.22	-0.07	-0.15	-0.02	0.07	-0.07	0.12	-0.05
	Attention shock, $A_t^{\perp}$	C5		***			*	**	***		*				**	***				***	–	0.09	0.04	0.04	0.06	0.02	-0.01	0.03	-0.02	0.03
Market controls	GVZ_t	C6		***	***	***		**	***		***	***	***	**	**				***	***	***	–	0.66	0.45	0.33	0.23	-0.05	0.48	-0.45	0.44
	VIX_t	D1		***	***	***					***	***	***		*				***	**	*	***	–	0.53	0.47	0.41	0.00	0.22	-0.60	0.20
	$COVOL_t$	D2		***	***	***					***	***	***					*	***			***	***	–	0.20	0.26	0.07	0.19	-0.21	0.13
	EPU_t	D3	**	***	***			**		**	***	***		*	***	**	*		***	***	**	***	***	**	–	0.63	0.09	-0.09	-0.46	-0.09
	EMU_t	D4	**	***	***		***		**		**	**		***	*	**	***		***			***	***	***	***	–	0.13	-0.19	-0.28	-0.20
	GPR_t	D5				***	***				***	***	***	***			***			**						***	–	-0.31	0.09	-0.36
	ILQ_t	D6		***	***	***	***				***	***	***	***	***		***					***		***		***	***	–	-0.24	0.89
	NSI_t	D7		***	***			***			***	***	**		**				***			***	***	***	***	***		*	–	-0.33
	$SRISK_t$	D8		***	***	***	***	*			***	***	***	***	***	*	***					***	*	***		***	***	***	**	–

Note: The values above the diagonal correspond to Pearson's correlation coefficients. Below the diagonal *, **, *** correspond to statistically significant coefficients at the 10%, 5% and 1% levels, where the significance is derived from a simple linear regression model of the two variables of interest, with heteroscedasticity and autocorrelation consistent estimator of the variance-covariance matrix of coefficient's standard errors, estimated with a quadratic spectral kernel weighting scheme and an automatic bandwidth selection procedure of Newey and West (1994). VIX is the CBOE implied volatility index. GVZ is the CBOE gold implied volatility index. $COVOL$ is a measure of global common volatility shocks. $SRISK$ is the expected capital shortfall of US financial institutions. ILQ measures market wide liquidity in US equities. EPU is the US economic policy uncertainty index. EMU captures news-based uncertainty index. NSI is a daily sentiment indicator. GPR is the geopolitical risk index.

Table 3: In-sample models for gold volatility

		$h = 1$			$h = 5$			$h = 22$			$h = 66$		
	Symbol	HAR	HAR-GSR	HAR-GSR-C	HAR	HAR-GSR	HAR-GSR-C	HAR	HAR-GSR	HAR-GSR-C	HAR	HAR-GSR	HAR-GSR-C
Const.		-0.12^d	0.02	-5.90^d	-0.06^c	0.25	-6.48^d	-0.06^b	0.06	-5.75^b	-0.08^a	0.12	-5.72
Panel A: Volatility persistence													
Daily variance	$\ln(RV_t^D)$	0.22^d	0.29^d	0.21^d	0.17^d	0.20^d	0.13^c	0.10^d	0.10^d	0.04	0.05^d	0.06^c	0.01
Weekly variance	$\ln(RV_t^W)$	0.38^d	0.34^d	0.18^d	0.32^d	0.30^d	0.13^b	0.20^d	0.19^d	0.04	0.11^d	0.11^d	-0.02
Monthly variance	$\ln(RV_t^M)$	0.28^d	0.24^d	-0.11^c	0.34^d	0.31^d	-0.07	0.40^d	0.38^d	0.12	0.34^c	0.33^c	0.24^b
Panel B: Valuation ratio terms													
Expected GSR	$\ln(E[GSR_t])$		-0.03	0.28^b		-0.07	0.38^a		-0.03	0.74^b		-0.05	1.17^a
Panel B1: Valuation channel [$\times 10^2$]													
Valuation shock	$GSR_t^\perp$		-1.44	-0.95		-0.07	0.32		-0.19	0.89		-2.45	-1.29
Valuation pressure	VP_t		1.60^d	1.42^d		1.33^c	1.13^d		1.79^c	1.38^c		1.49^d	0.94^b
Panel B2: Re-balancing channel [$\times 10^2$]													
Volume shock	$V_t^\perp$		21.10^d	15.04^d		10.43^c	5.23		4.43	1.37		5.33	7.40
Re-balancing pressure	RP_t		-3.15^b	-2.50^b		-2.33^b	-1.50		-1.87^a	-1.08		-2.35^b	-1.70
Panel B3: Attention channel [$\times 10^2$]													
Attention shock to gold and silver	$A_t^\perp$		0.13	0.42^a		0.26	0.49^b		0.59^a	0.61^b		0.57	0.57
Attention pressure	AP_t		0.13	0.06		0.14	0.06		0.30^a	0.20		0.15	0.08
Panel C: Control Variables													
Attention to gold	$\ln(A_t^G)$			0.13^c			0.10^b			0.02			0.06
Daily variance (silver)	$\ln(RV_t^{D,S})$			-0.04^a			-0.05			-0.02			0.04
Quoted spread (gold)	$\ln(QS_t)$			0.08			-0.04			-0.28			-0.62
Implied gold volatility	$\ln(GVZ_t)$			1.68^d			1.83^d			1.28^d			0.70^b
VIX [$\times 10^2$]	$\ln(VIX_t)$			-4.57			-10.89			8.14			19.79
COVOL	$\ln(COVOL_t)$			-0.01			0.02			0.09^a			0.11^a
EPU [$\times 10^2$]	$\ln(EPU_t)$			0.20			0.10			-1.19			-1.18
US EMU	$\ln(EMU_t)$			0.01			0.01			0.03			0.05
GPR [$\times 10^3$]	GPR_t			-0.05			-0.11			-0.26			0.01
Illiquidity	$\ln(ILQ_t)$			-0.07			-0.14			-0.50^c			-0.92^c
NSI	NSI_t			0.10			-0.03			-0.10			-0.16
SRISK[$\times 10^4$]	$SRISK_t$			-0.04			-0.11			-0.08			0.10
Time trend [$\times 10^4$]	t		0.02	-0.41		0.01	-1.08		0.00	-3.78^b		0.06	-6.95^b
R^2_{Adj}		50.5%	51.6%	56.4%	58.5%	59.1%	66.7%	48.3%	49.5%	57.6%	29.6%	30.8%	42.1%

Note: VIX is the CBOE implied volatility index. GVZ is the CBOE gold implied volatility index. $COVOL$ is a measure of global common volatility shocks. $SRISK$ is the expected capital shortfall of US financial institutions. ILQ measures market wide liquidity in US equities. EPU is the US economic policy uncertainty index. EMU captures news-based uncertainty index. NSI is a daily sentiment indicator. GPR is the geopolitical risk index. The subscripts d, c, b and a denote statistical significance of a coefficient at the 0.1%, 1%, 5% and 10% levels, respectively. The significance is derived from heteroskedasticity and autocorrelation consistent estimator of the variance-covariance matrix of coefficient's standard errors, estimated with a quadratic spectral kernel weighting scheme and an automatic bandwidth selection procedure of Newey and West [1994].

Table 4: In-sample models for silver volatility

	Symbol	h = 1 HAR	h = 1 HAR-GSR	h = 1 HAR-GSR-C	h = 5 HAR	h = 5 HAR-GSR	h = 5 HAR-GSR-C	h = 22 HAR	h = 22 HAR-GSR	h = 22 HAR-GSR-C	h = 66 HAR	h = 66 HAR-GSR	h = 66 HAR-GSR-C
Const.		0.04^b	0.48	-1.81^a	0.13^d	0.54	-2.98^a	0.25^d	0.00	-6.68^c	0.41^c	-0.94	-10.83^c
Panel A: Volatility persistence													
Daily variance	$\ln(RV_t^D)$	0.29^d	0.33^d	0.39^d	0.20^d	0.23^d	0.29^d	0.12^d	0.13^d	0.23^d	0.08^d	0.07^d	0.19^d
Weekly variance	$\ln(RV_t^W)$	0.34^d	0.31^d	0.20^d	0.31^d	0.28^d	0.15^c	0.19^c	0.17^c	0.04	0.17^d	0.16^d	0.04
Monthly variance	$\ln(RV_t^M)$	0.26^d	0.24^d	0.00	0.35^d	0.32^d	0.05	0.46^d	0.43^d	0.20^a	0.38^d	0.38^c	0.22^c
Panel B: Valuation ratio terms													
Expected GSR	$\ln(E[GSR_t])$		-0.11	-0.34^a		-0.10	-0.02		0.06	1.17^b		0.31	2.50^c
Panel B1: Valuation channel [$\times 10^2$]													
Valuation shock	$GSR_t^{\perp}$		-0.27	0.85		1.11	2.77		1.16	3.51		-1.43	0.54
Valuation pressure	VP_t		2.22^d	2.10^d		1.86^d	1.70^d		1.70^c	1.44^d		1.63^c	1.39^c
Panel B2: Re-balancing channel [$\times 10^2$]													
Volume shock	$V_t^{\perp}$		15.87^d	9.84^c		13.11^d	9.05^c		7.86^c	11.57^b		6.07	19.00^b
Re-balancing pressure	RP_t		-6.06^d	-5.51^d		-3.23^c	-2.67^c		-2.50^b	-1.87^b		-2.64^b	-1.97^a
Panel B3: Attention channel [$\times 10^2$]													
Attention shock to gold and silver	$A_t^{\perp}$		0.05	0.33		0.21	0.40^a		0.55	0.47^a		0.35	0.06
Attention pressure	AP_t		0.18^b	0.13		0.24^c	0.17^b		0.28	0.19		0.17	0.07
Panel C: Control Variables													
Attention to Silver	$\ln(A_t^S)$			0.12^c			0.08			-0.04			-0.09
Daily variance (gold)	$\ln(RV_t^{D,G})$			-0.18^d			-0.17^d			-0.14^c			-0.07^a
Quoted spread (silver)	$\ln(QS_t)$			0.10			-0.12			-0.82^b			-1.59^c
Implied gold volatility	$\ln(GVZ_t)$			1.20^d			1.29^d			1.02^d			0.63^b
VIX [$\times 10^2$]	$\ln(VIX_t)$			8.79^a			11.20			20.26^a			18.78
COVOL	$\ln(COVOL_t)$			-0.02			-0.01			0.04			0.07
EPU [$\times 10^2$]	$\ln(EPU_t)$			-1.60			-2.49			-5.28			-3.74
US EMU	$\ln(EMU_t)$			0.00			0.00			0.01			0.03
GPR [$\times 10^3$]	GPR_t			-0.48^c			-0.53^b			-0.63^b			-0.27
Illiquidity	$\ln(ILQ_t)$			-0.07			-0.23^b			-0.63^c			-1.09^d
NSI	NSI_t			0.10			0.02			-0.13			-0.51
SRISK[$\times 10^4$]	$SRISK_t$			0.47^b			0.74^b			1.32^c			1.62^b
Time trend [$\times 10^4$]	t		0.18	1.11^b		0.14	-0.03		0.08	-3.51^b		0.05	-7.69^c
R^2_{Adj}		58.9%	60.4%	63.6%	64.5%	65.6%	70.8%	57.3%	58.4%	65.9%	42.8%	44.5%	58.7%

Note: VIX is the CBOE implied volatility index. GVZ is the CBOE gold implied volatility index. $COVOL$ is a measure of global common volatility shocks. $SRISK$ is the expected capital shortfall of US financial institutions. ILQ measures market wide liquidity in US equities. EPU is the US economic policy uncertainty index. EMU captures news-based uncertainty index. NSI is a daily sentiment indicator. GPR is the geopolitical risk index. The subscripts d, c, b and a denote statistical significance of a coefficient at the 0.1%, 1%, 5% and 10% levels, respectively. The significance is derived from heteroskedasticity and autocorrelation consistent estimator of the variance-covariance matrix of coefficient's standard errors, estimated with a quadratic spectral kernel weighting scheme and an automatic bandwidth selection procedure of Newey and West (1994).

Table 5: Out-of-sample Gold and Silver Volatility Forecasts

	MSE				QLIKE			
Model	$h = 1$	$h = 5$	$h = 22$	$h = 66$	$h = 1$	$h = 5$	$h = 22$	$h = 66$
Panel A: Gold								
HAR	0.698	0.413	0.409	0.289	0.178	0.107	0.136	0.176
HAR + Valuation pressure	2.05%	2.28%	0.46%	0.14%	2.03%	2.27%	1.85%	0.21%
HAR + Rebalancing pressure	-0.23%	1.29%	1.45%	0.33%	-1.37%	-2.81%	-0.02%	-0.27%
HAR + Attention pressure	-0.60%	-0.99%	-1.35%	-0.49%	-1.63%	-6.04%	-1.51%	-1.51%
HAR + QS	-0.25%	-1.24%	1.65%†	0.63%	-3.92%	-2.47%	1.47%	0.96%
HAR + VIX	1.66%†	1.74%	0.28%†	-2.72%	0.89%	1.59%	-0.04%	-5.96%
HAR + GVZ	1.13%†	6.38%†	5.16%†	1.20%	3.85%	14.02%	10.98%†	1.55%
HAR + GVZ + Valuation pressure	2.76%†	8.32%†	5.28%†	1.26%	4.98%	15.60%	12.36%†	1.70%
HAR + GVZ + Rebalancing pressure	0.34%	6.67%†	6.09%†	1.52%	-5.72%	10.58%	10.38%†	1.32%
HAR + GVZ + Attention pressure	0.44%	5.34%†	3.61%†	0.83%	2.86%	12.03%	9.03%	0.41%
Panel B: Silver								
HAR	16.24	10.79	7.54	5.29	0.143	0.119	0.177	0.239
HAR + Valuation pressure	2.60%	2.01%	0.35%	1.64%	3.44%	3.01%	2.04%	2.86%
HAR + Rebalancing pressure	0.45%	1.71%	2.67%	1.49%	-1.51%	-1.17%	-1.25%	-0.21%
HAR + Attention pressure	-0.14%	-0.50%	0.65%	0.12%	-0.87%	-1.26%	-0.78%	-3.63%
HAR + QS	0.13%	0.27%	2.01%†	0.38%	0.77%	2.55%	3.86%	-1.11%
HAR + VIX	0.28%	0.18%	-0.14%	-2.95%	1.46%	4.51%†	1.53%	-6.78%
HAR + GVZ	-0.29%	4.07%†	4.44%†	3.13%†	5.21%†	15.07%†	19.46%†	10.05%
HAR + GVZ + Valuation pressure	2.26%	6.15%†	5.54%†	4.68%†	8.69%†	17.84%†	21.36%†	12.86%
HAR + GVZ + Rebalancing pressure	-0.01%	5.19%†	7.03%†	4.80%	4.33%	12.00%	17.43%	9.48%
HAR + GVZ + Attention pressure	-0.56%	3.42%†	4.57%†	2.95%†	4.23%†	13.68%†	16.77%†	7.97%

Notes: We report the average loss for the 'HAR' model and forecast improvements for competing models, with positive values indicating better forecasts. The symbol † is added if the given model belongs to the set of superior models as indicated by the Diebold and Mariano [1995] test run within the Romano and Wolf [2005] step-wise approach and with a 10% family-wise error. We use moving-block bootstrap of length h and 999 replications.

Table 6: Out-of-sample Gold and Silver Volatility Forecasts: Range-based Volatility

	MSE				**QLIKE**			
Model	$h=1$	$h=5$	$h=22$	$h=66$	$h=1$	$h=5$	$h=22$	$h=66$
Panel A: Gold								
HAR	2.064	0.891	0.642	0.390	0.514	0.198	0.184	0.199
HAR + Valuation pressure	1.92%	4.05%	2.04%	1.11%	2.10%	4.37%	4.83%	1.66%
HAR + Rebalancing pressure	-0.09%	1.24%	1.55%	0.49%	-3.64%	-2.01%	0.05%	-0.08%
HAR + Attention pressure	-0.72%	-0.30%	-1.15%	-0.61%	-0.66%	-6.30%	-1.29%	-2.00%
HAR + QS	-0.04%	-0.81%	0.88%†	0.92%†	-4.65%	-2.65%	1.00%	1.67%
HAR + VIX	1.34%†	2.87%†	0.31%†	-3.54%	2.46%	4.87%	1.05%	-6.95%
HAR + GVZ	1.07%	8.31%†	9.37%†	2.98%†	4.36%	21.69%	16.58%†	2.58%
HAR + GVZ + Valuation pressure	2.57%	11.54%†	10.53%†	4.15%†	4.80%	23.20%	19.05%†	4.37%
HAR + GVZ + Rebalancing pressure	0.78%	8.83%†	10.16%†	3.10%†	-2.13%	18.72%	15.77%†	2.06%
HAR + GVZ + Attention pressure	0.36%	7.90%†	8.08%†	2.21%†	3.78%	17.56%	15.20%†	0.32%
Panel B: Silver								
HAR	43.48	17.73	9.57	6.07	0.490	0.215	0.223	0.270
HAR + Valuation pressure	1.53%†	2.31%	0.14%	1.31%	2.57%	4.46%	3.32%	2.67%
HAR + Rebalancing pressure	1.12%†	2.10%	3.28%	2.25%	-6.90%	-2.62%	-0.72%	0.48%
HAR + Attention pressure	0.24%	-0.31%	1.49%	0.82%	-0.35%	-3.99%	-0.34%	-2.86%
HAR + QS	0.09%†	-0.07%	2.59%†	1.20%	0.57%	1.47%	3.90%	0.26%
HAR + VIX	0.20%†	0.51%	0.14%†	-3.07%	0.12%	5.30%	-0.77%	-9.54%
HAR + GVZ	-0.01%	4.37%†	7.56%†	4.59%†	6.30%	20.07%†	21.23%†	11.93%
HAR + GVZ + Valuation pressure	1.52%†	6.62%†	7.97%†	5.94%	8.54%	23.11%†	22.94%†	13.86%
HAR + GVZ + Rebalancing pressure	1.01%†	5.04%†	9.93%†	6.44%	-0.56%	15.89%	18.76%	11.35%
HAR + GVZ + Attention pressure	0.08%	3.75%†	8.13%†	4.56%	5.71%	15.53%	19.22%	9.13%

Notes: We report the average loss for the 'HAR' model and forecast improvements for competing models, with positive values indicating better forecasts. The symbol † is added if the given model belongs to the set of superior models as indicated by the Diebold and Mariano [1995] test run within the Romano and Wolf [2005] step-wise approach and with a 10% family-wise error. We use moving-block bootstrap of length h and 999 replications.

A. Supply and demand for gold and silver

Table A1: Global supply and demand for gold and silver

	2016	2017	2018	2019	2020	2021	2022	2023	2024
Panel A: Gold									
Supply									
Mine Production	3,553.88	3,555.04	3,646.78	3,612.24	3,446.12	3,569.78	3,638.79	3,710.42	3,591.68
Recycling	1,232.14	1,112.39	1,131.70	1,275.66	1,293.06	1,135.76	1,136.10	1,233.62	1,366.79
Total Supply	4,786.02	4,667.44	4,778.47	4,887.90	4,739.18	4,705.54	4,774.89	4,944.05	4,958.47
Demand									
Jewelry	2,023.22	2,267.26	2,297.54	2,162.20	1,331.80	2,252.53	2,208.57	2,208.40	2,026.79
Technology	329.37	339.44	341.72	332.68	308.99	337.17	314.79	305.16	326.23
Investment	1,622.09	1,321.94	1,167.13	1,281.80	1,805.33	1,006.86	1,125.37	950.89	1,181.88
Central Banks and Other Inst.	394.86	378.56	656.23	605.41	254.94	450.11	1,080.01	1,050.81	1,089.37
Total Demand	4,369.54	4,307.20	4,462.63	4,382.10	3,701.07	4,046.67	4,728.74	4,515.26	4,624.26
Panel B: Silver									
Supply									
Mine Production	900.10	863.90	850.80	837.40	783.80	830.80	839.40	812.70	819.70
Recycling	156.30	160.20	162.30	163.80	180.50	190.70	193.50	183.50	193.90
Other	1.10	1.00	1.20	14.90	9.70	1.50	1.70	1.60	1.50
Total Supply	1,057.50	1,025.10	1,014.30	1,016.10	974.00	1,023.00	1,034.60	997.80	1,015.10
Demand									
Industrial	491.00	528.00	525.80	525.40	511.90	564.10	592.30	657.10	680.50
Photography	34.70	32.40	31.40	30.70	26.90	27.70	27.70	27.30	25.50
Jewelry	189.10	196.20	203.20	201.60	150.90	182.00	234.50	203.10	208.70
Silverware	53.50	59.40	67.10	61.30	31.20	40.70	73.50	55.10	54.20
Investment	224.90	156.90	173.30	187.00	208.10	287.80	356.20	255.80	195.20
Total Demand	993.30	972.90	1,000.80	1,006.40	929.00	1,102.40	1,284.20	1,198.50	1,164.10

Sources: World Gold Council and The Silver Institute. Gold data is in tonnes. Silver records are in million ounces.

B. Continuous and jump variation volatility model results

Appendix B presents impulse response functions calculated from a volatility model, where future volatility is predicted by continuous and discontinuous (jump) trading-period components. The estimate of the continuous component of the quadratic variation is the median realized variance of Andersen et al. [2012] with a correction for zero returns of Kolokolov and Renò [2024], as zero returns might be more frequent with 1-minute sampling and a 23-hour trading period. Let $\{P_{t,j}\}_{j=0}^{N}$ denote intraday prices at 1-minute sampling frequency and $K = 5$ the number of sub-sampled grids. For each offset $\delta \in \{0, 1, \ldots, K-1\}$, the K-minute sub-sampled log returns are

$$r_{t,\delta,j} = 100 \times \ln \frac{P_{t,\delta+Kj}}{P_{t,\delta+K(j-1)}}, \qquad j = 1, \ldots, \left\lfloor \frac{N-\delta}{K} \right\rfloor. \tag{B.1}$$

For the correction, we identify the ordered indices of non-zero returns, $\{\tau_{t,\delta,i}\}_{i=1}^{N_{t,\delta}}$, such that $r_{t,\delta,\tau_{t,\delta,i}} \neq 0$ (with $\tau_{t,\delta,0} = 0$), where $N_{t,\delta}$ denotes the number of non-zero returns. The durations between consecutive non-zero returns are

$$\Delta_{t,\delta,i} = \tau_{t,\delta,i} - \tau_{t,\delta,i-1}, \qquad i = 1, \ldots, N_{t,\delta}. \tag{B.2}$$

The average duration is

$$\overline{\Delta}_{t,\delta} = N_{t,\delta}^{-1} \sum_{i=1}^{N_{t,\delta}} \Delta_{t,\delta,i}. \tag{B.3}$$

The duration-corrected non-zero returns are

$$\tilde{r}_{t,\delta,i} = \frac{\left| r_{t,\delta,\tau_{t,\delta,i}} \right|}{\sqrt{\Delta_{t,\delta,i}}}, \qquad i = 1, \ldots, N_{t,\delta}. \tag{B.4}$$

The median realized variance using corrected returns and averaged across five sub-samples is given by

$$CC_t = K^{-1} \sum_{\delta=0}^{K-1} \left\{ \frac{\pi}{6 - 4\sqrt{3} + \pi} \, \overline{\Delta}_{t,\delta} \, \frac{N_{t,\delta}}{N_{t,\delta} - 2} \sum_{i=2}^{N_{t,\delta}-1} \operatorname{med}\left(\tilde{r}^2_{t,\delta,i-1}, \, \tilde{r}^2_{t,\delta,i}, \, \tilde{r}^2_{t,\delta,i+1} \right) \right\}, \tag{B.5}$$

provided that $N_{t,\delta} \geq 3$. Let IV_t denote the trading-period realized variance, the trading-period variation from Eq. (13) after applying the sub-sampling average. The jump component is given by

$$JC_t = \max\{IV_t - CC_t, \, 0\}. \tag{B.6}$$

The $\boldsymbol{RM}_t$ vector in Eq. 15 now includes lagged and log-scaled CC_t, JC_t and corresponding average weekly and monthly components, instead of realized variance. The resulting impulse responses are in Figure B1.

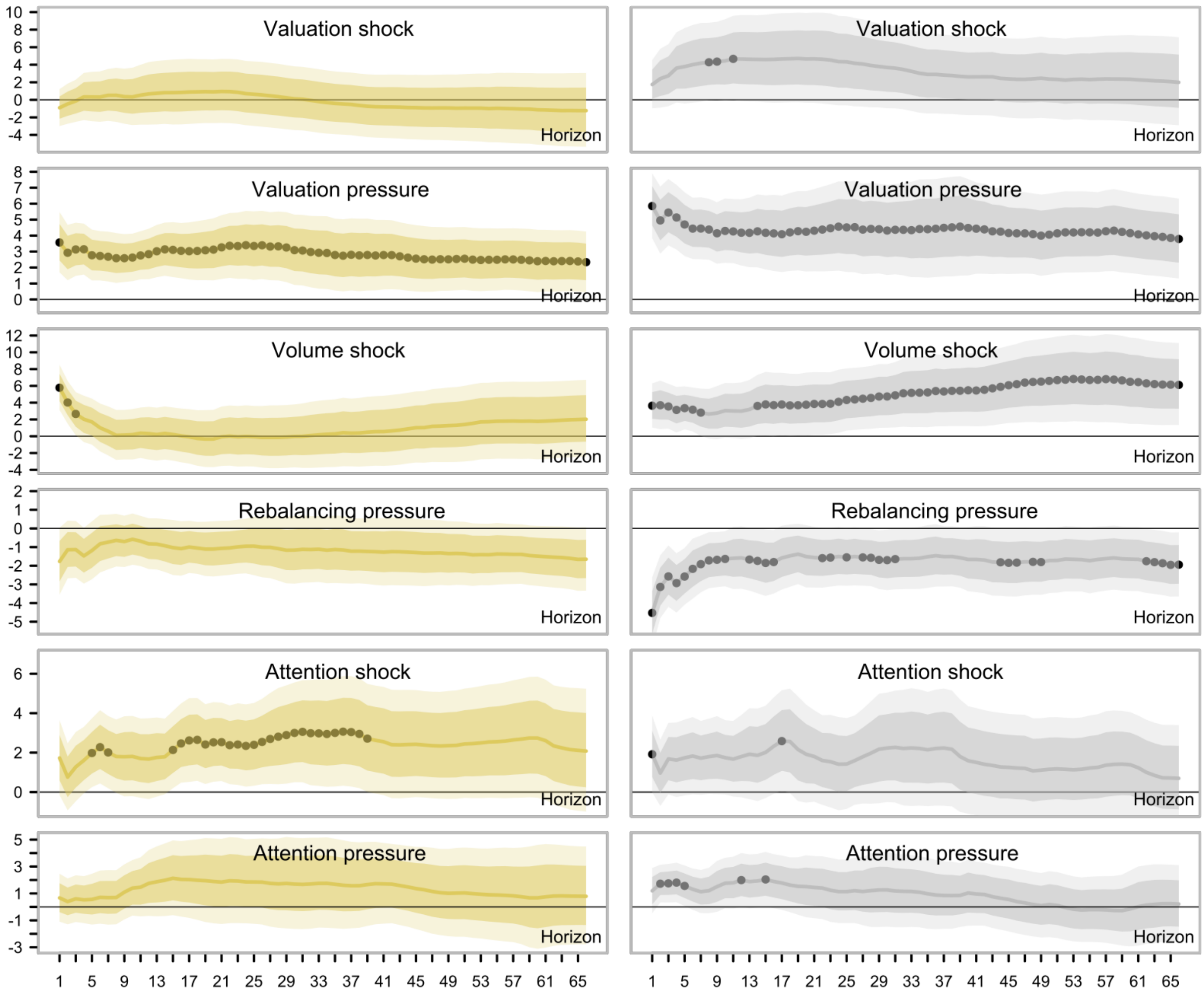

Figure B1: Local projection analysis: Continuous and jump variation model
Note: The graphs show the coefficients for the corresponding terms and their respective 68% (darker) and 90% (lighter) confidence intervals for up to three months ahead gold and silver realized variance for a variation of the model (15), which includes lagged continuous and jump variation components instead of realized variances. The valuation pressure is a term considering the squared shocks in GSR (Eq. 6). The re-balancing pressure is the product of a shock in trading volume and the valuation pressure (Eq. 10). The attention pressure is the product of a shock in the attention to gold and silver, and the valuation pressure (Eq. 12). The coefficient estimates are obtained by running in-sample models spanning the period from 19 August 2012 to 18 June 2025. The statistically significant coefficients are marked with thick dots.

C. Semi-variance volatility model results

Appendix C presents impulse response functions calculated from a volatility model, where future volatility is predicted by positive and negative semi-variances. The semi-variances follow Patton and Sheppard [2015],

$$
\begin{aligned}
PV_t &= K^{-1} \sum_{\delta=1}^{K} \sum_{j=1}^{\lfloor (N-\delta+1)/K \rfloor} \left[r_{t,\delta,j} \, \mathbb{I}(r_{t,\delta,j} > 0) \right]^2 , \\
NV_t &= K^{-1} \sum_{\delta=1}^{K} \sum_{j=1}^{\lfloor (N-\delta+1)/K \rfloor} \left[r_{t,\delta,j} \, \mathbb{I}(r_{t,\delta,j} \leq 0) \right]^2 ,
\end{aligned}
\tag{C.1}
$$

with $r_{t,\delta,j} = 100 \ln \frac{P_{t,\delta+Kj-1}}{P_{t,\delta+K(j-1)-1}}$. The $\boldsymbol{RM}_t$ vector in Eq. 15 now includes lagged and log-scaled PV_t, NV_t and corresponding weekly and monthly average components, instead of realized variance. The resulting impulse responses are shown in Figure C1.

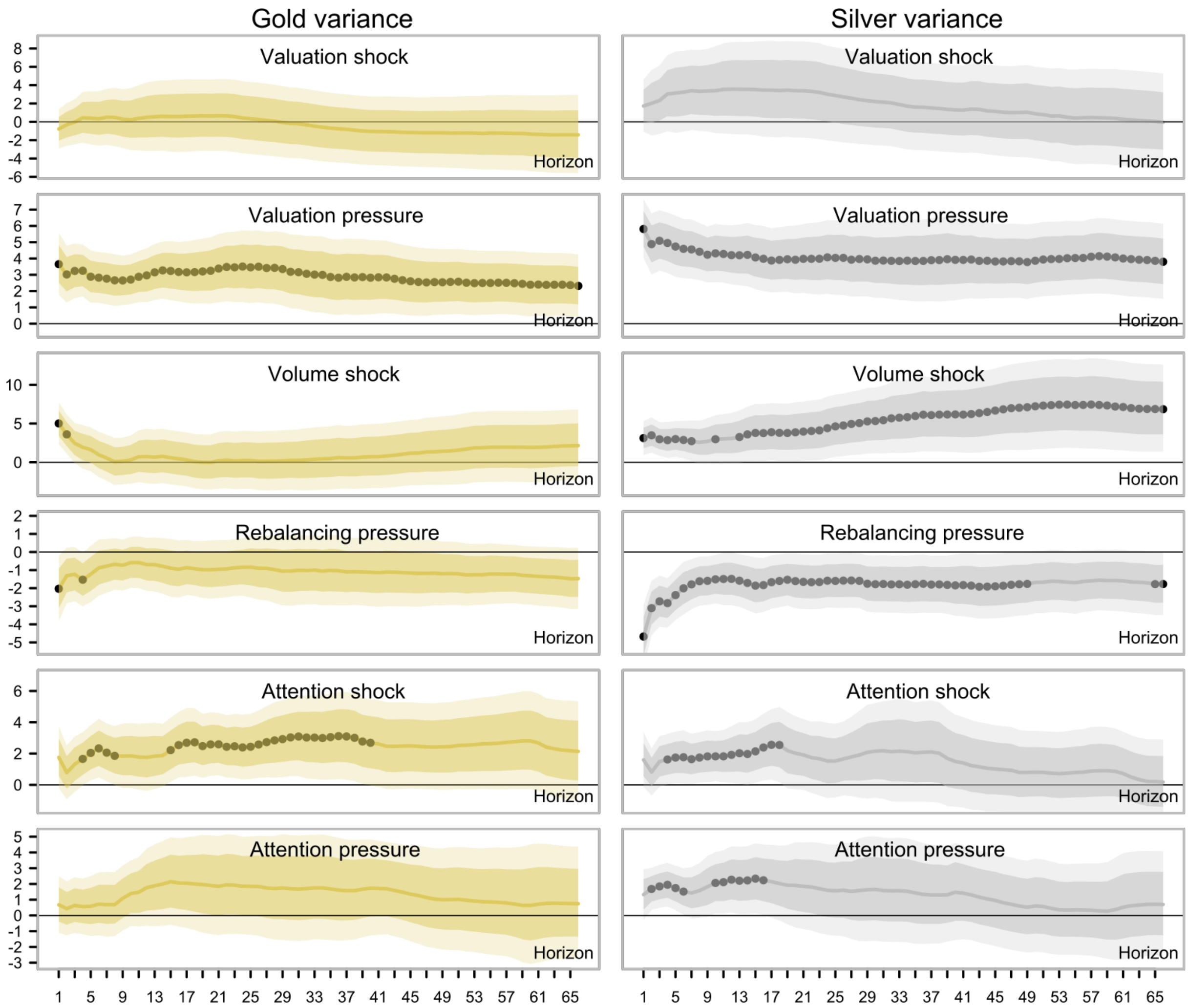

Figure C1: Local projection analysis: Semi-variance volatility model

Note: The graphs show the coefficients for the corresponding terms and their respective 68% (darker) and 90% (lighter) confidence intervals for up to three months ahead gold and silver realized variance for model (15), which includes lagged positive and negative semi-variances instead of realized variances. The valuation pressure is a term considering the squared shocks in GSR (Eq. 6). The re-balancing pressure is the product of a shock in trading volume and the valuation pressure (Eq. 10). The attention pressure is the product of a shock in the attention to gold and silver, and the valuation pressure (Eq. 12). The coefficient estimates are obtained by running in-sample models spanning the period from 19 August 2012 to 18 June 2025. The statistically significant coefficients are marked with thick dots.

D. Full out-of-sample results

Appendix D presents joint out-of-sample results across all models that include also other potential predictors. The results provide benchmarks against our main models, especially against the: i) HAR + GVZ + Valuation pressure and ii) HAR + GVZ + Rebalancing pressure that led to most accurate forecasts.

Table D1: Out-of-sample Gold Volatility Forecasts

	MSE				**QLIKE**			
Model	$h = 1$	$h = 5$	$h = 22$	$h = 66$	$h = 1$	$h = 5$	$h = 22$	$h = 66$
HAR	0.698	0.413	0.409	0.289	0.178	0.107	0.136	0.176
HAR + Valuation pressure	2.05%	2.28%	0.46%	0.14%	2.03%	2.27%	1.85%	0.21%
HAR + Rebalancing pressure	-0.23%	1.29%	1.45%	0.33%	-1.37%	-2.81%	-0.02%	-0.27%
HAR + Attention pressure	-0.60%	-0.99%	-1.35%	-0.49%	-1.63%	-6.04%	-1.51%	-1.51%
HAR + All pressures	0.99%	0.52%	-1.73%	-0.81%	1.31%	0.43%	1.59%	-1.73%
HAR + GVZ	1.13%	6.38%†	5.16%†	1.20%	3.85%	14.02%	10.98%	1.55%
HAR + GVZ + Valuation pressure	2.76%†	8.32%†	5.28%†	1.26%	4.98%	15.60%	12.36%	1.70%
HAR + GVZ + Rebalancing pressure	0.34%	6.67%†	6.09%†	1.52%	-5.72%	10.58%	10.38%	1.32%
HAR + GVZ + Attention pressure	0.44%	5.34%†	3.61%†	0.83%	2.86%	12.03%	9.03%	0.41%
HAR + GVZ + All pressures	1.53%†	6.52%†	2.38%	0.62%	-4.45%	14.42%	11.66%	0.48%
HAR + SRISK	-0.12%	-0.36%	0.85%	-5.65%	-5.79%	-6.86%	-3.94%	-19.08%
HAR + EPU	0.62%†	0.41%	0.33%	1.00%	1.12%	0.52%	2.01%	1.64%
HAR + COVOL	0.24%	-0.99%	-0.75%	0.07%	0.22%	-0.88%	-2.65%	-0.13%
HAR + QS	-0.25%	-1.24%	1.65%	0.63%	-3.92%	-2.47%	1.47%	0.96%
HAR + VIX	1.66%†	1.74%	0.28%	-2.72%	0.89%	1.59%	-0.04%	-5.96%
HAR + US EMU	0.16%	0.18%	-0.22%	-0.30%	-0.38%	-0.68%	-0.48%	-0.02%
HAR + Attention to gold	0.33%†	0.39%†	-0.80%	-11.47%	1.92%†	1.93%	-3.33%	-21.37%
HAR + RV of Silver	0.30%	0.10%	-2.32%	-3.41%	0.66%	0.49%	-4.94%	-7.70%
HAR + Illiquidity	-0.79%	-0.90%	0.93%†	5.00%†	-5.64%	-4.10%	1.61%	16.10%
HAR + NSI	-0.24%	-0.19%	-0.96%	-5.22%	-0.83%	-2.73%	-3.81%	-8.81%
HAR + GVZ + SRISK	1.11%	5.89%†	6.72%†	-5.61%	3.97%	14.61%	10.26%	-22.66%
HAR + GVZ + EPU	1.48%†	6.29%†	4.72%†	0.87%	4.39%	13.44%	10.71%	1.78%
HAR + GVZ + COVOL	1.12%	4.81%†	4.29%†	1.76%	3.68%	12.70%	8.26%	1.75%
HAR + GVZ + QS	1.75%†	5.83%†	6.33%†	0.26%	2.87%	13.65%	11.19%	-1.97%
HAR + GVZ + VIX	1.66%†	6.04%†	4.09%†	-0.69%	4.18%	13.23%	8.65%	-3.49%
HAR + GVZ + US EMU	1.11%†	6.41%†	4.75%†	0.76%	2.19%	12.92%	9.93%	1.48%
HAR + GVZ + Attention to gold	1.47%†	6.40%†	4.48%†	-6.25%	6.91%	15.38%	7.23%	-16.69%
HAR + GVZ + RV of silver	0.58%	4.94%†	3.07%†	-1.52%	4.11%	13.96%	7.63%	-4.90%
HAR + GVZ + Illiquidity	1.34%	6.23%†	7.67%†	4.92%†	5.80%	15.92%	14.01%	14.90%
HAR + GVZ + NSI	1.09%	6.23%†	4.40%†	-2.75%	4.21%	13.46%	8.44%	-4.55%

Notes: We report the average loss for the 'HAR' model and forecast improvements for competing models, with positive values indicating better forecasts. The symbol † is added if the given model belongs to the set of superior models (across all models in a given row) as indicated by the Diebold and Mariano [1995] test run within the Romano and Wolf [2005] step-wise approach and with a 10% family-wise error. We use moving-block bootstrap of length h and 999 replications. Similarly to attention pressure, EPU, US EMU, Attention to gold and NSI enter the models with an extra lag as their values are known only at the end of the calendar day.

Table D2: Out-of-sample Silver Volatility Forecasts

	MSE				QLIKE			
Model	$h = 1$	$h = 5$	$h = 22$	$h = 66$	$h = 1$	$h = 5$	$h = 22$	$h = 66$
HAR	16.24	10.79	7.54	5.29	0.143	0.119	0.177	0.239
HAR + Valuation pressure	2.60%	2.01%	0.35%	1.64%	3.44%	3.01%	2.04%	2.86%
HAR + Rebalancing pressure	0.45%	1.71%	2.67%	1.49%	-1.51%	-1.17%	-1.25%	-0.21%
HAR + Attention pressure	-0.14%	-0.50%	0.65%	0.12%	-0.87%	-1.26%	-0.78%	-3.63%
HAR + All pressures	2.49%	2.63%	2.64%	2.68%	2.67%	3.21%	1.30%	-0.51%
HAR + GVZ	-0.29%	4.07%	4.44%	3.13%	5.21%†	15.07%†	19.46%	10.05%
HAR + GVZ + Valuation pressure	2.26%	6.15%	5.54%	4.68%	8.69%†	17.84%†	21.36%	12.86%
HAR + GVZ + Rebalancing pressure	-0.01%	5.19%	7.03%	4.80%	4.33%	12.00%	17.43%	9.48%
HAR + GVZ + Attention pressure	-0.56%	3.42%	4.57%	2.95%	4.23%	13.68%†	16.77%	7.97%
HAR + GVZ + All pressures	2.01%	6.16%	7.81%	6.01%	7.31%†	17.03%†	17.99%	10.26%
HAR + SRISK	-0.12%	-0.09%	-0.89%	-15.67%	-4.02%	-3.47%	-5.49%	-68.66%
HAR + EPU	0.02%	0.02%	0.20%	0.60%	0.69%	0.82%	3.24%	2.53%
HAR + COVOL	0.05%	-0.57%	-0.16%	0.03%	0.01%	-0.72%	-1.05%	-1.01%
HAR + QS	0.13%	0.27%	2.01%	0.38%	0.77%	2.55%	3.86%	-1.11%
HAR + VIX	0.28%	0.18%	-0.14%	-2.95%	1.46%	4.51%†	1.53%	-6.78%
HAR + US EMU	-0.03%	-0.23%	-0.92%	-1.66%	0.34%	-0.55%	-1.05%	-1.14%
HAR + Attention to silver	0.08%	0.34%	-1.18%	-10.73%	2.10%†	2.21%	-2.86%	-18.86%
HAR + RV of gold	-0.22%	-0.86%	-5.10%	-7.32%	0.05%	-0.77%	-5.36%	-14.09%
HAR + Illiquidity	-0.26%	0.04%	1.66%	6.35%	-3.49%	-0.72%	4.36%	16.48%
HAR + NSI	-0.13%	0.10%	-0.09%	-6.36%	-0.60%	-1.67%	0.64%	-8.43%
HAR + GVZ + SRISK	-0.41%	3.85%	3.06%	-14.40%	5.12%†	14.45%†	15.12%	-42.77%
HAR + GVZ + EPU	-0.29%	3.81%	3.68%	2.73%	5.18%†	13.92%†	17.76%	9.72%
HAR + GVZ + COVOL	-0.40%	3.68%	4.53%	2.90%	4.73%†	13.54%†	17.76%	4.76%
HAR + GVZ + QS	-0.19%	4.06%	4.83%	-0.22%	4.85%†	14.91%†	18.79%	4.25%
HAR + GVZ + VIX	-0.32%	3.32%	3.90%	-1.18%	4.24%	13.39%†	16.65%	-3.90%
HAR + GVZ + US EMU	-0.35%	3.75%	3.52%	1.44%	5.18%†	14.10%†	17.72%	7.89%
HAR + GVZ + Attention to silver	-0.23%	4.45%	3.11%	-7.74%	6.93%†	16.24%†	16.04%	-0.17%
HAR + GVZ + RV of gold	-0.42%	4.27%	1.84%	-2.28%	5.36%†	14.81%†	16.21%	-1.04%
HAR + GVZ + Illiquidity	-0.35%	4.70%	7.87%†	7.94%	3.67%	14.87%†	23.86%	20.15%
HAR + GVZ + NSI	-0.32%	3.87%	3.07%	-4.19%	5.20%†	13.02%†	16.13%	2.32%

Notes: We report the average loss for the 'HAR' model and forecast improvements for competing models, with positive values indicating better forecasts. The symbol † is added if the given model belongs to the set of superior models (across all models in a given row) as indicated by the Diebold and Mariano [1995] test run within the Romano and Wolf [2005] step-wise approach and with a 10% family-wise error. We use moving-block bootstrap of length h and 999 replications. Similarly to attention pressure, EPU, US EMU, Attention to silver and NSI enter the models with an extra lag as their values are known only at the end of the calendar day.