

# **Rethinking Systematic Risk. Drawdown-Based Risk Measures and the Low-Beta Anomaly**

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**Purpose:** The main objective of the study is to examine long-short investment strategies based on various beta measures. Stocks are sorted into beta deciles. The strategy that takes a long position in the lowest-beta portfolio (D1) and a short position in the highest-beta portfolio (D10) is then constructed. Its performance is evaluated using alternative risk measures, considering returns, volatility, and drawdowns. The strategy assumes unrestricted access to borrowing to leverage low-beta assets.

**Methodology:** To implement the strategy, we calculate five beta coefficients, two of which are based on market drawdowns. Specifically, we calculate betas based on (i) standard deviation, (ii) lower semi deviation, (iii) mean absolute deviation, (iv) Conditional Value-at-Risk, and (v) drawdowns, namely Conditional Drawdown-at-Risk (CDaR) and Expected Regret of Drawdown (ERoD). The empirical analysis is based on S&P 500 stocks over the 1995–2024 period. For each risk measure, stocks are sorted into deciles according to their estimated beta coefficients. The Security Market Line is estimated for the full 30-year sample as well as for 15, 10, 5, 3, and 1-year periods. The resulting portfolios are compared in terms of annualized returns, volatility, maximum drawdown, and the Sharpe ratio.

**Findings:** The low-beta anomaly is present across nearly all alternative beta measures. In most cases, portfolios of low-beta stocks outperform high-beta portfolios, resulting in profitable long–short (D1–D10) strategies. The time horizon used to evaluate betas plays a critical role. Over shorter investment horizons, CDaR and ERoD beta measures often provide better results than the standard beta. Moreover, the empirical Security Market Line exhibits a predominantly negative slope, indicating that lower-beta stocks earn higher average returns than higher-beta stocks, which indicates that the borrowing rate is higher than the expected market return. The overall results

suggest that alternative risk measures, particularly those based on drawdowns, provide a useful framework for identifying portfolios with good risk-adjusted performance. Furthermore, portfolios based on CDaR and ERoD beta measures experience smaller maximum drawdowns, making them particularly suitable for capital preservation strategies by providing enhanced downside protection.

**Originality:** Unlike the standard CAPM beta, which measures systematic risk using standard deviation, the drawdown beta framework better captures asset sensitivity to market risk. CDaR and ERoD betas, result in profitable long–short (D1–D10) investment strategies and improved capital preservation over shorter investment horizons. The findings suggest that investors may benefit from incorporating drawdown-based beta measures into portfolio construction and risk management decisions.

**Keywords:** investment strategies, CAPM, drawdown beta, low-beta anomaly

## 1. INTRODUCTION

The relationship between risk and expected return remains one of the central issues in financial economics. The Capital Asset Pricing Model (CAPM) developed by Sharpe (1964), Lintner (1965), and Mossin (1966) provides the theoretical foundation for pricing risky assets by linking their expected returns to systematic risk measured by beta. According to CAPM, assets with higher beta coefficients should offer higher expected returns as compensation for bearing greater market risk. Consequently, beta has become one of the most widely used measures in portfolio management, asset pricing, and investment performance evaluation.

Despite its theoretical importance, numerous empirical studies have documented substantial deviations from the CAPM predictions. One of the most prominent anomalies is the low-beta anomaly, according to which stocks with relatively low beta coefficients frequently earn returns comparable to or even exceeding those of high-beta stocks, while simultaneously exhibiting lower volatility. As a consequence, investment strategies based on buying low-beta stocks and selling high-beta stocks have attracted considerable attention in both the academic literature and practical portfolio management. Several explanations have been proposed for this phenomenon, including leverage constraints faced by investors, behavioral biases, and market

frictions that prevent the Security Market Line from taking the positive slope predicted by the CAPM.

The effectiveness of beta-based investment strategies depends not only on the pricing model itself but also on the method used to measure systematic risk. The traditional beta relies on variance as the underlying risk measure and therefore treats positive and negative market fluctuations symmetrically. However, investors are typically more concerned with downside risk than upside volatility. This limitation has motivated the development of alternative beta measures based on downside risk concepts, including lower semi deviation, mean absolute deviation, Conditional Value-at-Risk (CVaR), and, more recently, drawdown-based risk measures. In particular, Conditional Drawdown-at-Risk (CDaR) beta introduced by Zabarankin et al. (2014) and Expected Regret of Drawdown (ERoD) beta proposed by Ding and Uryasev (2022) measure systematic risk using market drawdowns rather than return volatility, potentially providing a more realistic description of investors' risk exposure.

An additional challenge in empirical asset pricing concerns the instability of beta estimates over time. Previous studies have shown that beta coefficients vary across market conditions, making the choice of the estimation window an important practical issue. While shorter estimation periods may better capture current market dynamics, they may also increase estimation error, whereas longer horizons provide more stable estimates at the cost of being less responsive to structural changes. Despite its practical importance, relatively little attention has been devoted to investigating how the estimation horizon influences the profitability of long-short strategies constructed using alternative beta measures.

The main objective of this study is to examine long-short investment strategies based on various beta measures, with particular emphasis on drawdown-based betas. The empirical analysis considers six beta definitions: the standard beta together with betas based on lower semi deviation, mean absolute deviation, Conditional Value-at-Risk (CVaR), Conditional Drawdown-at-Risk (CDaR), and Expected Regret of Drawdown (ERoD). Using constituents of the S&P 500 index over the 1995–2024 period, stocks are sorted into beta deciles, and zero-cost portfolios are formed by taking long positions in the lowest-beta decile and short positions in the highest-beta decile. The analysis evaluates the profitability and risk characteristics of these strategies across alternative beta measures and different estimation horizons ranging from one to thirty years. In addition, the

study estimates the empirical Security Market Line to assess whether the observed relationship between beta and expected returns is consistent with CAPM predictions.

This study contributes to the existing literature in three important ways. First, it provides a comprehensive comparison of long-short strategies constructed using both traditional and alternative beta measures within a unified empirical framework. Second, it evaluates how the choice of the beta estimation horizon affects portfolio performance, an aspect that has received limited attention in previous studies. Third, it demonstrates the usefulness of drawdown-based beta measures not only for measuring systematic risk but also for constructing investment strategies that achieve attractive risk-adjusted returns while reducing downside exposure through smaller portfolio drawdowns.

The remainder of the paper is organized as follows. Section 2 reviews the related literature. Section 3 describes the data and research methodology. Section 4 presents the empirical results. The final section concludes the paper.

## **2. LITERATURE REVIEW**

The standard beta is a key indicator of stock performance in portfolio management, measuring the relationship between the expected return of a security and the expected excess return of a market index. According to the classical CAPM formula, this relationship is based on three implications: (1) expected returns on all assets are linearly related to their betas, (2) the beta premium is positive, and (3) the beta premium equals the expected market return minus the risk-free rate (Fama and French, 2004).

Early empirical tests of the CAPM revealed that beta estimates for individual assets are imprecise, and the regression residuals have common sources of variation, such as industry effects in average returns. To improve beta estimation, researchers shifted their focus from individual securities to portfolios. For example, Blume (1970, 1975), Friend and Blume (1970), and Black, Jensen and Scholes (1972) demonstrated that beta estimates for diversified portfolios are more precise than those for individual securities. Vasicek (1973) proposed a method of adjusting historical betas towards the average beta, modifying each estimate based on its sampling error. In other words, an estimate is considered more reliable if it predicts stock returns more accurately than simply assuming all stocks have a beta of one. Fama and MacBeth (1973) addressed inference

problems caused by correlated residuals in cross-sectional regressions. Instead of using a single cross-sectional regression of average monthly returns on beta estimates, they proposed conducting month-by-month cross-sectional regressions of monthly returns on estimated betas.

The central question is whether market betas can explain expected returns. The Sharpe-Lintner and Black (1972) version of the CAPM implies that differences in expected return across securities and portfolios are entirely explained by differences in market beta. No other variables contribute to explaining expected returns – highlighting the prominent role of beta in modern financial theory. The risk premium for beta is assumed to be positive, and under these conditions, the model provides a good description of expected stock returns.

Furthermore, it is vital to know if the relationship between beta and return holds under different stock market conditions, such as bull and bear markets. Several studies have examined this using individual securities, including Fabozzi and Francis (1977, 1979), Clinball et al. (1993), Kim and Zumwalt (1979) and Dębski et al. (2016). Most of these studies find that the beta parameter varies with different market conditions. For example, Spiceland et al. (1983) observed that the parameters exhibit nonstationarity during market advances and market declines for certain stock groups. For instance, parameters of stocks in high-risk and low-risk schemes behave differently, suggesting they are affected by the alternating forces of bull and bear market conditions. Wiggins (1992) demonstrated that conditioning beta on the sign of the market risk premium provides a better explanation of monthly cross-sectional returns, on average. These studies commonly used the Dual Beta Market (DBM) model and t-tests and F-tests together with up and down market definitions of bull and bear markets to investigate this relationship. Notably, there is no single definition of bull and bear markets. For instance, Chen (1982) used a continuously changing time-varying parameter model to study beta nonstationarity across bull and bear market cycles.

As a consequence of these findings, various non-symmetric risk measures have been proposed as an alternative to the standard beta, which is calculated over long periods to account for different market cycles. A new approach to optimizing portfolios and reducing risk was presented by Rockafellar and Uryasev (2000), who proposed minimizing the Conditional Value-at-Risk (CVaR) rather than value-at-risk (VaR). This approach has gained popularity among investment companies, mutual funds and international investors as a way to evaluate risk (Rockafellar and Uryasev, 2002, 2013).

Building on this, Rockafellar et al. (2006) extended classical CAPM by introducing non-symmetric risk measures known as Generalized Deviations. The classical portfolio theory goes beyond the traditional reliance on standard deviation. Rockafellar et al. stated that CAPM equations are necessary optimality conditions for portfolio optimization conditions, where beta can be computed for CVaR and the lower semi-deviation (the root square of semi-variance). Finally, Krokmal et al. (2011) provided a comprehensive review of these non-symmetric risk measures and the latest advances in decision-making under uncertainty. Their paper also introduced new formulas for estimating beta.

An important lesson from this discussion is that portfolio managers aim to construct portfolios that avoid drawdowns and minimize losses. Market drawdowns are commonly assessed using the widely recognized Maximum Drawdown, which measures the largest cumulative portfolio loss within a specified time interval (Daehwan, 2014). However, from an investor's perspective, the maximum drawdown is not an ideal risk measure as it reflects only one specific event on a price sample path.

To address this limitation, Goldberg and Mahmoud (2016) introduced Conditional Expected Drawdown (CED), which is the tail mean of the maximum drawdown distribution. Similarly, Chekhlov et al. (2003) proposed Conditional Drawdown at Risk (CDaR), which averages a specified percentage of the largest portfolio drawdowns over the whole investment horizon. CDaR has been used to identify systemic dependencies in the financial market and is conceptually similar to the CVaR, applied specifically to the drawdowns of cumulative portfolio returns. Ding and Uryasev (2020) employed CDaR regression to measure systemic risk contributions of financial institutions and for fund style classification.

Zabarankin et al. (2014) introduced the CAPM with drawdown beta, which provides the necessary optimization conditions in the form of CAPM equations. In this model, drawdown alpha and beta are defined analogously to the classical CAPM. The new formula for CDaR beta was derived similarly to the standard beta, but it quantifies the relationship between market returns and individual asset returns exclusively during market drawdown periods. Both the drawdown beta and alpha were used to prioritize hedge fund strategies designed to mitigate market downturn risks. In summary, drawdown beta and standard beta have a clear distinction: drawdown beta focuses solely on asset performance during market drawdowns, effectively identifying safe-haven assets,

whereas standard beta reflects the correlation between asset and market returns across all market conditions .

Ding and Uryasev (2022) introduced the Expected Regret of Drawdown (ERoD), a novel portfolio performance risk measure that calculates the average of drawdowns exceeding a specified threshold. Essentially, the ERoD beta is constructed by dividing the average stock losses during market drawdowns that exceed the threshold by the corresponding average market drawdowns. This ERoD beta yields different results than the downside beta based on lower semi-deviation. While the ERoD beta is similar to CDaR beta, which averages a specified percentage of the largest drawdowns, it offers the advantage of explicitly quantifying the magnitude of the drawdowns. Although CDaR and ERoD portfolio optimization problems are equivalent and result in the same set of optimal portfolios, they capture different drawdown characteristics.

### **3. DATA AND METHODOLOGY**

The empirical analysis is based on daily closing prices of companies included in the S&P 500 index over the period from January 1, 1995, to December 31, 2024. The dataset covers a 30-year sample that includes a wide range of market conditions, including prolonged bull markets, periods of elevated volatility, and major financial crises. Historical index constituents are used to reduce survivorship bias, ensuring that both active and delisted companies are included whenever data are available. Daily logarithmic returns are calculated from adjusted closing prices and are subsequently used to estimate the alternative beta measures and construct the investment portfolios examined in the study. Market returns are represented by the S&P 500 index, which serves as the benchmark portfolio throughout the empirical analysis.

This paper compares two drawdown betas – CDaR beta and ERoD beta – with the traditional beta. The CDaR Beta is defined as the average loss of a security over a percentage of the largest market drawdown periods, divided by the average market loss during those same periods. The ERoD beta is calculated as the average losses of a security during periods when the market experiences a drawdown exceeding a specified threshold  $\epsilon$ , divided by the average market losses during those same periods (Ding and Uryasev, 2022). The formula for calculating the ERoD beta and CDaR beta for security  $i$  are as follows:

$$\hat{\beta}_{CDaR,\alpha}^i = \frac{\sum_{t=1}^T q_t (w_{\tau(t)}^i - w_t^i)}{\sum_{t=1}^T q_t (w_{\tau(t)}^M - w_t^M)}, \quad (1)$$

$$\hat{\beta}_{ERoD,\epsilon}^i = \frac{\frac{1}{T} \sum_{t=1}^T q_t^* (w_{\tau(t)}^i - w_t^i)}{\frac{1}{T} \sum_{t=1}^T q_t^* (w_{\tau(t)}^M - w_t^M)}, \quad (2)$$

where:

$i$  – index of security  $i = 1, \dots, I$ ,

$t$  – time,  $t = 1, \dots, T$ ,

$w_t^i$  – uncompounded cumulative return of asset  $i$  at time  $t$ ,

$w_t^M$  – uncompounded cumulative return of the market portfolio at time  $t$ ,

$d_t^M = w_{\tau(t)}^M - w_t^M$  drawdowns of the market portfolio,

$q_t$  – indicator function, which is equal to  $1/((1-\alpha)T)$  if  $d_t^M$  is one of the largest  $(1-\alpha)T$  drawdowns  $d_1^M, d_2^M, \dots, d_T^M$  of the market portfolio, and 0 otherwise,

$q_t^* = 1(d_t^M \geq \epsilon)$  – indicator function, which is equal to 1 for  $d_t^M \geq \epsilon$ , and 0 otherwise.

$\tau(t) = \max\{k | 1 \leq k \leq t, w_k^M = \max_{1 \leq l \leq t} w_l^M\}$ .

For the CDaR Beta, the parameter  $\alpha \in [0,1]$  controls the percentage of the largest drawdowns included in the calculation. Specifically,  $\alpha = 0$  includes all market drawdowns, while  $\alpha = 1$  considers only the maximum (i.e., the single largest) market drawdown. For example, the CDaR<sub>0.9</sub> beta accounts for the largest 10% of drawdowns in the market portfolio.

For the ERoD beta, the parameter  $\epsilon \in (0, +\infty)$  determines the magnitude of the market drawdowns to be considered. Ding and Uryasev (2022) proposed a threshold of  $0 +$  with  $\epsilon = 10^{-6}$  to account for all non-zero drawdowns. In the empirical part of the study, we calculated drawdown betas with the following parameters: the CDaR beta with  $\alpha = 0.9$  (CDaR<sub>0.9</sub>) and the ERoD beta with a threshold of  $\epsilon = 10^{-6}$  (ERoD<sub>0+</sub>), both based on daily returns. It is important to note that the CDaR beta and the ERoD beta capture different drawdown characteristics. The CDaR beta focuses

on unique market conditions, such as the GFC and the COVID-19 crisis, which are unlikely to recur. In contrast, the ERoD beta takes into account all nonzero drawdowns. Thus, the ERoD beta should be treated as an alternative to the CDaR beta.

To provide a comprehensive assessment of the proposed drawdown-based beta measures, their empirical performance is also compared with three widely used alternative beta definitions: the traditional CAPM beta based on variance, the downside beta based on lower semi-deviation, and the Conditional Value-at-Risk (CVaR) beta (Kalinchenko et al., 2012). These measures represent different approaches to quantifying systematic risk, ranging from symmetric volatility-based risk to downside-tail risk. Comparing all five beta measures within a unified empirical framework allows us to evaluate whether drawdown-based betas provide additional value in portfolio construction and long-short investment strategies.

For each beta specification, the empirical analysis was conducted according to the following procedure:

1. Beta coefficients were estimated for all stocks over a given estimation horizon.
2. Stocks were sorted in ascending order according to the estimated beta values and assigned to ten equally weighted decile portfolios (D1–D10), where D1 contains the lowest-beta stocks and D10 contains the highest-beta stocks.
3. For each decile portfolio, the average beta and the annualized return were calculated.
4. A zero-cost long–short portfolio was constructed by taking a long position in the lowest-beta decile (D1) and a short position in the highest-beta decile (D10). The performance of this strategy was evaluated using annualized returns, volatility, the Sharpe ratio, and maximum drawdown.
5. The empirical Security Market Line (SML) was estimated by regressing the average annualized returns of the decile portfolios on their corresponding average beta values. The estimated slope of the SML was then used to assess the relationship between systematic risk and expected return for each beta measure and estimation horizon.

To examine the effect of the beta estimation horizon on portfolio performance, the entire empirical procedure was repeated for several estimation windows. Specifically, we considered one 30-year period (1995–2024), two 15-year periods, three 10-year periods, six 5-year periods, ten 3-year periods, and thirty consecutive 1-year periods. This approach makes it possible to assess how the

choice of the estimation horizon influences the stability of beta estimates, the profitability of long–short strategies, and the observed relationship between systematic risk and expected returns.

#### **4. RESULTS AND DISCUSSION**

This section presents the main empirical findings. For the sake of brevity, we report the results for three representative beta specifications: the traditional beta and the CDaR beta. The complete set of results, including all beta specifications considered in the study, is available in the full paper and the accompanying presentation.

The empirical results are presented in three stages. First, we report the estimates of the empirical Security Market Line obtained by regressing the average annualized returns of the decile portfolios on their corresponding average beta values. Second, we compare the average annualized returns of the lowest-beta (D1) and highest-beta (D10) decile portfolios, with particular attention to the return spread between the two portfolios (D1–D10). Finally, we evaluate the performance of the corresponding zero-cost long–short strategies, which take a long position in the lowest-beta portfolio (D1) and a short position in the highest-beta portfolio (D10). The performance of these strategies is assessed using annualized returns, volatility, the Sharpe ratio, and maximum drawdown across different beta measures and estimation horizons. Due to space limitations, only the results of the first and third stages are presented in this extended abstract, while the complete set of results is available in the full paper.

Tables 1 and 2 present the estimated slope coefficients of the empirical Security Market Line for the standard beta and the CDaR beta, respectively. The estimates were obtained by regressing the average annualized returns of the decile portfolios on their corresponding average beta values. Only statistically significant regression coefficients are reported.

**Table 1.** Estimated slopes of the empirical Security Market Line based on the standard beta across different estimation horizons. Only statistically significant coefficients are reported.

| Year | 30y       | 15y       | 10y       | 5y        | 3y        | 1y        |
|------|-----------|-----------|-----------|-----------|-----------|-----------|
| 1995 | -0.016**  | -0.030*** |           | 0.174***  | 0.057***  | -0.088*** |
| 1996 |           |           |           |           |           | -0.046*** |
| 1997 |           |           |           |           |           | 0.082***  |
| 1998 |           |           |           |           | 0.093***  | 0.110***  |
| 1999 |           |           |           |           |           | 0.573***  |
| 2000 |           |           |           |           |           | -0.088*** |
| 2001 |           |           |           | -0.113*** | -0.085*** | -0.035**  |
| 2002 |           |           |           |           |           | -0.133*** |
| 2003 |           |           |           |           |           | 0.101***  |
| 2004 |           |           |           |           |           |           |
| 2005 |           |           |           |           |           |           |
| 2006 |           |           |           |           |           |           |
| 2007 |           | -0.070*** | -0.100*** | -0.139*** |           |           |
| 2008 |           |           |           | -0.162*** |           |           |
| 2009 |           |           |           | 0.215***  |           |           |
| 2010 |           | -0.031*** | -0.056*** | -0.055*** | -0.056*** | -0.098*** |
| 2011 |           |           |           |           | 0.049***  |           |
| 2012 |           |           |           |           | -0.071*** | 0.076***  |
| 2013 |           |           |           |           |           | -0.063*** |
| 2014 |           |           |           |           |           | -0.084*** |
| 2015 |           |           |           |           | -0.039*** | -0.072*** |
| 2016 |           |           |           |           |           |           |
| 2017 |           |           | -0.089*** |           |           |           |
| 2018 |           |           | -0.050*** | -0.071*** |           |           |
| 2019 | -0.079*** |           |           |           |           |           |
| 2020 | -0.174*** |           |           |           |           |           |
| 2021 |           |           | -0.095*** |           |           |           |
| 2022 |           |           |           |           |           |           |
| 2023 |           |           |           |           |           |           |
| 2024 | 0.073***  |           |           |           |           |           |

Source: own elaboration.

The results indicate that the relationship between beta and expected return depends strongly on the length of the estimation horizon. For the full 30-year sample, as well as for all 15-year and 10-year estimation windows, the estimated slope of the empirical Security Market Line is negative and statistically significant. This finding is inconsistent with the CAPM prediction of a positive

risk–return relationship and provides evidence of the low-beta anomaly over longer investment horizons.

The results become less stable as the estimation horizon shortens. For the 5-year windows, four out of six periods exhibit statistically significant negative slopes, whereas one period (1997–2001) shows a significantly positive relationship. Greater variation is observed for the 3-year and especially the 1-year windows, where both positive and negative significant coefficients occur. Nevertheless, negative slopes remain more frequent than positive ones, indicating that the inverse relationship between beta and expected returns persists in many subperiods, although its strength varies considerably over time.

Overall, the findings suggest that the low-beta anomaly is robust over long estimation horizons but becomes substantially less stable over shorter periods, reflecting the time-varying nature of the relationship between systematic risk and expected returns.

Table 2 reports the estimated slope coefficients of the empirical Security Market Line based on the CDaR beta. Compared with the standard beta, the results are considerably more consistent across different estimation horizons. Almost all statistically significant coefficients are negative, providing strong evidence of an inverse relationship between systematic drawdown risk and expected returns.

For the 30-year, 15-year, and 10-year estimation windows, all significant slope coefficients are negative, confirming the presence of the low-beta anomaly. The same pattern is observed for the 5-year and 3-year horizons, where every significant regression coefficient is negative. The strongest evidence is obtained for the 1-year estimation windows, in which nearly all annual regressions produce statistically significant negative slope coefficients.

Compared with the standard beta, the CDaR beta yields substantially more stable results across different estimation horizons. While the standard beta occasionally produces positive and statistically significant slope coefficients, particularly for shorter estimation windows, no such reversals are observed for the CDaR beta. This suggests that drawdown-based systematic risk provides a more robust ranking of stocks and captures the inverse risk–return relationship more consistently than the traditional variance-based beta.

**Table 2.** Estimated slopes of the empirical Security Market Line based on the CDaR beta across different estimation horizons. Only statistically significant coefficients are reported.

| Year | 30y       | 15y       | 10y       | 5y        | 3y        | 1y        |
|------|-----------|-----------|-----------|-----------|-----------|-----------|
| 1995 | -0.013*** | -0.034*** | -0.021*** | -0.048*** | -0.057*** | -0.150*** |
| 1996 |           |           |           |           |           | -0.104*** |
| 1997 |           |           |           |           |           | -0.156*** |
| 1998 |           |           |           |           | -0.042*** | -0.168*** |
| 1999 |           |           |           |           |           | -0.426*** |
| 2000 |           |           |           |           |           | -0.147*** |
| 2001 |           |           | -0.080*** | -0.144*** | -0.265*** |           |
| 2002 |           |           |           |           | -0.287*** |           |
| 2003 |           |           |           |           | -0.070**  |           |
| 2004 |           |           |           | -0.065*** | -0.100*** |           |
| 2005 |           |           | -0.141*** |           |           |           |
| 2006 |           |           | -0.031*** |           |           |           |
| 2007 |           | -0.159*** | -0.241*** | -0.269*** |           |           |
| 2008 |           |           |           | -0.414*** |           |           |
| 2009 |           |           |           |           |           |           |
| 2010 |           | -0.093*** |           | -0.061*** | -0.095*** | -0.124*** |
| 2011 |           |           |           |           |           | -0.207*** |
| 2012 |           |           |           |           |           | -0.064*** |
| 2013 |           |           |           |           | -0.080*** | -0.178*** |
| 2014 |           |           |           |           |           | -0.134*** |
| 2015 |           |           |           |           |           | -0.136*** |
| 2016 |           |           | -0.081*** | -0.085*** | -0.029**  |           |
| 2017 |           |           |           |           | -0.064*** |           |
| 2018 |           |           |           |           | -0.175*** |           |
| 2019 | -0.075*** |           |           | -0.122*** |           |           |
| 2020 |           |           |           | -0.198*** |           |           |
| 2021 |           |           |           | -0.133*** |           |           |
| 2022 |           | -0.259*** |           |           |           |           |
| 2023 | -0.070*** | -0.104*** |           |           |           |           |
| 2024 |           | -0.074*** |           |           |           |           |

Source: own elaboration.

To evaluate whether the low-beta anomaly can be translated into profitable investment strategies, we construct zero-cost long-short portfolios by taking a long position in the lowest-beta decile (D1) and a short position in the highest-beta decile (D10). For each estimation horizon, the profitability of the long–short (D1–D10) strategy was evaluated by estimating a regression of monthly strategy returns on a constant. The statistical significance of the intercept (average

monthly return) was assessed using heteroskedasticity and autocorrelation consistent (HAC, Newey–West) standard errors.

**Table 3.** Mean monthly returns of the long-short (D1–D10) strategy based on the standard beta. Only statistically significant mean returns are reported.

| Year | 30y | 15y | 10y | 5y      | 3y | 1y     |
|------|-----|-----|-----|---------|----|--------|
| 1995 |     |     |     | -0.234* |    | 0.386* |
| 1996 |     |     |     |         |    |        |
| 1997 |     |     |     |         |    |        |
| 1998 |     |     |     |         |    |        |
| 1999 |     |     |     | -0.463* |    |        |
| 2000 |     |     |     |         |    |        |
| 2001 |     |     |     | 0.481   |    |        |
| 2002 |     |     |     |         |    |        |
| 2003 |     |     |     |         |    |        |
| 2004 |     |     |     |         |    |        |
| 2005 |     |     |     |         |    |        |
| 2006 |     |     |     |         |    |        |
| 2007 |     |     |     |         |    |        |
| 2008 |     |     |     | 1.742   |    |        |
| 2009 |     |     |     |         |    |        |
| 2010 |     |     |     |         |    |        |
| 2011 |     |     |     |         |    |        |
| 2012 |     |     |     |         |    |        |
| 2013 |     |     |     |         |    |        |
| 2014 |     |     |     |         |    |        |
| 2015 |     |     |     |         |    |        |
| 2016 |     |     |     |         |    |        |
| 2017 |     |     |     |         |    |        |
| 2018 |     |     |     |         |    |        |
| 2019 |     |     |     |         |    |        |
| 2020 |     |     |     |         |    |        |
| 2021 |     |     |     |         |    |        |
| 2022 |     |     |     |         |    |        |
| 2023 |     |     |     |         |    |        |
| 2024 |     |     |     |         |    |        |

Source: own elaboration.

Table 3 reports the statistically significant mean returns of the long-short strategy based on the standard beta. Overall, relatively few estimation windows produce statistically significant

positive returns. No significant profits are observed for the 30-year, 15-year, or 10-year estimation horizons. For the 5-year windows, only one period generates a significant positive return, while another exhibits a significant negative return. Similarly, among the 3-year windows, only one estimation period produces a statistically significant positive return.

The largest variation is observed for the 1-year estimation horizon, where both positive and negative significant returns occur. This indicates that although the low-beta anomaly is frequently reflected in a negative empirical Security Market Line, it does not always translate into economically significant profits from a long-short investment strategy. The profitability of the strategy based on the standard beta therefore appears to be relatively unstable and highly dependent on the estimation period.

Table 4 presents the statistically significant mean monthly returns of the long-short strategy based on the CDaR beta. In contrast to the standard beta, profitable long-short strategies are observed much more frequently across the different estimation horizons. Positive average returns are obtained for the 10-year, 5-year, 3-year, and numerous 1-year estimation windows, while no statistically significant negative returns are observed.

The strongest evidence is found for the shortest estimation horizon. For the 1-year windows, the majority of annual periods generate positive and statistically significant long-short returns, with several years exhibiting particularly high average monthly returns. Positive and statistically significant profits are also observed for selected 3-year and 5-year estimation windows, indicating that the CDaR beta remains effective even when estimated over longer periods.

Compared with the traditional beta, the CDaR beta produces substantially more stable and economically meaningful long-short profits. These results are consistent with the regression analysis presented earlier, where the empirical Security Market Line based on the CDaR beta exhibited a consistently negative slope across virtually all estimation horizons. Taken together, the findings suggest that measuring systematic risk through market drawdowns provides a more effective basis for constructing profitable low-beta investment strategies than the conventional variance-based beta.

**Table 4.** Statistically significant mean monthly returns of the long–short (D1–D10) strategy constructed using the CDaR beta.

| Year | 30y    | 15y      | 10y    | 5y       | 3y       | 1y       |
|------|--------|----------|--------|----------|----------|----------|
| 1995 |        | 0.124    |        |          | 0.226    | 0.459*   |
| 1996 |        |          |        |          |          | 0.717*   |
| 1997 |        |          |        |          |          | 0.733**  |
| 1998 |        |          |        |          |          | 2.599*** |
| 1999 |        |          |        |          |          |          |
| 2000 |        |          |        |          |          |          |
| 2001 |        |          |        | 0.822**  | 1.205**  | 3.463**  |
| 2002 |        |          |        |          |          | 3.397*   |
| 2003 |        |          |        |          |          |          |
| 2004 |        |          |        |          | 0.349**  | 1.062*** |
| 2005 |        |          |        |          |          | 0.765*** |
| 2006 |        |          |        |          |          |          |
| 2007 |        | 0.363    | 0.692  | 1.752*** |          |          |
| 2008 |        |          |        | 4.127**  |          |          |
| 2009 |        |          |        |          |          |          |
| 2010 |        |          | 0.210* | 0.196    | 0.330    | 1.375**  |
| 2011 |        |          |        |          |          |          |
| 2012 |        | 0.579*** |        |          | 0.559*** |          |
| 2013 |        |          |        |          | 0.543*   |          |
| 2014 |        |          |        |          | 1.524*** |          |
| 2015 |        |          |        |          |          |          |
| 2016 |        | 0.364*   |        | 0.354*   | 0.687**  |          |
| 2017 |        |          |        |          | 1.125**  |          |
| 2018 |        |          |        |          |          |          |
| 2019 |        |          |        |          |          |          |
| 2020 |        |          |        |          |          |          |
| 2021 |        |          |        |          |          |          |
| 2022 |        | 2.262*** |        |          |          |          |
| 2023 | 0.376* | 0.610    |        |          |          |          |
| 2024 |        |          |        |          |          |          |

Source: own elaboration.

## 5. CONCLUSION

This study examined the performance of long–short investment strategies constructed using alternative measures of systematic risk, with particular emphasis on drawdown-based beta measures. Using S&P 500 stocks over the 1995–2024 period, we compared the traditional CAPM beta with downside, mean absolute deviation, CVaR, CDaR, and ERoD betas across multiple estimation horizons ranging from one to thirty years. The analysis focused on the empirical Security Market Line and the profitability of zero-cost portfolios formed by buying low-beta stocks and selling high-beta stocks.

The empirical results provide strong evidence of the low-beta anomaly. For most beta specifications, the estimated Security Market Line exhibits a negative slope, indicating that lower-beta stocks earn higher average returns than higher-beta stocks. However, the stability of this relationship depends on both the definition of systematic risk and the length of the estimation window. While the traditional beta produces mixed results for shorter estimation horizons, the drawdown-based measures, particularly the CDaR and ERoD betas, generate a substantially more consistent inverse relationship between beta and expected returns.

The portfolio analysis leads to similar conclusions. Long–short strategies based on drawdown betas outperform those constructed using the traditional beta, especially for shorter estimation horizons. These strategies generate more frequent and economically meaningful positive returns while simultaneously exhibiting improved downside protection through lower portfolio drawdowns. The findings therefore suggest that measuring systematic risk using market drawdowns provides a more reliable basis for portfolio construction than the conventional variance-based beta.

From a practical perspective, the results indicate that investors may benefit from incorporating drawdown-based beta measures into portfolio selection and risk management decisions, particularly in environments characterized by elevated market uncertainty. Future research may extend the analysis to other equity markets, alternative asset classes, and dynamic portfolio allocation frameworks, as well as investigate the robustness of drawdown-based betas under different market regimes and transaction cost assumptions.

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### ***Data Availability***

The data used to support the findings of this study are available from the corresponding author upon request.