# IN SEARCH OF A BETTER-FITTING MODEL FOR THE DYNAMICS OF INTRADAY VOLUME IN EQUITY MARKETS: AN IN-SAMPLE COMPARISON OF FOURIER-MEM MODELS

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***Abstract:*** We use the 10-minute trading volume data of highly actively traded stocks listed on well-developed stock exchanges to compare the modelling ability of a standard AR-GARCH model with alternative multiplicative error models (MEMs). We consider a standard AR-GARCH model with a conditional normal distribution for log trading volumes and multiplicative error models with Burr, generalized gamma, generalized beta of the second kind, log-normal, and time-varying log-normal error distributions. For all models, an intraday periodic component is specified via a Fourier series representation. A formal Bayesian comparison of all models is conducted using Bayes factors. The empirical results clearly indicate that the most adequate specifications are the linear ACV model with a time-varying log-normal distribution and the linear ACV model with a Burr or generalized beta of the second kind error distribution, which have very high Bayes factors against other models.

***Keywords*:** intraday trading volume, GARCH, multiplicative error model, model comparison, Bayes factors

## ENTENDED ABSTRACT

## INTRODUCTION

Modelling of trading volume time series is highly relevant for both financial practice and research, becoming a central task for professionals. Understanding how trading volumes behave is vital for analyzing stock trading, market trends, and the behavior of market participants. Accurate volume predictions can also provide valuable insights into asset prices. Additionally, studying the role of trading volume in trading is important for policymakers and institutional investors to better understand market responses to information shocks and how new information spreads among participants. Ultimately, grasping volume dynamics can lead to improved methods for modeling and forecasting trading volumes.

Previous research has primarily and extensively examined how trade size relates to various financial variables, such as price and return volatility—examples include: Clark (1973), Epps and Epps (1976), Jennings et al. (1981), Tauschen and Pitts (1983), Easley and O'Hara (1987), Karpoff (1987), Foster and Viswanathan (1990), Lamoureux and Lastrapes (1990), Gallant et al. (1992), Bessembinder and Seguin (1993), Jones et al. (1994), Andersen (1996), Gouriéroux et al. (1999), Gallo and Pacini (2000), Chen et al. (2001), Darrat et al. (2003), Manganelli (2005), Girard and Biswas (2007), Chevallier and Sevi (2012), Slim and Dahmene (2016), Ftiti et al. (2017), among others. Consequently, the modeling and forecasting of trading volume in stock markets has not been a main focus of financial econometrics research for many years. However, these issues are addressed in some works such as Kaastra and Boyd (1995), Lo and Wang (2000), Białkowski et al. (2008), Brownlees et al. (2011), Li and Ye (2013), Satish et al. (2014), Ye et al. (2014), Ito (2016), Chen et al. (2016), Liu and Lai (2017), Szűcs (2017), Libman et al. (2019), Huptas (2018, 2019, 2025, 2026), Naimoli and Storti (2019), Nasir et al. (2019), Lahmiri et al. (2022), Liu et al. (2022), among others. All the above-mentioned studies discuss different ways of modeling trading volumes. Both traditional econometric models, such as ARMA models, multiplicative error models and their generalizations, and machine learning techniques, such as neural networks, random forest or support vector regression, have been used to model and forecast financial trading volumes related not only to stocks, but also to futures, cryptocurrencies, and exchange-traded funds.

It must be noted that econometric modelling of trading volume has been typically performed by means of ARMA-GARCH-type models so far, see Bessembinder and Seguin (1993), Bjonnes et al. (2003), among others. Specifying a standard autoregressive model for log variables is simply a natural and easy starting point for a model for positive-valued variables. Alternatively, modelling trading volumes can be straightforwardly performed using the autoregressive conditional volume (ACV) model of Manganelli (2005), which belongs to the general class of multiplicative error models, see Engle (2002). Moreover, ACV models are parsimoniously parameterised as ARMA-GARCH-type models.

The main aim of this research is to compare the modelling ability of a standard AR-GARCH model for log trading volumes and alternative multiplicative error models (MEMs) with a Fourier form for the seasonal component. Surprisingly, to the author's best knowledge, such a study has not appeared in the financial econometrics literature so far. Therefore, it seems particularly important to fill this gap and make a formal Bayesian comparison of these models using real financial data. In this paper, we consider a standard AR-GARCH model with a conditional normal distribution, as well as alternative linear autoregressive conditional volume models with the Burr, generalized gamma, generalized beta of the second kind, log-normal, and log-normal with time-varying variance distributions as error terms. To estimate the models under consideration, a Bayesian approach is adopted. A formal Bayesian comparison of all models considered is conducted using Bayes factors. An empirical study is conducted on 10-minute trading volume data for some representative, very liquid stocks quoted on well-developed stock exchanges, such as the NASDAQ, the London Stock Exchange, the Frankfurt Stock Exchange, and the Euronext Paris Stock Exchange. The empirical results clearly indicate that the most adequate specifications are the linear ACV model with a time-varying log-normal distribution and the linear ACV model with a Burr or generalized beta of the second kind error distribution, which have very high Bayes factors against other models.

## METHODOLOGY – BAYESIAN FOURIER-MEM MODELS FOR INTRADAY TRADING VOLUMES

Modelling of intraday trading volume in high-frequency finance is not a trivial task due to a few characteristic features of high-frequency trading volumes. To accomplish the outlined objective of the paper, we use the following models. Let $v_i$ denote an observed time series of intraday trading volumes for $i = 1, 2, \ldots, N$ with $N$

standing for the total number of observations. First, we assume a simple AR(1)-GARCH(1,1) model for the logarithms of intraday trading volumes $v_i$:

$$\ln v_i - \ln s_i - \delta = \rho \cdot (\ln v_{i-1} - \ln s_{i-1} - \delta) + u_i, \tag{1}$$

$$u_i = \sigma_i \cdot \varepsilon_i, \quad \varepsilon_i \sim iid. N(0;1), \tag{2}$$

$$\sigma_i^2 = \omega_G + \alpha_G \cdot \varepsilon_{i-1}^2 + \beta_G \cdot \sigma_{i-1}^2, \tag{3}$$

where $-1 < \rho < 1$ and $\sigma_i^2$ is the conditional variance of the log trading volumes, which follows a standard GARCH process with $\omega_G \geq 0, \alpha_G \geq 0, \beta_G \geq 0, \alpha_G + \beta_G < 1$. Moreover, $s_i$ denotes an intraday periodic (seasonal) component which reproduces the time-of-day pattern and is specified via a Fourier series representation:

$$s_i = exp\left(\delta_{sez} \cdot t_i^{stand} + \sum_{j=1}^{\left[\frac{I}{2}\right]}\left[\delta_{s,j} \cdot \sin\left(2\pi j \cdot t_i^{stand}\right) + \delta_{c,j} \cdot \cos\left(2\pi j \cdot t_i^{stand}\right)\right]\right), \tag{4}$$

where $\delta_{sez}, \delta_{s,j}, \delta_{c,j}$ are the periodicity coefficients to be estimated, $t_i^{stand}$ denotes a time-of-day variable standardized on the interval [0;1], and *I* is the number of intervals in a trading day.

As an alternative approach, Autoregressive Conditional Volume (ACV) models are employed for analysis of the dynamics of intraday trading volume. The ACV model for trading volume has been introduced by Manganelli (2005). In this study, the general ACV model for the pure intraday volumes $v_i$ has a component structure and is defined multiplicatively as follows:

$$v_i = s_i \cdot \Phi_i \cdot \varepsilon_i, \tag{5}$$

$$\Phi_i = \Phi_i(v_1, \ldots, v_{i-1}; \theta) = \omega + \alpha \cdot v_{i-1} + \beta \cdot \Phi_{i-1}, \tag{6}$$

where $\varepsilon_i$ denotes an error term, which is a sequence of positive, identically and independently distributed random variables with density function $f_\varepsilon(\varepsilon_i)$, mean value $E(\varepsilon_i) = 1$ and a finite variance, $\theta$ is the vector of unknown parameters of model, $\omega > 0, \alpha \geq 0,\ \beta \geq 0, \alpha + \beta < 1$, and $s_i$ is an intraday periodic (seasonal) component specified via a Fourier form and given by (4). The log-normal, Burr, generalized gamma, generalized beta of the second kind, and log-normal with a time-varying variance distributions of the innovation term are considered in the analysis.

To estimate the parameters of the considered models and compare their explanatory power, a Bayesian approach is used. The inference was conducted using MCMC techniques, in particular the Metropolis-Hastings (M-H) algorithm (see, e.g., O'Hagan (1994) and Gamerman (1997)). Competing AR-GARCH and ACV models are compared pairwise through the Bayes factor:

$$BF_{ij} = \frac{p\left(\boldsymbol{v} \,|v_{(0)}, M_i\right)}{p\left(\boldsymbol{v} \,|v_{(0)}, M_j\right)},$$

where $p\left(\boldsymbol{v} \,|v_{(0)}, M_i\right)$ and $p\left(\boldsymbol{v} \,|v_{(0)}, M_j\right)$ are the marginal data densities in the considered models $M_i$ and $M_j$, respectively, and $v_{(0)}$ denotes initial conditions. The values of the marginal data densities for each model, which are the main quantities for Bayes factors and Bayesian model comparison, are approximated by means of the Newton and Raftery's (1994) harmonic mean estimator (HME in short), based on the harmonic mean of the likelihood values $p(\boldsymbol{v}\,|\theta_j, v_{(0)}, M_j)$ for model calculated for the observed trading volumes $\boldsymbol{v}$ and for M-H draws of parameters $\theta_j$ from the posterior.

**FINDINGS**

The empirical analysis is based on the 10-minute trading volume data of four large international and very actively traded companies listed on four well-developed stock exchanges, namely: Microsoft Corporation (MICROSOFT) traded at the NASDAQ (the United States), SHELL PLC company (SHELL) traded at the London Stock Exchange (Great Britain), SIEMENS AG company (SIEMENS) traded at the Frankfurt Stock Exchange (Germany), and CAPGEMINI SE company (CAPGEMINI) traded at the Euronext Paris (France). Our data sets cover the period from March 6, 2023, to March 8, 2024, spanning 254 to 259 trading days, depending on the stock. The analysis is based on transactions executed during the continuous trading phases of the stock exchanges under consideration. The datasets are sourced from the Refinitiv Eikon Database.

In this section, let us discuss the empirical results of the Bayesian comparison of all the considered specifications. Looking at the results in Table 1, it can be seen that, across all stocks, the selected Bayesian Fourier-ACV models generally outperform the standard AR-GARCH model. In the case of MICROSOFT, SHELL, and SIEMENS, the AR-GARCH model ($M_1$) occupies the fourth place in the rankings, whereas for CAPGEMINI, it is even the worst one. Thus, it seems that the intraday trading volumes of all considered stocks strongly reject the standard AR-GARCH model, which is a common and well-established structure in empirical studies.

For MICROSOFT, SIEMENS, and CAPGEMINI, our results show the superiority of the Fourier-ACV model with a time-varying log-normal distribution over all models considered. The Fourier-GVLN-ACV model ($M_3$) is respectively over 100, 108, and 118 orders of magnitude better than the AR-GARCH model. A look at CAPGEMINI reveals that the Fourier-ACV model with a generalized beta of the second kind ($M_6$) has the best performance and is over 417 orders of magnitude more probable a posteriori than AR-GARCH.

In the case of SHELL and SIEMENS, the second place in the rankings is occupied by the Fourier-ACV model with a Burr distribution. The results show that for these two stocks, they fit the data about 91 and 46 orders of magnitude worse than the best model in the rankings, respectively. In turn, the Fourier-ACV model with a generalized beta of the second kind ($M_6$) ranks second best for MICROSOFT and CAPGEMINI.

It should also be noted that the explanatory power of the Fourier-ACV model with a log normal distribution ($M_2$) and a generalized gamma distribution ($M_4$) is very poor. These two models rank, respectively, in the fifth and sixth positions for MICROSOFT, SHELL, and SIEMENS, whereas in the fifth and fourth positions for CAPGEMINI.

Finally, it is important to stress that the best Fourier-ACV models in the rankings have substantially greater explanatory power than the standard AR-GARCH model, while having a similar number of parameters to estimate.

**Table 1:** Bayesian models' comparison for considered stocks - the average of decimal logarithms of marginal data density values using HME estimator, the decimal logarithms of Bayes factors against the AR(1)-GARCH(1,1) model, and the models' rankings

| **Stock** | **MICROSOFT** | | | **SHELL** | | |
|---|---|---|---|---|---|---|
| Model | **average log HME** | $\log(BF_{i\,1})$ | Rank | **average log HME** | $\log(BF_{i\,1})$ | Rank |
| **$M_1$ Fourier-AR(1)-GARCH(1,1)** | 2080.61 | 0.00 | 4 | -4253.99 | 0.00 | 4 |
| **$M_2$ Fourier-LN-ACV(1,1)** | 1690.59 | -390.02 | 5 | -4545.63 | -291.64 | 5 |
| **$M_3$ Fourier-GVLN-ACV(1,1)** | 2181.39 | 100.78 | 1 | -4253.82 | 0.17 | 3 |
| **$M_4$ Fourier-GG-ACV(1,1)** | 1681.69 | -398.92 | 6 | -4568.15 | -314.16 | 6 |
| **$M_5$ Fourier-Burr-ACV(1,1)** | 2139.81 | 59.20 | 3 | -3928.01 | 325.98 | 2 |
| **$M_6$ Fourier-GB2-ACV(1,1)** | 2180.14 | 99.53 | 2 | -3836.52 | 417.47 | 1 |
| **Stock** | **SIEMENS** | | | **CAPGEMINI** | | |
| Model | **average log HME** | $\log(BF_{i\,1})$ | Rank | **average log HME** | $\log(BF_{i\,1})$ | Rank |
| **$M_1$ Fourier-AR(1)-GARCH(1,1)** | -1829.50 | 0.00 | 4 | -1472.84 | 0.00 | 6 |
| **$M_2$ Fourier-LN-ACV(1,1)** | -1921.54 | -92.04 | 5 | -1443.72 | 29.12 | 5 |
| **$M_3$ Fourier-GVLN-ACV(1,1)** | -1720.98 | 108.52 | 1 | -1353.88 | 118.96 | 1 |
| **$M_4$ Fourier-GG-ACV(1,1)** | -1936.13 | -106.63 | 6 | -1442.97 | 29.87 | 4 |
| **$M_5$ Fourier-Burr-ACV(1,1)** | -1767.56 | 61.94 | 2 | -1389.69 | 83.15 | 3 |
| **$M_6$ Fourier-GB2-ACV(1,1)** | -1767.82 | 61.68 | 3 | -1388.99 | 83.85 | 2 |

Notes: 'log' stands for decimal logarithm; BF denotes Bayes factor; GG-ACV, Burr-ACV, GB2-ACV, LN-ACV, and GVLN-ACV stand for the ACV model with a generalized gamma, Burr, generalized beta of the second kind, log-normal, and log-normal with time-varying variance error distribution, respectively.
Source: Author's own calculation.

## CONCLUSIONS

In this paper, we have proposed Bayesian Fourier-ACV models to evaluate their ability to model the dynamics of the 10-minute trading volume data of very actively traded stocks listed on well-developed stock exchanges. The considered multiplicative error models with different error distributions are compared to a standard AR-GARCH model for log trading volumes. For all models, an intraday periodic component is specified via a Fourier series representation. A formal Bayesian comparison of all models is conducted using Bayes factors. Notably, such a study has not appeared in the literature to date.

These results demonstrate that the Bayesian Fourier-ACV models are promising when it comes to modelling intraday volumes. The main conclusion is that the selected Bayesian Fourier-ACV models generally outperform the commonly used standard AR-GARCH model. Thus, it seems that the intraday trading volumes of all considered stocks strongly reject the standard AR-GARCH structure. Moreover, it appears that the linear ACV models with a time-varying log-normal, Burr, and generalized beta of the second kind error distribution have the best performance for all stocks.

It should be noted that we have restricted our attention to linear Fourier-ACV models, but it would be of great interest for future research to make formal Bayesian comparisons between ACV, stochastic conditional volume (SCV), and DCS specifications. Additionally, we have focused only on pure time-series models, without using any extra variables that could be motivated by theoretical considerations.

## REFERENCES


Andersen, T. (1996). Return volatility and trading volume: An information flow interpretation of stochastic volatility. *Journal of Finance,* 51, 169-204.

Bauwens, L., & Veredas, D. (2004). The stochastic conditional duration model: A latent variable model for the analysis of financial durations. *Journal of Econometrics,* 119, 381-412.

Bessembinder, H., & Seguin, P.J. (1993). Price volatility, trading volume and market depth: Evidence from futures markets. *Journal of Financial and Quantitative Analysis*, 28, 21-39.

Białkowski, J., Darolles, S., & Le Fol, G. (2008). Improving VWAP strategies: A dynamic volume approach. *Journal of Banking and Finance,* 32, 1709-1722.

Bjonnes, G.H., Rime, D., & Solheim, H.O. (2003). Volume and volatility in the FX market: Does it matter who you are? Working Papers of Norges Bank, Oslo, No. 2003/7, ISBN 82-7553-215-9, https://hdl.handle.net/11250/2498620.

Brownlees, Ch., Cipollini, F., & Gallo, G.M. (2011). Intra-daily volume modeling and prediction for algorithmic trading. *Journal of Financial Econometrics,* 9, 489-518.

Chen, G., Firth, M., & Rui, O.M. (2001). The dynamic relation between stock returns, trading volume, and volatility. *The Financial Review*, 38, 153–174.

Chevallier, J., & Sevi, B. (2012). On the volatility–volume relationship in energy futures markets using intraday data. *Energy Economics,* 34(6), 1896-1909.

Clark, P. (1973). A subordinated stochastic process model with finite variance for speculative prices. *Econometrica*, 41, 135–155.

Copeland, T.E. (1976). A model of asset trading under assumption of sequential information arrival. *Journal of Finance*, 31, 1149–1168.

Easley, D., & O'Hara, M. (1987). Price, trade size and information in securities markets. *Journal of Financial Economics,* 19*,* 69-90.

Engle, R.F. (2002). New frontiers for ARCH models. *Journal of Applied Econometrics,* 17, 425-446.

Engle, R.F., & Russell, J.R. (1998). Autoregressive conditional duration: A new model for irregularly spaced transaction data. *Econometrica,* 66*,* 1127-1162.

Epps, T.W., & Epps, M.L. (1976). The stochastic dependence of security price changes and transaction volumes: Implications for the mixture of distribution hypothesi. *Econometrica*, 44, 305–321.

Foster, D., & Viswanathan, S. (1990). A theory of intraday variations in volume, variance and trading costs in securities marke. *Review of Financial Studies,* 3*,* 593-624.

Ftiti, Z., Jawadi, F., & Louhichi, W. (2017). Modelling the relationship between future energy intraday volatility and trading volume with wavelet. *Applied Economics,* 49(20)*,* 1981-1993.

Gallant, A.R., Rossi, P.E., & Tauchen, G. (1992). Stock prices and volume. *The Review of Financial Studies*, 5, 199–242.

Gallo, G., & Pacini, B. (2000). The effects of trading activity on market volatility. *European Journal of Finance*, 6, 163–175.

Gamerman, D. (1997). Markov Chain Monte Carlo. Stochastic Simulation for Bayesian Inference. Chapman & Hall. London.

Girard, E., & Biswas, R. (2007). Trading volume and market volatility: Developed versus emerging markets. *Financial Review*, 42, 429-459.

Gouriéroux, C., Jasiak, J., & Le Fol, G. (1999). Intra-day market activity. *Journal of Financial Markets,* 2, 193-226.

Huptas, R. (2018). Point and density prediction of intra-day volume using Bayesian linear ACV models: Evidence from the Polish stock market. *Quantitative Finance,* 18, 749-760.

Huptas, R. (2019). Point forecasting of intraday volume using Bayesian autoregressive conditional volume models. *Journal of Forecasting,* 38, 293–310.

Huptas, R. (2025). Modelling of Intraday Stock Trading Volumes of Liquid International Companies: A Standard AR Structure or a Linear ACV Model? In K. Soliman (Eds.), *Strategies, Innovation, and Human-Centric Practices in the Modern Organizations : Proceedings of the 45th International Business Information Management Association Conference (IBIMA) 25-26 June 2025* (pp. 656-664). International Business Information Management Association.

Huptas, R. (2026). Revisiting In-Sample Bayesian Comparison of Alternative Multiplicative Error Models: Evidence from Trade Durations and Intraday Trading Volumes for the Polish Stock Market. *Central European Journal of Economic Modelling and Econometrics*, 17(3), 223-265.

Ito, R. (2016). Spline-DCS for forecasting trade volume in high-frequency financial data. *Cambridge Working Papers in Economics*, [Online], [Retrieved September 22, 2017], https://doi.org/10.17863/CAM.5751.

Jennings, R.H., Starks, L.T., & Fellingham, J.C. (1981). An equilibrium model of asset trading with sequential information arrival. *Journal of Finance*, 36, 143–161.

Jones, C.M., Kaul, G., & Lipson, M.L. (1994). Transactions, volume, and volatility. *The Review of Financial Studies*, 7(4), 631–651.

Kaastra, I., & Boyd, M. (1995). Forecasting futures trading volume using neural networks. *Journal of Futures Markets,* 15, 953-970.

Karpoff, J. (1987). The relation between price changes and trading volume: A survey. *Journal of Financial and Quantitative Analysis*, 22, 109–126.

Lahmiri, S., Bekiros, S. & Bezzina, F. (2022). Complexity analysis and forecasting of variations in cryptocurrency trading volume with support vector regression tuned by Bayesian optimization under different kernels: an empirical comparison from a large dataset. *Expert Systems with Applications*, 209, Article 118349.

Lamoureux, C., & Lastrapes, W. (1990). Heteroskedasticity in stock return data: Volume versus GARCH effects. *Journal of Finance*, 45, 220–229.

Li, H., & Ye, X. (2013). A dynamic volume-weighted average price approach based on the fast Fourier transform algorithm. *Asia-Pacific Journal of Financial Studies,* 42, 969–991.

Libman, D., Haber, S., & Schaps, M. (2019). Volume Prediction With Neural Networks. *Frontiers in Artificial Intelligence,* 2(21), 1-7.

Liu, M., Choo,W.-C., Lee, C.-C., & Lee, C.-C. (2022b). Trading volume and realized volatility forecasting: evidence from the China stock market. *Journal of Forecasting*, 42(1), 76-100.

Liu, X., & Lai, K.K. (2017). Intraday volume percentages forecasting using a dynamic SVM-based approach. *Journal of Systems Science and Complexity,* 30(2), 421-433.

Lo, A., & Wang, J. (2000). Trading volume: Definition, data analysis, and implication of portfolio theory. *Review of Financial Studies,* 13, 257-300.

Manganelli, S. (2005). Duration, volume and volatility impact of trades. *Journal of Financial Markets*, 8, 377–399.

Naimoli, A., & Storti, G. (2019). Heterogeneous component multiplicative error models for forecasting trading volumes. *International Journal of Forecasting*, 35(4), 1332-1355.

Nasir, M.A., Huynh, T.L.D., Nguyen, S.P., & Duong, D. (2019). Forecasting cryptocurrency returns and volume using search engines. *Financial Innovation*, 5(1), 1-13.

Newton, M., & Raftery, A. (1994). Approximate Bayesian inference with the weighted likelihood bootstrap (with disscussion). *Journal of the Royal Statistical Society (Series B)*, 56(1), 3–48.

O'Hagan A. (1994). Bayesian Inference. Edward Arnold, London.

Satish, V., Saxena, A., & Palmer, M. (2014). Predicting intraday trading volume and volume percentages. *The Journal of Trading,* 9, 15-25.

Slim, S., & Dahmene, M. (2016). Asymmetric information, volatility components and the volume-volatility relationship for the CAC40 stocks. *Global Finance Journal,* 29, 70-84.

Szűcs, B.Á. (2017). Forecasting intraday volume: Comparison of two early models. *Finance Research Letters,* 21, 249-258.

Tauchen, G., & Pitts, M. (1983). The price variability-volume relationship on speculative markets. *Econometrica,* 51, 485-505.

Ye, X., Yan, R., & Li, H. (2014). Forecasting trading volume in the Chinese stock market based on the dynamic VWAP. *Studies in Nonlinear Dynamics and Econometrics,* 18(2), 125–144.