Option-implied Forecasts of Exchange Rate Dependence

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Extended Abstract

Abstract

This paper extracts forward-looking measures of exchange rate dependence from currency option prices and evaluates their information content for the joint dynamics of exchange rate returns. Whereas the option-implied correlations used in prior work rely solely on at-the-money options and deliver a single scalar measure of association, the proposed methodology exploits the entire cross-section of option prices, thereby enabling the recovery of not only the overall degree of covariation but also any asymmetric dependence for joint positive and joint negative returns. Empirically, forecasting models augmented with option-implied dependence measures deliver more accurate out-of-sample density forecasts, as well as superior Value-at-Risk and Expected Shortfall forecasts for currency portfolios.

JEL: F31, F37, G14, G15, G17.
Keywords: Currency options; Density forecasts; Tail dependence; Copulas

1.Introduction

Understanding how exchange rates co-move is central to international portfolio allocation, hedging, and risk management, because currency dependence shapes the joint distribution of returns on foreign-currency investments. Standard time-series approaches to modeling exchange rate dependence (e.g., correlation and multivariate volatility models) are primarily backward-looking, as they update dependence using past returns and may adjust slowly when dependence changes.

This paper extracts forward-looking, option-implied dependence estimates from exchange rate option prices. We then use these measures to augment dynamic copula models that capture the bivariate conditional distribution of exchange rate returns. The approach is designed to capture both (i) time variation in overall dependence and (ii) asymmetric dependence for joint positive versus joint negative returns, which linear correlation cannot capture.

2. Methodology

Dataset. The empirical application focuses on the USD–EUR–JPY currency trio. Option-based dependence measures are constructed from OTC FX implied volatility quotes (ATM volatilities, 25-delta risk reversals, and 25-delta butterfly spreads) at multiple maturities. Data are sourced from Bloomberg. These quotes are converted into option prices and used to infer risk-neutral densities and option-implied dependence parameters.

Extracting option-implied dependence. We employ a copula representation of a bivariate risk-neutral density that respects triangular no-arbitrage across a currency trio and infer copula dependence parameter(s) by matching observed cross-rate option prices. Operationally, we proceed in two steps.

Step 1 (marginals). We calibrate the stochastic volatility model of Heston (1993) to daily panels of option data to estimate (univariate) risk-neutral densities for the relevant exchange rates, for arbitrary horizons. The calibration is repeated each day in the sample.

Step 2 (copula dependence). Option-implied dependence is estimated by matching observed cross-rate option prices to those generated by the copula-based bivariate risk-neutral representation of Taylor and Wang (2010). Let $X_T = S_T^{\mathrm{USD/JPY}}$ and $Y_T = S_T^{\mathrm{EUR/JPY}}$, and denote their risk-neutral densities by $f_X(\cdot)$ and $f_Y(\cdot)$ with CDFs $F_X(\cdot)$ and $F_Y(\cdot)$. Let $c(\cdot,\cdot)$ be the copula density. Then the USD-denominated call price on the cross rate $S_T^{\mathrm{EUR/USD}} = Y_T/X_T$ with strike K can be written as

$$C_{\mathrm{USD}}(K) = \frac{e^{-r_{\mathrm{JPY}}T}}{S_0^{\mathrm{USD/JPY}}} \int_0^\infty \int_0^\infty (y - Kx)^+ \, f_X(x)\, f_Y(y)\, c(F_X(x), F_Y(y))\; dx\, dy, \tag{1}$$

where $(a)^+ \equiv \max(a, 0)$.

Copula parameters are estimated by minimizing (average) squared pricing errors across observed cross-rate option strikes. Writing $C_t^{\mathrm{obs}}(K_i)$ for observed call prices of options written on the USD/EUR exchange rate and $C_t^{\mathrm{mod}}(K_i;\theta_{c,t}^Q)$ for option prices generated from (1), we estimate

$$\widehat{\theta}_{c,t}^Q = \arg\min_{\theta_c^Q} \frac{1}{N_t} \sum_{i=1}^{N_t} \Big(C_t^{\mathrm{obs}}(K_i) - C_t^{\mathrm{mod}}(K_i;\theta_c^Q) \Big)^2. \tag{2}$$

Time-series models for the conditional joint distribution. Following Creal, Koopman, and Lucas (2013), we model time-varying dependence using generalized autoregressive score (GAS) dynamics and consider (i) *historical* specifications that use only past exchange rate return information and (ii) *augmented* specifications that include option-implied dependence estimates as additional explanatory variables. The augmented approach is analogous in spirit to the GRAS framework of Salvatierra and Patton (2015), but we use option-implied (risk-neutral) dependence measures rather than realized measures.

Normal copula. For the Normal copula, the dependence parameter ρ_t evolves as

$$\rho_t = \Lambda\Big(\omega_\rho + \beta_\rho \rho_{t-1} + \alpha_\rho FV_{t-1} + \gamma_\rho \rho_{t-1}^Q\Big), \tag{3}$$

where FV_{t-1} is the GAS forcing variable, ρ^Q_{t-1} is the option-implied dependence proxy, and

$$\Lambda(x) \equiv 2\left(1+e^{-x}\right)^{-1} - 1 \tag{4}$$

maps $\mathbb{R}$ to $(-1, 1)$.

Dropping γ_ρ yields the baseline "historical" specification without option-implied dependence.

Gumbel mixture copula. For the Gumbel mixture copula, upper and lower tail dependence parameters τ_t^U and τ_t^L are modeled as

$$\tau_t^U = \bar{\Lambda}\Big(\omega_U + \beta_U \tau_{t-1}^U + \alpha_U FV_{t-1}^U + \gamma_U \tau_{t-1}^{Q,U}\Big), \tag{5}$$

$$\tau_t^L = \bar{\Lambda}\Big(\omega_L + \beta_L \tau_{t-1}^L + \alpha_L FV_{t-1}^L + \gamma_L \tau_{t-1}^{Q,L}\Big), \tag{6}$$

where $\tau_{t-1}^{Q,U}$ and $\tau_{t-1}^{Q,L}$ are option-implied upper/lower tail dependence indicators and $\bar{\Lambda}(x) \equiv (1+e^{-x})^{-1}$ maps $\mathbb{R}$ to $(0, 1)$. Dropping γ_U and γ_L yields the baseline "historical" specification.

3. Findings

Asymmetric risk-neutral dependence. We show that option-implied dependence estimates reveal substantial asymmetry in the risk-neutral dependence of the exchange rates under study. Figure 1 illustrates two dates (09/17/2003 and 11/08/2006) for which the Normal copula dependence parameter is approximately $\rho \approx 0.55$, while the Gumbel mixture parameters differ markedly: on 09/17/2003, $(\tau^U, \tau^L) = (0.29, 0.56)$, whereas on 11/08/2006, $(\tau^U, \tau^L) = (0.54, 0.33)$. Panels (a)–(c) display copula density contours and panels (d)–(f) show the corresponding joint density contours, using standard normal margins to isolate dependence effects. The Gumbel mixture plots differ substantially across the two dates, reflecting opposite directions of asymmetry: stronger joint downside dependence in case (A) versus stronger joint upside dependence in case (B). This indicates that it is important to look beyond scalar measures of option-implied dependence.

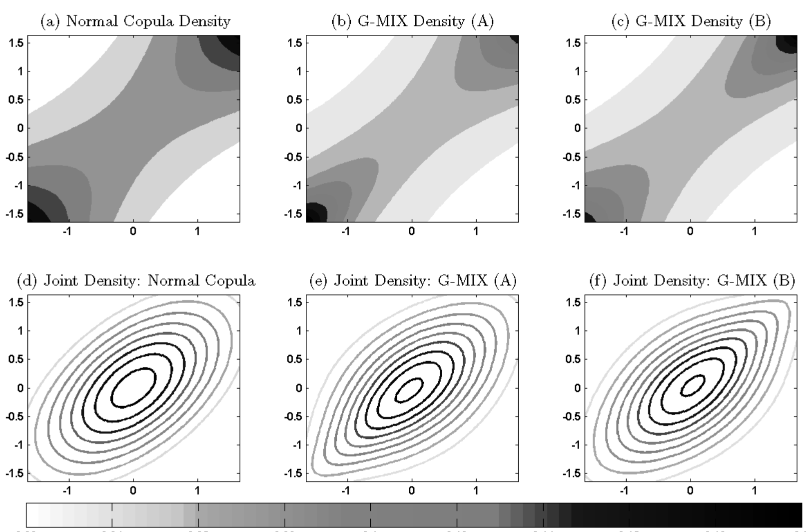

Figure 1: *Copula density contours (a–c) and implied joint density contours (d–f) on 09/17/2003 (A) and 11/08/2006 (B), using standard normal margins.*

Forecasting performance and risk measures. We conduct out-of-sample comparisons between parsimonious historical" models and augmented" specifications that incorporate option-implied dependence estimates as additional explanatory variables.

The models are first evaluated on the basis of out-of-sample log-likelihood comparisons using the scoring rules of Amisano and Giacomini (2007) and Bao, Lee, and Saltoglu (2007). A summary of these results is presented in Table 1. We then examine the accuracy of the models' Value-at-Risk and Expected Shortfall forecasts for currency portfolios across a range of Value-at-Risk levels, using the loss function proposed by Fissler and Ziegel (2016). A summary of these results is presented in Table 2.

Overall, the empirical evidence indicates that models incorporating option-implied dependence measures outperform specifications that rely solely on past exchange rate returns.

Table 1: Out-of-Sample Results

Panel A. Out-of-Sample Log-likelihoods

	Normal Copula			Gumbel Mixture Copula		
	Constant	*Time-Varying*	*Augmented*	*Constant*	*Time-Varying*	*Augmented*
OLL_c	222.12	319.65	334.81†	251.68	324.16	338.54†

Panel B. Diebold–Mariano Tests of Equal Predictive Ability

	Normal Copula			Gumbel Mixture Copula		
	Constant	*Time-Varying*	*Augmented*	*Constant*	*Time-Varying*	*Augmented*
Normal Copula						
Constant	–					
Time-Varying	0.0748***	–				
Augmented	0.0864***	0.0116	–			
Gumbel Mixture Copula						
Constant	0.0227***	−0.0521***	−0.0637***	–		
Time-Varying	0.0783***	0.0035	−0.0082	0.0556***	–	
Augmented	0.0893***	0.0145*	0.0029	0.0666***	0.0110***	–

Panel A displays the out-of-sample log-likelihood statistics of the alternative copula models. *Constant* and *Time-Varying* refer to the constant and conditional copula specifications estimated on historical returns, while *Augmented* refers to the time-varying specification augmented with option-implied dependence. † indicates that the model is included in the 95% model confidence set of Hansen et al. (2011). **Panel B** reports the average difference in out-of-sample log-likelihood between the model in the row and the model in the column, so that a positive entry indicates that the row model outperforms the column model. Asterisks *, **, and *** denote rejection of the null of equal predictive ability of the Diebold–Mariano test at the 10%, 5%, and 1% significance level, respectively.

Table 2: Tail Risk Forecasts: VaR and Expected Shortfall

Panel A. Lower Tail

	Normal Copula			Gumbel Mixture Copula		
VaR Level	*Constant*	*Time-Varying*	*Augmented*	*Constant*	*Time-Varying*	*Augmented*
0.05	−580.59	−580.37	−581.05	−580.94	−580.77	−580.47
0.10	−609.72	−610.09	−610.89	−610.02	−610.30	−610.51

Panel B. Upper Tail

	Normal Copula			Gumbel Mixture Copula		
VaR Level	*Constant*	*Time-Varying*	*Augmented*	*Constant*	*Time-Varying*	*Augmented*
0.90	−611.10	−611.80	−612.09	−611.07	−611.58	−611.84
0.95	−581.74	−582.29	−582.44	−581.92	−581.97	−582.05

The table reports the average realized losses, multiplied by 100, from the joint Value-at-Risk and Expected Shortfall loss function of Fissler and Ziegel (2016) for the alternative copula models, computed over the out-of-sample period at each VaR level. **Panel A** corresponds to the lower tail of the return distribution, and **Panel B** to the upper tail. *Constant* and *Time-Varying* refer to the constant and conditional copula specifications estimated on historical returns, while *Augmented* refers to the time-varying specification augmented with option-implied dependence. Lower values indicate more accurate tail risk forecasts.

4. Conclusion

The paper contributes in three ways. First, it moves beyond scalar measures of dependence, notably option-implied correlation, by extracting separate upper- and lower-tail dependence estimates from option prices. Second, it integrates option-implied dependence measures into score-driven dynamic copula models, in the spirit of Salvatierra and Patton (2015), who augment generalized autoregressive score models with realized correlation measures. Third, it evaluates competing models in a bivariate density-forecasting setting, including out-of-sample analyses of tail risk in foreign-currency portfolios.

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