Three-Criteria Portfolio Selection Model Incorporating Semi-Variance and Accounting Multiples

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Abstract:

This study aims to establish whether incorporating accounting information along with a downside-risk measure into the portfolio optimization process improves the portfolio performance. Using stocks from the Dow Jones Industrial Average Index, we form optimal portfolios that either minimize variance or semi-variance. The optimization problem includes additional constraints, which are defined by five accounting multiples: Earnings/Price Ratio, Book Value/Price Ratio, Sales Price/Price Ratio, Cash Flow/Price Ratio, and Dividend Yield. Due to the complexity of semi-variance optimization, we develop an iterative algorithm for generating efficient portfolios within a multiple-criteria decision-making framework. We then evaluate the performance of each portfolio for one-month, three-month, and six-month investment horizons, utilizing several performance metrics. Furthermore, we conduct a comparative analysis of portfolio performance in three sub-periods, reflecting different states of the market. Our results indicate that semi-variance minimizing portfolios significantly outperform variance-based portfolios regardless of the performance metric and investment horizon used. The performance superiority of semi-variance minimizing portfolios is especially stronger during the COVID-19 pandemic and underpins the benefit of focusing on downside risk. Lastly, we find that the inclusion of accounting constraints in addition to traditional constraints enhances the long-term performance of portfolios. Specifically, the earnings-to-price ratio proves to be the best predictor of future stock price performance.

Keywords: portfolio optimization; multicriterial choice; accounting multiples; downside risk; semi-variance; portfolio performance evaluation; NYSE

JEL Classification: G11; G12

1. Introduction

The original classical theories for portfolio selection were developed by Markowitz (1952) and furthered by Sharpe (1963). The theory was based on the stock price changes that represent a company's market performance. The two most important factors used to evaluate portfolios are the expected rate of return, which represents the rate at which you expect your investment to grow, and risk, which has traditionally been defined by the variance or standard deviation of returns. These models, however, do not incorporate alternative measures of risk and supplementary criteria that might yield deeper insights into a company's financial condition or future prospects—factors that may exert a considerable influence on the market valuation of its securities.

It should be noted that Markowitz (1959) proposed a downside version of variance, which he termed semi-variance. He also developed the Critical Line Algorithm to identify efficient portfolios under the semi-variance criterion. This algorithm was unique because standard quadratic or nonlinear programming techniques were unable to obtain the optimal semi-variance solution. However, one limitation of the Critical Line Algorithm (Markowitz, 1959; Markowitz et al., 2020) is that it is not well-suited to multicriteria decision-making problems. The complexity of the method was probably one of the main reasons why the variance-based model became dominant in practice. Although numerous studies have demonstrated the advantages of downside risk measures over traditional risk metrics, downside approaches remain rarely used in practice. Measures such as the Sharpe ratio, beta coefficient, and variance have remained largely unchanged for decades and continue to be widely applied. These metrics for individual stocks are routinely reported in professional financial databases.

In recent years, increasing attention has been directed toward portfolio analysis methods that explore new approaches to portfolio construction. Classical portfolio theory has been extended in various directions. A line of research adopts alternative risk measures beyond the variance or standard deviation of returns in the minimization of portfolio risk. For instance, semi-variance, conditional value-at-risk, or higher moments of return distributions, such as skewness or kurtosis, are among the alternative risk metrics that are used in the objective function (Maier-Paape et al., 2019; Fabozzi et al., 2007; Briec et al., 2007; Rodriguez et al., 2011). Ballestro et al. (2012) have incorporated other factors beyond financial performance, such as environmental, social, and governance factors, in their portfolio optimization problem. As well as changing the objective function used to optimize portfolios, there is also another line of research where different constraints other than the traditional ones (i.e., expected returns and weights) are employed to achieve the desired portfolio outcomes. In particular, liquidity, dividends, leverage, and the book-to-market ratio are used to investigate if alternative constraints would lead to better portfolio choices (Lo et, 2003; Xidonas et al., 2010; Jacobs and Levy, 2013). Lastly, there are also studies that modified both the objective function and constraints (Rutkowska-Miczka et al., 2024; Wu et al., 2022).

This study combines these strands of literature and aims to examine the empirical properties of portfolios constructed using non-classical methods. More specifically, we address the following research questions. 1) Does the inclusion of information about the fundamental accounting characteristics of companies in the portfolio optimization problem improve portfolio performance? 2) Does using semi-variance as a measure of risk give better results than using variance in portfolio selection? 3) If so, are there specific periods or market conditions during which this performance difference is more pronounced? 4) Do the performances of optimized portfolios that integrate fundamental accounting criteria vary across different investment horizons?

To address the first question, we incorporate an additional constraint into the portfolio optimization framework to consider the fundamental value of a portfolio consisting of stocks in the US Dow Jones Industrial Average Index. Specifically, we impose a requirement that the fundamental value of the portfolio, as measured by a selected accounting multiple, must not fall below a specified benchmark level. Consistent with the extensive empirical literature on the predictive capacity of accounting multiples for subsequent performance, we employ the commonly used valuation metrics of the earnings-to-price ratio, book value-to-price ratio, sales-to-price ratio, cash flow-to-price ratio, and dividend yield.

Fundamental analysis is mainly a long-term investment strategy, so the information it offers is likely to show up in asset prices over time. To examine this conjecture, we assess the performance of optimized portfolios over three specific investment horizons: one month, three months, and six months. This design lets us see if the effect of accounting information gets stronger as the investment horizon gets longer.

In addition to the traditional risk metric of variance, we employ semi-variance as an alternative measure of risk. Unlike variance, semi-variance considers only the negative deviations of returns below a predefined threshold. Therefore, it focuses specifically on downside risk. One other benefit to using semi-variance is that it does not require any assumptions regarding either the distribution of investment returns or an investor's preferences via their utility function, as stated by Harlow and Rao (1989). The commonly used quadratic utility function in the mean-variance framework has some theoretical drawbacks. This function reaches a maximum after which returns provide decreasing utility. Thus, the common investor preferences for higher returns are not reflected in this utility function.

Using semi-variance as a risk measure might be especially beneficial during market downturns, such as the COVID-19 outbreak in early 2020. As semi-variance-based portfolios are set up to lower downside risk while allowing for upside risk, they could exhibit unique performance patterns in very volatile markets. To investigate this possibility, we divide the sample period into three sub-periods, pre-pandemic, pandemic, and post-pandemic, and re-estimate portfolio performance separately for each. This approach enables us to assess whether the advantages of downside-risk optimization vary across different market conditions characterized by varying levels of volatility.

Our findings can be summarized as follows. We find that portfolios built using semi-variance minimization consistently outperform those based on traditional variance minimization, no matter how the portfolio performance measure is evaluated. Whether we measure risk-adjusted returns with the Sharpe ratio, Sortino ratio, or the Fama–French–Carhart six-factor alpha, the semi-variance-optimized portfolios deliver stronger results across all investment horizons considered. This finding applies to both unconstrained portfolios and those with additional restrictions on fundamental characteristics, underscoring the limitations of variance that penalize upside volatility. It also shows how important it is to account for downside risk when building a portfolio.

Our sub-period analysis shows that the overperformance of semi-variance–minimizing portfolios is most pronounced during times of market turmoil. During the COVID-19 pandemic, variance-based and equal-weighted portfolios experienced large risk-adjusted losses. On the other hand, in addition to outperforming traditional variance-based portfolio strategies by producing significant positive abnormal returns for the three- and six-month holding periods, these semi-variance-based portfolio strategies also demonstrate greater resiliency than their variance-based counterparts under adverse market conditions. The superior performance of semi-variance-based portfolios post-pandemic supports the position that semi-variance is an effective and reliable risk measure across all types of markets.

Finally, including fundamental accounting characteristics as additional constraints in the optimization process improves portfolio performance, especially over longer investment horizons. At the one-month horizon, although semi-variance-minimizing portfolios outperform their variance-based counterparts, neither approach surpasses the equal-weighted benchmark portfolio and achieves positive risk-adjusted returns. Over three months, the portfolios that achieve the highest risk-adjusted returns are those that minimize semi-variance without constraints (the SV portfolio) and those that minimize semi-variance subject to an expected-return constraint (the SV-E portfolio). Other semi-variance-minimizing portfolios that impose further constraints based on fundamental accounting variables underperform relative to SV and SV-E. This implies that the informational advantages of fundamental analysis require time to materialize. However, over the six-month horizon, the SV-E-EP portfolio, which is constructed by adding the earnings-to-price (EP) ratio as an additional constraint, outperforms all other portfolio configurations. This aligns with the idea that fundamental analysis is inherently a long-term investment strategy. Furthermore, among the various accounting characteristics examined, the EP ratio appears to contain the most relevant information for explaining expected returns.

Our study contributes to the literature in several ways. First, we make a methodological contribution. Portfolio optimization based on semi-variance is inherently more complex than variance-based approaches. Conventional solver software is insufficient for identifying minimum semi-variance portfolios because semi-covariance calculations require knowledge of the specific periods when portfolio returns fall below the target benchmark. This interdependence on portfolio composition renders semi-variance optimization computationally challenging. In the classical Markowitz model, the covariance matrix does not depend on the portfolio composition or the assumed rate of return. In contrast, when optimizing based on semi-variance, the semi-covariance matrix depends both on the assumed target return and on the portfolio composition. This means that the semi-covariance matrix must be recalculated separately for each assumed target return. Furthermore, it also depends on the portfolio composition, since the portfolio itself determines the periods in which the overall return falls below the target return. Therefore, one must initially assume a certain portfolio composition (e.g., the Markowitz solution) in order to obtain the first semi-covariance matrix for a given target return. Then, as the portfolio composition changes during optimization, the elements of the semi-covariance matrix must be recomputed until a self-consistent solution is reached.

The present study proposes an alternative approach to the Critical Line Algorithm introduced by Markowitz (1959) for portfolio optimization and later extended to the mean–semivariance framework by Markowitz et al. (2020). The proposed method is based on an iterative algorithm that repeatedly updates the elements of the semivariance–semicovariance matrix in response to changes in portfolio composition, as outlined above. Its key advantage lies in the possibility of employing standard nonlinear or quadratic programming solvers to determine minimum semi-variance portfolios. Moreover, the iterative framework can be adapted to multicriteria portfolio optimization problems by incorporating additional investor-relevant criteria, such as market indicators, firms' financial condition, or ESG factors.

A further complication arises from the fact that the efficient frontier may contain an infinite set of optimal solutions, which introduces ambiguity in selecting boundary conditions in the bi-criteria and tri-criteria formulations. In such cases, specifying the risk-free rate as the required rate of return at a given point in time provides a natural and economically justified boundary condition. Likewise, setting the boundary value of a given value-based indicator (accounting multiple) at the level corresponding to the equally weighted portfolio at a given point in time is also a reasonable choice. To address these complexities, we develop a proprietary optimization algorithm, which iteratively solves quadratic optimization problems subject to linear constraints.

Our other contributions are mainly empirical. We enhance the current literature by incorporating one of the most extensive collections of accounting characteristics as supplementary restrictions within the portfolio optimization framework, with a downside-risk metric. By assessing the performance of optimal portfolios created based on diverse accounting criteria, we offer fresh perspectives on enduring discussions in portfolio management and investment analysis concerning the information content of firm fundamentals. Specifically, our approach sheds light on whether a company's accounting characteristics contain predictive information about future stock returns, and, if so, which specific accounting variable conveys the most significant signal for expected returns. This method not only connects fundamental analysis and modern portfolio theory but also shows how important it is to include firm-level information in risk-sensitive investment strategies, especially when investors are mainly worried about downside risk. Next, we directly compare the performance of semi-variance- and variance-based portfolios using a long–short portfolio that takes a long position in the semi-variance-based portfolio and a short position in the variance-based portfolio. The return on this long–short portfolio shows the difference in returns between the two strategies. A significantly positive alpha for the long-short portfolio indicates that the semi-variance-based portfolio attains a superior risk-adjusted return compared to its variance-based counterpart, thereby validating its outperformance. Prior studies have assessed the performance of semi-variance- and variance-based portfolios separately. However, there has been limited explicit testing to determine if the disparity in their risk-adjusted performances is statistically significant.

Our final contribution is to assess the performance of optimal portfolios through varying investment horizons. This type of analysis has implications for investors who use fundamental analysis to help generate superior returns. In essence, we have demonstrated that fundamental accounting variables are increasingly relevant to the forecasting of portfolio returns with increasing length of holding periods. This indicates that long-term investors should consider including fundamental information in their long-term strategic investment decisions. This finding provides further support for the view that accounting information is gradually integrated into prices.

2. Literature Review

Recently, classical portfolio theory has been extended in various directions. Kolm et al. (2014) provided a comprehensive review of the major developments in portfolio theory since its inception. One direction of extension is the use of a wide range of risk measures in contemporary portfolio management (Fabozzi, et al. 2007). For example, Maier-Paape et al. (2019) incorporated drawdown risk measures as a more practical tool for investors and further generalized the traditional one-period analysis framework to a multi-period portfolio selection model. Pla-Santamaría and Bravo (2013) showed that portfolios built using the mean-semi-variance frontier can differ substantially from those derived from the traditional mean–variance frontier. Their results suggest that downside-focused risk measures provide more realistic portfolio choices. The idea here was extended by Kassimatis (2021), who used a constant relative risk aversion (CRRA) model. He found that portfolios developed using the CRRA model have much less risk on the downside compared to those developed with traditional mean-variance models.

Another area for extension is to use multicriteria methods in this field (Doumpos and Zopounidis, 2014). Some studies have incorporated other characteristics of the return distribution as additional criteria for evaluating portfolio performance, such as skewness and kurtosis (Briec et al., 2007; Nakajima, 2020). Additionally, Lo et al. (2003) considered stock liquidity as another important variable affecting investment results.

Theoretical models and empirical evidence support the importance of fundamentals in determining returns (Fama & French, 1992, 2015). Research has confirmed that across all major financial markets, specific accounting indicators substantially impact stock performance. Using machine learning techniques, Cakici and Zaremba (2025) demonstrated for the US stock market that strategies focused on using accounting indicators generate more consistent and profitable returns. At the same time, the use of accounting information in portfolio construction proves particularly effective over longer investment horizons, exceeding one month, whereas in shorter horizons, technical analysis may yield superior results. Consequently, the inclusion of fundamental accounting information in portfolio construction models appears well justified.

A number of studies have focused on developing a framework that integrates portfolio optimization with fundamental analysis. The work by Xidonas et al. (2010) utilized dividend payment data as proxies for assessing firm-level performance. Jacobs and Levy (2013) used financial leverage as a proxy for default-related risks. More recent research emphasizes the need for developing effective methods of dynamically rebalancing portfolios based upon current market conditions. One promising direction involves the application of the Taxonomic Measure of Investment Attractiveness (TMAI) as a synthetic indicator for assessing firms' economic and financial condition, which can be incorporated as an additional portfolio selection criterion. The TMAI framework quantifies the Euclidean distance between each firm and a predefined benchmark representing an "ideal" firm in a multidimensional space of financial indicators. The original formulation and subsequent developments of the model are presented in Tarczyński and Tarczyńska-Łuniewska (2018). The development of additional versions of the TMAI methodology includes replacing variance with semi-variance as an alternative risk metric to represent downside risk (Rutkowska-Ziarko & Garsztka, 2014; Rutkowska-Miczka, Kliber & Szydłowski, 2024) and using the Mahalanobis distance to describe dependencies between financial indicators (Rutkowska-Ziarko, 2013). Research conducted by Tarczyńska-Łuniewska (2022) supports the use of portfolios formed using fundamental analysis as tools to enhance portfolio diversification and provide valuable decision support during periods of heightened risk.

Another important development in this area was a study undertaken by Wu et al. (2022), which employed a multi-criteria method to optimize equity portfolios. Their method included minimizing portfolio risk, measured as the absolute deviation of portfolio returns, while simultaneously maximizing portfolio expected return in a manner similar to that utilized within the Markowitz Method. In addition, two further criteria were introduced: corporate social responsibility (CSR) and the financial condition of the firms. Both CSR and financial condition were assessed using synthetic measures derived from multidimensional comparative analysis. Extending the idea of incorporating non-financial sustainability-related criteria into portfolio construction, Garcia-Bernabeu et al. (2024) integrated environmental, social, and governance (ESG) metrics into the conventional portfolio optimization model. However, they continued to utilize the traditional measure of variance in assessing risk, and they have not taken into account risk asymmetry or higher-order moments.

We build on previous research regarding portfolio optimization in several ways. First, we develop an optimization technique that is capable of handling the computational challenges associated with optimizing portfolios using semi-variance. A key issue is that the semi-covariance matrix used within this method depends upon both the target return and the evolving composition of the portfolio. Our algorithm iteratively calculates the semi-covariance matrix until it reaches a consistent equilibrium point while applying reasonable boundary conditions. This makes it possible to use downside-risk optimization more accurately. Second, we empirically build on previous studies by incorporating a comprehensive array of firm-level accounting characteristics as additional constraints within the portfolio optimization framework, thereby offering new evidence regarding the predictive significance of fundamental information for expected returns. Third, we also add to the body of knowledge by directly comparing the performance of semi-variance- and variance-based portfolios using a long-short strategy. This shows that the former consistently gives better risk-adjusted returns, which is a difference that hasn't been tested in previous studies. Finally, by looking at how a portfolio performs over different time periods, we show that the predictive power of fundamental information gets stronger over time. This supports the idea that this accounting information is progressively integrated into asset prices.

3. Accounting multiples and their importance in developing an investment strategy

Accounting multiples indicate how investors value a publicly traded company. These multiples are derived from a combination of market data and a company's financial performance. Empirical studies show that accounting multiples are associated with the future performance of firms, measured through stock returns. Currently, many market analysts rely on fundamental price ratios to enhance their investment strategies. Accounting multiples are widely used by investors and market makers due to their simple calculation. In addition, they rely on up-to-date data to obtain accurate results (Damodaran, 2006).

Extensive research has been conducted on whether accounting multiples influence stock returns.[1] In this study, we use the most widely used accounting multiples in the literature. Breen (1968) and Basu (1977) examined how the price-to-earnings ratio (P/E) influences the future profitability of firms. This ratio indicates how many times the market price of a share exceeds the net profit per share and is formalized as follows:

[1] As an example, Barbee et al. (2008) analyzed the relationship between market multiple values and subsequent stock prices. Loughran & Wellman (2011) and Crawford et al. (2019) provided evidence that enterprise multiple plays a key role in explaining stock returns. In turn, Vafaeikhah et al. (2025) propose a multi-objective model for portfolio rebalancing based on the constant proportion portfolio insurance strategy.

$$P/E = \frac{stock\ price\ per\ share}{earnings\ per\ share}$$

One can associate share prices not only with profit indicators but also with other attributes describing the company's financial position. It can be valuable for investors to assess how the market valuation of a company's share capital corresponds to the net value of its assets (Rosenberg, et al., 1985; Fama and French, 1992, Bird and Casavecchia). The price-to-book value per share ratio shows how many monetary units investors pay for each unit of a company's book value. The formula for the ratio is presented below:

$$\frac{P}{BV} = \frac{stock\ price\ per\ share}{book\ value\ per\ share},$$

where:

$$book\ value\ per\ share = \frac{assets - liabilities}{number\ of\ shares}.$$

Barbee et al. (2008) examined the relationship between various accounting ratios and subsequent annual returns for portfolios of liquid U.S. stocks. In univariate regressions, the price-to-sales (P/S) ratio exhibits the most consistently significant negative relationship with future returns and the highest explanatory power among the variables considered. The P/S ratio is calculated as follows:

$$P/S = \frac{stock\ price\ per\ shar}{sales\ per\ share}.$$

A related measure to the price-to-earnings ratio is the price-to-cash-flow-per-share ratio (P/CF), for which Chan and Chen (1991) and Sloan (1996) reported a positive correlation with stock returns. The PCF ratio is defined as follows:

$$P/CF = \frac{stock\ price\ per\ share}{operating\ cash\ flow\ per\ share}.$$

Many studies consider dividend yield an important factor influencing stock prices (Baker et al., 2020, and Ang and Bekaert, 2007). The dividend yield ratio (DY), calculated as dividends per share relative to the share's market price, indicates how much a company distributes in dividends compared with its share price. In general, a higher dividend yield makes a company more attractive to investors. The formula for the indicator is presented below:

$$DY = \frac{annual\ dividends\ per\ share}{stock\ price\ per\ share}$$

In this study, the indicator values are calculated for the entire portfolio, using its individual components as the basis. Therefore, the indicators used should be additive (that is, the indicator of a 'sum' of two assets should be equal to the sum of the indicators of individual assets). This condition is fulfilled when the indicators are expressed per share price, as is the case with the dividend yield (DY). For P/E, P/BV, P/S, and P/CF, their reciprocals are used instead of their original values. Accordingly, we use the following indicators in our analyses:

$$EP = \frac{1}{P/E} = \frac{earnings\ per\ share}{stock\ price\ per\ share}$$

$$BVP = \frac{1}{P/BV} = \frac{book\ value\ per\ share}{price\ of\ the\ share}$$

$$SP = \frac{1}{P/S} = \frac{sales\ per\ share}{stock\ price\ per\ share}$$

$$CFP = \frac{1}{P/CF} = \frac{operating\ cash\ flow\ per\ share}{stock\ price\ per\ share}$$

This study focuses on a single country and the constituents of a specific industrial index to ensure the reliability and comparability of our analysis. Cross-country comparisons are difficult as accounting practices may vary considerably across countries. Moreover, sector-specific financial characteristics result in different types of financial ratio information. For example, manufacturing businesses typically have lower quick ratios than do service or trading businesses. The use of a single industrial index eliminates some of the variability inherent in using a broad group of firms to evaluate the relationship between accounting metrics and stock price performance.

4. Downside risk in portfolio choice

From the very early stages of portfolio theory, variance has been the dominant risk measure (Markowitz, 1952). Yet, its validity has been questioned almost as long. The key limitation of variance lies in its treatment of all deviations from the mean return as equally undesirable. In reality, negative deviations refer to losses, and positive deviations offer the opportunity to achieve greater profits. The lack of symmetry in investors' preference for these two types of deviation lays the foundation for the development of the semi-variance as a risk measure. Semi-variance is calculated as the mean of the returns that fall under an investor's benchmark. Like many other downside-oriented risk metrics, such as lower partial moments, semi-variance focuses on the left-hand tail of the return distribution (Markowski, 2024). Depending on the context, the benchmark may be the expected return or another target value[2].

An investor who wishes to avoid portfolio returns falling below a target threshold will naturally gravitate toward portfolio strategies that minimize downside risk (Klebaner, Landsman, et al., 2017). While it is often argued that, under symmetric distributions, variance performs no worse than semi-variance as a risk measure (Galagedera & Brooks, 2007), empirical evidence suggests that asset returns in financial markets are rarely normally distributed or symmetric (Post & van Viet, 2006; Sun & Yan, 2003). In such cases, the relevance of downside risk measures becomes more pronounced.

For right-skewed distributions, most of the variance arises from positive deviations, which correspond to higher returns, whereas the contribution of downside deviations is relatively limited. This explains why investors typically favor assets whose returns are skewed to the right (Galagedera & Brooks, 2007; Peiro, 1999; Kassimatis, 2021). Thus, skewness has become an important factor in determining risk[3].

In addition, this preference toward downside-oriented measures also aligns with the Prospect Theory (Kahneman & Tversky, 1979), where it is well established that investors tend to be more concerned by potential losses than they would be concerned about corresponding gains. Furthermore, Nawrocki and Viole (2024) demonstrated that partial moments are less susceptible to both statistical and estimation errors than the risk measures of the traditional mean-variance analysis. Therefore, these alternative risk measures are prioritized in many statistical applications, especially when the true underlying distribution is unknown.

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[2] The reliance on variance is only defensible under specific conditions: when returns are normally or symmetrically distributed, or when investors possess quadratic utility functions. The violation of these assumptions limits the applicability of the classical Markowitz framework and may render its results misleading.

[3] For assets with skewed returns, the mean–variance model tends to overestimate risk by penalizing upside volatility equally with downside risk, thereby excluding portfolios that may in fact be attractive from a downside-efficiency perspective (Foo & Eng, 2000).

In Figure 1, it is shown that even under the assumption of normally distributed returns, symmetric and downside risk measures do not convey identical information to investors. Consider the returns of two portfolios with normal distributions that share the same standard deviation but differ in their expected rates of return. It is clear that Investment A is more profitable than Portfolio B. The risk of both investments, when measured by standard deviation or any other symmetric risk measure, appears identical. However, in the case of Investment A, the investor always achieves a positive return, whereas in the case of Investment B, there is a possibility of incurring a loss, illustrated by the shaded area. This essential difference in the risk of the two investments can be captured by a downside risk measure such as semi-variance.

< Insert Figure 1 about here >

All these arguments point to the superiority of downside risk metrics over their classical counterparts. As a result, in this study, we adopt downside risk measures as the primary risk metric for portfolio optimization. This choice allows us to account for the empirical features of return distributions and to align the risk assessment more closely with investor behavior.

In its discrete case, semi-variance is defined as the average of squared shortfalls of returns below a specified target level γ. Mathematically, it can be expressed as:

$$dS_i^2(\gamma) = \frac{\sum_{t=1}^{m} \ d_{it}{}^2(\gamma)}{m-1}, \quad t = (1,2,\dots,m),$$

where,

$$d_{it}(\gamma) = \begin{cases} 0 & for \ R_{it} \geq \gamma \\ R_{it} - \gamma & for \ \ R_{it} < \gamma \end{cases}$$

R_{it} – return on stock i in period t,

$dS_i^2(\gamma)$ – semi-variance for stock i.

m – the number of observations,

γ – mean return or any target return, such as the risk-free rate, selected by the investor.

An extension of semi-variance as a risk measure is provided by lower partial moments, introduced by Bawa (1975) and Fishburn (1977). Lower partial moments generalize the concept of downside risk by allowing the emphasis on losses to be adjusted through the order n, where higher orders give greater weight to larger shortfalls. Formally, the lower partial moment of order n can be expressed as:

$$LPM_i^n(\gamma) = \frac{1}{m-1}\sum_{t=1}^{m} lpm_{it}^n(\gamma),$$

where

$$lpm_{it}(\gamma) = \begin{cases} 0 & for\ R_{it} \geq \gamma \\ R_{it} - \gamma & for\ \ R_{it} < \gamma \end{cases}.$$

Note that when $n = 2$, the lower partial moment reduces to semi-variance.

The semi-variance, $dS_P^2(x;\gamma)$, of a portfolio consisting of k assets $x = (x_1, \dots, x_k)^T$ is defined by:

$$dS_P^2(x;\gamma) = \sum_{i=1}^{k}\sum_{j=1}^{k} x_i x_j d_{ij}(x;\gamma),$$

where x_i is the weight of stock i in the portfolio, the symbol $d_{ij}(x;\gamma)$ denotes the semi-covariance between returns on stocks i and j, which is represented by:

$$d_{ij}(x;\gamma) = \frac{1}{m-1}\sum_{t=1}^{m} d_{ijt}(x;\gamma), \tag{1}$$

where:

$$d_{ijt}(x;\gamma) = \begin{cases} 0 & for\ R_{pt}(x) \geq \gamma \\ (R_{it} - \gamma)(R_{jt} - \gamma) & for\ R_{pt}(x) < \gamma \end{cases}$$

$$R_{pt}(x) = \sum_{i=1}^{k} x_i R_{it},\ t = (1,2,\dots,m).$$

It should be noted that a portfolio's semi-variance depends on the chosen target return γ and must be recalculated whenever γ is adjusted. If the reference level γ is set equal to the portfolio's mean return, it becomes portfolio-dependent and must be recomputed each time the portfolio composition changes. Unlike variance, which treats deviations symmetrically and does not depend on a reference point, semi-variance focuses only on downside deviations. This recalculation is therefore essential in portfolio optimization, as the measure of downside risk directly affects asset selection and the determination of the efficient frontier. Moreover, this dependency increases computational complexity, particularly for large portfolios, since semi-variance must be updated repeatedly during iterative optimization procedures.

5. Portfolio Selection with Semi-variance and Accounting Multiples

We consider a portfolio that contains k different assets. The mean return of this portfolio is then given by the formula:

$$\mu_P = \sum_{i=1}^{k} x_i \mu_i$$

where μ_i is the mean return on asset i and x_i represents the market-capitalization-based weight of the investment in stock i.

The return variance of the portfolio rate of return is denoted by:

$$S_P^2(x) = \sum_{i=1}^{k} \sum_{j=1}^{k} x_i x_j \sigma_{ij}.$$

where σ_{ij} shows the covariance between returns on stocks i and j.

Semi-variance of the portfolio is expressed as:

$$dS_P^2(x;\gamma) = \sum_{i=1}^{k} \sum_{j=1}^{k} x_i x_j d_{ij}(x;\gamma),$$

where the semi-covariance of stock returns is given by (1).

When choosing a portfolio, we also consider an accounting multiple. This can be connected with one of the five multiples described in Section 3 (EP, BVP, SP, CFP, or DY). β_i denotes the value of this criterion for stock i. The value of this multiple for the entire portfolio is given by:

$$\beta_P = \sum_{i=1}^{k} x_i \beta_i.$$

In the empirical analysis, we examine portfolios obtained as solutions to the following optimization problems:

(a) The minimum-variance portfolio, defined as the solution to

$$\min_{x_1,\dots,x_k} S_P^2(x) = \sum_{i=1}^{k}\sum_{j=1}^{k} x_i x_j \sigma_{ij} \tag{2}$$

with the restriction that

$$\sum_{i=1}^{k} x_i = 1, \tag{3}$$

$$x_1, x_2, \dots x_k \geq 0. \tag{4}$$

(b) A portfolio that minimizes variance under the condition that its expected return is at least μ_0. The problem is specified by Eqns. (2)–(4), with the following additional restriction:

$$\mu_P = \sum_{i=1}^{k} x_i \mu_i \geq \gamma \tag{5}$$

(c) A variance-minimizing portfolio with restrictions on mean return and fundamental value, where the portfolio's fundamental value, measured by a chosen market multiple, must be at least β_0. This gives a problem (2)-(5) with an additional restriction:

$$\beta_P = \sum_{i=1}^{k} x_i \beta_i \geq \beta_0 \tag{6}$$

From a mathematical point of view, problems (a)-(c) are problems of quadratic optimization with linear constraints (given by equations and inequalities). They can be solved using standard algorithms.

The second set of portfolios is based on semi-variance optimization, defined by the following problems:

(d) The unconditional semi-variance–minimizing portfolio, obtained by solving:

$$\min_{x_1,\dots,x_k} dS_P^2(x;\gamma) = \sum_{i=1}^{k}\sum_{j=1}^{k} x_i x_j d_{ij}(x;\gamma), \tag{7}$$

with restrictions (3)-(4).

(e) A portfolio that minimizes semi-variance under the condition that its mean return is at least μ_0. The problem is specified by Eqns. (3)–(5), and the goal function is given by Eqn. (7).

(f) A semi-variance-minimizing portfolio with restrictions on mean return and accounting multiple. The problem is specified by Eqns. (3)-(6), and the goal function is given by Eqn. (7).

Problems (d)-(f) are not standard optimization problems because semi-covariances $d_{ij}(x;\gamma)$ in (7) change with the changes in the portfolio structure x_1, …, x_k. Using semi-variance to determine effective portfolios creates considerable difficulties because calculating semi-covariances $d_{ij}(x;\gamma)$ requires knowing the periods when the portfolio return is below the target γ, a determination that depends on the portfolio's composition. This makes calculations of effective portfolios with semi-variance as a risk measure more complicated than in the case when variance is used. Each time a portfolio with the minimum semi-variance is determined, the composition of the portfolio changes. Therefore, the semi-covariances $d_{ij}(x;\gamma)$ should be re-estimated.

To solve these problems, the numerical algorithm depicted in Figure 2 is used. We start with an initial portfolio, $x^{(0)}$, which is a variance-minimizing portfolio. For each iteration n, we re-estimate semi-covariance terms $d_{ij}(x^{(n)};\gamma)$ based on the portfolio $x^{(n)}$. Then, we solve each of the problems (d)-(f) as a quadratic programming problem to obtain the next portfolio weights, $x^{(n+1)}$, using the Goldfarb and Idnani (1983) algorithm.[4] Next, we re-estimate the semi-covariances, $d_{ij}(x;\gamma)$, and resolve the optimization problem using the updated data. This iterative process continues until convergence is achieved. In other words, the changes in portfolio composition between successive iterations become sufficiently small (i.e., $\|x^{(n+1)} - x^{(n)}\| < \varepsilon$, where ε is 0.01).

< Insert Figure 2 about here >

A further complexity emerges from the possibility that the efficient frontier may encompass infinitely many optimal solutions. This creates an ambiguity in the selection of boundary conditions within bi-criteria and tri-criteria formulations. In these instances, designating the risk-free rate as the requisite rate of return at a specific moment establishes a logical and economically substantiated border condition. Similarly, establishing the boundary value of a specific value-based indicator (market multiple) at the level aligned with the equally weighted

[4] The optimizations problems that are considered in the paper are problems of minimizing quadratic forms (without linear element, i.e. the goal function is strictly convex), defined in Eqn. (2) or Eqn. (7), given some linear restrictions. The Goldfarb and Idnani algorithm is the standard tool for this kind of problems.

portfolio at a particular moment is a sensible decision. In computation, we use specially developed software written in R.

5. Data

The study analyzes companies listed on the New York Stock Exchange (NYSE) and included in the Dow Jones Industrial Average (DJIA) Index. Financial institutions, as well as companies with quotation series of fewer than 500 trading days, were excluded to enable a reliable estimation of portfolio analysis parameters (particularly correlation coefficients). The final research sample comprised 20 firms. As the risk-free rate, the yield on the 13-week Treasury Bill is adopted.

Portfolio performance is assessed using closing share prices, with returns measured at a monthly frequency. Firm-level financial data, such as earnings, book values of equity, cash flows, dividends, and closing prices, are retrieved from Thomson Reuters Refinitiv Eikon. Based on these data, accounting multiples are calculated for each trading day, and monthly stock returns are derived for all firms. From the return series, expected returns, variances, and covariances are subsequently estimated.

The parameters used in portfolio construction are estimated on the basis of the preceding 500 trading days. Consequently, the estimation period extends from 11 May 2017 to 4 March 2025. For each trading day, 15 distinct portfolios are constructed by solving optimization problems (a)–(f) as outlined in Section 5. Investment horizons are defined ex ante, with portfolios purchased on a given trading day assumed to be liquidated after one month (1M portfolios), three months (3M portfolios), or six months (6M portfolios). In this study, 15 portfolio types are examined. Table 1 presents their descriptions along with the symbols used for reference.

< Insert Table 1 about here >

In portfolios subject to restrictions on mean return and/or on one of the accounting indicators (EP, BVP, SP, DY, CFP), the following constraints are imposed. For accounting indicators, the portfolio indicator is required to be no lower than the corresponding average value for all companies in the sample. This condition ensured that the financial performance of the portfolio would not fall below the market average. With respect to expected return, the mean portfolio return is required to be no lower than the risk-free rate, thereby guaranteeing that portfolios provide sufficiently high returns.

All 15 portfolio types (see Table 1) are constructed for each trading day between 4 November 2019 and 4 March 2025 in the case of one-month investment horizons. For the three-month investment horizon, the sample period ended on 30 December 2024, while for the six-month horizon, it ended on 27 September 2024. In total, 23,085 portfolios are created for 1,539 trading days under the one-month horizon; 22,395 portfolios for 1,493 trading days under the three-month horizon; and 21,405 portfolios for 1,427 trading days under the six-month horizon.

6. Empirical Results

Using a rolling window of the most recent 500 trading days, semi-covariance and covariance matrices are estimated and employed in the portfolio optimization procedure. The performance of the optimization models is evaluated ex-post, with risk measures and adjusted risk measures computed from realized returns. Due to the large number of portfolios analyzed in this study (66,885), details regarding portfolio composition and other elements of the ex-ante analysis, such as expected risk or mean market ratios, are not presented. Instead, the characteristics reported in tables pertain exclusively to realized returns (ex-post analysis). Thus, they capture the actual investment outcomes and the corresponding risks borne by investors.

6.1. The Performance Evaluation of Optimized Portfolios

For each of the three investment horizons, the realized holding-period returns are computed. Based on these returns, the following distributional characteristics are derived: the mean and median rate of return, standard deviation, minimum return, Value-at-Risk (VaR, specified as the 10% quantile of returns), semi-deviation, skewness (asymmetry), the Sharpe ratio (mean return divided by standard deviation), and the Sortino ratio (mean return divided by semi-deviation). It is worth emphasizing that both the Sharpe and Sortino ratios provide particularly informative criteria for portfolio comparison in instances where the realized excess rate of return is positive.

To assess differences in portfolio returns generated by various construction methods, we conduct a series of statistical tests. As a first step, we examine the distributional properties of the returns using the Shapiro–Wilk and Jarque–Bera tests. For all portfolio construction methods and investment horizons, both tests reject the null hypothesis of normality (p-values < 0.03). Consequently, standard parametric analysis of variance (ANOVA), which relies on the assumption of normally distributed data, cannot be applied.

Instead, we employ the non-parametric Kruskal–Wallis test. This test evaluates whether the central tendency of the distributions differs between groups and is commonly interpreted as a test for differences in medians (though it may also reflect differences in means under certain conditions). In addition, to investigate potential differences in the dispersion of returns, we conduct Levene's test. The null hypothesis of this test is that all groups being compared have equal population variances. We use this test, as it is robust against departures from normality.

< Insert Table 2 about here >

The summary statistics of the one-month portfolios in Table 2 show that the equally weighted portfolio has the greatest mean returns. However, it is also the riskiest. It has the largest standard deviation and the lowest minimum realized return among all portfolios. Its return distribution also displays considerable positive skewness. Concerning performance efficiency, the equally weighted portfolio generated the greatest Sharpe ratio and Sortino ratio, thereby suggesting that it provides better risk-adjusted returns than each of the other portfolios. Comparing the rest of the portfolio types, portfolios that are optimized based on semi-variance show consistently better performance than portfolios that are optimized based on variance. The inclusion of an additional optimization criterion relating to EP and CFP improves portfolio performance slightly. This improvement is most pronounced for the three-criterion portfolios that include the CFP constraint.

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< Insert Table 3 about here >

Table 3 displays the pairwise comparisons of the median returns for the one-month portfolios. Each entry is the difference between the median return of the portfolio in the row and the median return of the portfolio in the column. A negative value would represent the relative under-performance of the row portfolio. The results reveal that the equally weighted portfolio consistently outperforms all competing strategies. Variance-minimizing portfolios generally exhibit slightly lower median returns compared to the other optimization methods. The p-values from the Kruskal–Wallis tests, shown in parentheses beneath the median differences, indicate that the differences between the equally weighted portfolio and all alternatives (first column) are statistically significant at the 5% level. In addition, statistically significant differences are observed between the semi-variance–minimizing portfolio with the cash-flow-to-price ratio constraint (the SV-E-CFP portfolio) and the variance-minimizing portfolios.

< Insert Table 4 about here >

Table 4 presents the summary statistics for the three-month portfolios. The highest rates of return are achieved by portfolios minimizing semi-variance and those minimizing semi-variance with an additional restriction on mean return. These strategies are also associated with the highest levels of risk, as measured by the standard deviation of returns. Semi-variance minimizing portfolios that include mean return and accounting multiple restrictions (EP, BVP, SP, and CFP) exhibit relatively stronger performance. These portfolios also have better efficiency than their peers, as evidenced by higher Sharpe and Sortino ratios. In stark contrast, in the three-month horizon, the equally weighted portfolio produces poor returns and also maintains high levels of risk. This results in less efficient performance as indicated by lower Sharpe and Sortino ratios.

< Insert Table 5 about here >

Table 5 displays the pairwise differences in median returns for 3-month portfolios. In contrast to Table 3, which reports median differences for a one-month horizon, Table 5 shows that all portfolios constructed to minimize semi-variance tend to achieve higher median returns than both variance-minimizing and equal-weighted portfolios. Kruskal–Wallis tests indicate that these median differences between semi-variance-based portfolios and the variance-based and equal-weighted portfolios are significantly positive, reinforcing the finding that semi-variance-based portfolios outperform the alternatives. However, the median return differences within the group of semi-variance-based portfolios are generally not statistically significant, suggesting that no particular semi-variance-based portfolio dominates the others. However, it is noteworthy that the SV-E-DY portfolio consistently underperforms the other semi-variance–based portfolios.

< Insert Table 6 about here >

Table 6 presents the descriptive properties of the six-month portfolios. Extending the investment period substantially increases realized returns across all portfolio types. In addition, both the volatility of returns (measured by standard deviation) and efficiency measures (Sharpe and Sortino ratios) rise markedly relative to the one-month portfolios. These findings are consistent with the patterns observed for the three-month horizon. Once again, the highest returns are generated by portfolios minimizing semi-variance and semi-variance with a restriction on mean return, while semi-variance-based portfolios incorporating individual accounting multiples also deliver the strongest results. Particularly, the SV-E-EP portfolio exhibits the highest Sharpe (1.21) and Sortino (6.55) ratios. Conversely, the equally weighted portfolio and variance-minimizing portfolios with restrictions on mean return and market multiples produce comparatively lower returns.

< Insert Table 7 about here >

The median raw return differences for six-month portfolios are displayed in Table 7. The results closely mirror those of the three-month portfolios reported in Table 5. In particular, the semi-variance–minimizing portfolios consistently achieve higher median returns than both the variance-based and equal-weighted portfolios, and the Kruskal–Wallis tests confirm that this outperformance is statistically significant. However, performance differences within the group of semi-variance–based portfolios are generally not significant. Hence, none of them is clearly superior to the other.

Overall, extending the investment horizon from one to three and six months altered the level of returns across portfolio types. In particular, semi-variance-based strategies consistently dominate in terms of return and risk-adjusted performance, whereas the equally weighted portfolio and variance-based strategies become progressively less efficient as the horizon lengthens.

We also examine the differences in distributional characteristics of optimized portfolios. Table A1 in the Online Appendix presents results from Levene's test, which evaluates the homogeneity of variances across portfolio types. The outcomes provide strong evidence of heterogeneity in return variance across portfolio types, with particularly pronounced differences between semi-variance-minimizing (SV) and variance-minimizing (V) portfolios. Consistent with the findings reported in Tables 2, 4, and 6, variance-minimizing portfolios exhibit systematically lower return dispersions (as measured by standard deviation and thus variance), which corroborates the outcomes of the Levene test. As investment horizon lengthens, portfolios within the semi-variance group also exhibit distinct return distributions. These results call for a deeper performance assessment, as comparisons of median differences are meaningful only when the portfolios exhibit similar variance or overall risk characteristics. In the next subsection, we conduct performance evaluation tests that account for the effects of several risk factors.

6.2. Alphas as Risk-adjusted Performance Indicators

The results of the Levene test in the preceding subsection indicate that the portfolios under comparison exhibit differing levels of variance. As the returns on these portfolios do not share the same variance, directly comparing their raw median returns does not clarify whether a return gap arises from true performance differentials or from variations in risk (i.e., variance). Moreover, risk factors beyond variance or semi-variance may also influence portfolio performance. To account for these effects, we also estimate Jensen's alpha using the Fama and French (2015) five-factor model, extended with Carhart's (1999) momentum factor, to obtain a more comprehensive measure of risk-adjusted performance:

$$R_t - R_{ft} = \alpha + \beta_{mkt}\, MKT_t + \beta_{smb}\, SMB_t + \beta_{hml}\, HML_t + \beta_{cma}\, CMA_t + \beta_{rmw}\, RMW_t + \beta_{mom}\, MOM_t + \varepsilon_t \quad (8)$$

where R_t denotes the return on one of the 15 portfolios over one-, three-, or six-month holding periods on day t. *MKT* represents the market factor, *SMB* captures the size effect, *HML* reflects the value effect, *CMA* symbolizes the investment factor, and *RMW* corresponds to the profitability factor. Finally, *MOM* refers to the momentum factor proposed by Carhart (1999).

All six US factors are obtained from Kenneth French's data library.[5] As we rebalance the portfolios daily based on the optimization results for each day within the research period, our one-, three-, and six-month holding periods do not overlap with calendar months. As an illustration, the holding periods initiated on May 12 terminate on June 11, August 11, and November 11 for the one-, three-, and six-month horizons, respectively, thereby diverging markedly from conventional calendar month boundaries. Consequently, we cannot directly use the monthly Fama-French-Carhart (FFC) six factors from Kenneth French's data library, which are constructed for calendar months. Instead, we download the daily FFC six factors and compound them over one-, three-, and six-month intervals to compute the monthly, quarterly, and semiannual risk factors for each trading day in the sample period, assuming 22 trading days per month. After constructing these risk factors for each holding period and each day, we estimate Eqn. (8) and report the results in Table 8.

[5] Please see https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html for the details of factor formation.

<Insert Table 8 about here>

The one-month holding period results show that all portfolios, except for the equal-weighted portfolio, have negative alphas. Portfolios that minimize semi-variance show less negative alphas than those that minimize variance. This suggests that strategies based on semi-variance are more effective than those based on variance. Nonetheless, since all portfolios optimized on the basis of either semi-variance or variance yield negative alphas, neither approach produces positive abnormal returns over the one-month horizon. In contrast, the equal-weighted portfolio generates a small positive alpha (0.09%). However, this value is not statistically significant (t-stat = 0.79). As a result, no trading strategy examined provides statistically significant abnormal returns in the short run.

In contrast, the results for the three- and six-month holding horizons sketch a completely different picture. Portfolios based on semi-variance minimization produce alphas that are both economically more significant than those of variance-based portfolios. For instance, the SV and SV-E portfolios deliver the largest three-month alphas of 1.71%, each with a t-statistic of 4.17, while the SV-E-EP portfolio achieves the highest six-month alpha of 7.55% with a highly significant t-statistic of 9.94. None of the portfolios minimizing variance generates statistically significant alphas over the three-month holding period. While variance-minimizing portfolios show significantly positive alphas over six months, the magnitude of these alphas is still lower than that of the semi-variance-minimizing portfolios. Hence, semi-variance-based strategies outperform variance-based strategies.

Overall, semi-variance-minimizing portfolios provide economically larger alphas than variance-minimizing portfolios for all holding periods examined. The dominance of semi-variance-minimizing portfolios strengthens as the holding period elongates. These findings support our earlier results based on raw returns as well as Sharpe and Sortino ratios.

6.3. Alphas on Zero-Cost Long-Short Optimal Portfolios

The previous subsection identifies the variance- or semi-variance-minimizing portfolios, with or without constraints, that earn significantly positive abnormal returns. In this section, we examine whether each optimal portfolio outperforms or underperforms the others. In other words, we test whether the pair-wise differences between alphas on competing portfolios are different from zero or not. To this end, we analyze the alpha on a zero-cost trading strategy that takes a long position in one optimal portfolio and a short position in another. Specifically, the raw return difference between two optimal portfolios is regressed on the six FFC factors, and the intercept term (alpha) of the regression, along with its t-statistics, is reported in Tables 10, 11, and 12 for the one-, three-, and six-month holding periods, respectively. In other words, the alphas reported in these tables correspond to the difference portfolios, constructed as the row portfolio minus the column portfolio. For instance, the alpha of -0.0053 (t-stat = -3.86) in the top left corner of Table 9 is the alpha on a strategy that goes long the row portfolio V and shorts the column portfolio EW. A statistically significant negative alpha indicates that the risk-adjusted return on V is significantly lower than that on EW, implying that EW outperforms V.

<Insert Table 9 about here>

A key point to note about Table 9 is that the EW portfolio significantly outperforms all of the optimal portfolios. This means that trading optimal portfolios is not a profitable strategy for the one-month holding period. Simply holding an equal-weighted portfolio is the best short-term strategy. Another important finding is that portfolios that minimize semi-variance usually fail to outperform portfolios that minimize variance. The only exception is the SV-E-CFP portfolio. It weakly outperforms variance-minimizing portfolios, with alphas ranging from 0.15% to 0.23% and t-statistics varying from 1.53 to 1.97. The comparable performance of semi-variance- and variance-minimizing portfolios is not surprising, given that both groups of portfolios generated negative one-month alphas, as shown previously in Table 8. Overall, these results show that risk-adjusted strategies based on portfolio optimization do not pay off in the short run.

<Insert Table 10 about here>

Conversely, the three-month alphas on long-short portfolios, presented in Table 10, reveal a clear dominance of semi-variance-minimizing portfolios over their variance-minimizing counterparts. Every portfolio that minimizes semi-variance significantly outperforms all variance-minimizing portfolios as well as the equal-weighted portfolio. This naturally raises the question of which portfolio performs best among the semi-variance-based (SV) portfolios. The row minus column portfolios indicate that the SV and SV-E portfolios are the top-performing optimal portfolios for the three-month holding period.

<Insert Table 11 about here>

A similar analysis for the six-month holding period, reported in Table 11, produces qualitatively consistent results. All SV-based portfolios outperform both the V-based portfolios and the equal-weighted portfolio. However, the best performing SV-based portfolio over six months is the SV-E-EP portfolio.

In short, optimal portfolios minimizing semi-variance yield the highest abnormal returns over three- and six-month holding periods. Hence, semi-variance is a more effective risk metric than traditional variance for medium term on long term horizons. Furthermore, incorporating accounting information as an additional constraint also improves portfolio performance in holdings periods longer than one month.

6.4. Sub-period Results

Our research period covers the COVID-19 pandemic. There is substantial evidence in the literature that stock return behavior changes during crisis periods (Harb and Umutlu, 2024). To check whether the portfolio performance is affected during a global instability period, we split our research period into three sub-periods: pre-COVID, COVID, and post-COVID periods. The pre-COVID period covers April 2019 to February 2020. The COVID period spans from March 2020 through January 2022. Finally, the post-COVID period is from February 2022 to March 2025. Holding periods will be categorized in one of those sub-periods based on whether more than fifty percent of a particular holding period is included in that sub-period.

< Insert Table 12 about here >

Table 12 Panel A presents the results by sub-period for one-month alphas. All t-statistic values are insignificant during the Pre-COVID period, which means none of the portfolio alphas significantly deviate from zero. Although insignificant, variance-minimizing portfolios exhibit negative alphas, whereas semi-variance-minimizing portfolios deliver positive alphas. In the COVID period, all portfolios yield significantly negative alphas, suggesting that there is not a remarkable performance difference between the variance- and semi-variance-based portfolios. During the post-COVID period, almost all portfolio alphas turn positive, but they remain statistically insignificant. In summary, the sub-period results for one-month alphas do not point out a substantial difference in the performance of the two portfolio groups.

We next examine the three-month alphas for each subperiod, the results of which are reported in Panel B. During the pre-COVID period, alphas are not statistically different than zero, both for the semi-variance- and variance-based portfolios. The most striking differences between these two groups of portfolios are observed during the COVID period. Portfolios constructed using semi-variance deliver highly significant three-month alphas, whereas those based on variance or equal weighting generate significantly negative alphas. Hence, semi-variance-based strategies are especially useful for long-term investments and/or in times of extreme market turmoil because they provide enhanced portfolio performance at exactly the time when resiliency is required the most.

In the post-COVID period, portfolios that minimize semi-variance keep performing strongly, as evidenced by significantly positive high alphas. This indicates that semi-variance-based portfolios yield abnormal returns not only during turbulent market conditions but also in tranquil periods. During the same sub-period, variance-based and equally weighted portfolios also generate positive and significant alphas. However, portfolios with the highest alphas, specifically the SV and SV-E portfolios, still belong to the group of semi-variance-minimizing portfolios. Therefore, semi-variance-minimizing portfolios provide a persistent advantage relative to variance-minimizing portfolios regardless of market conditions, especially for extended holding periods.

The six-month alphas for each subperiod, presented in Panel C, indicate that the patterns seen in the three-month holding period are even more pronounced over six months. Specifically, the superior performance of semi-variance-minimizing portfolios, both economically and statistically, strengthens during the COVID and post-COVID periods. Consistent with the pre-COVID results in Panel B, the six-month holding period in the pre-COVID subperiod also shows that none of the portfolios earn abnormal returns. In sum, portfolios that aim to minimize semi-variance always dominate portfolios that aim to minimize variance during both market downturns and recovery periods for three- and six-month holding periods.

7. Conclusions

This study contributes to the portfolio optimization literature through the development and evaluation of a framework for creating fundamental portfolios using both variance and semi-variance-based optimization frameworks. It builds on existing multiple criteria portfolio selection models to develop a customized model that integrates expected return, downside risk, and firm-specific accounting information simultaneously. The methodology addresses significant computational problems associated with multiple criteria programming and specifically tackles semi-variance, a type of asymmetric risk measure. A method for generating efficient portfolios within the semi-variance optimization setting was developed, along with a means of adding additional accounting-based constraints to the portfolio optimization process.

Empirical results show that portfolio optimization based on semi-variance produces better results than traditional variance-based approaches. Semi-variance minimizing portfolios outperform variance-minimizing portfolios in terms of all performance metrics (Sharpe Ratio, Sortino Ratio, Fama-French-Carhart Six Factor Alpha) and across all investment horizons. Results confirm that variance has limited use as a risk metric. It cannot distinguish between negative volatility and positive volatility. Also, results indicate that there is a significant advantage in explicitly accounting for downside risk in portfolio selection.

In addition, the optimization framework is extended to include accounting-based criteria measured by five market-related valuation ratios (earnings, sales revenue, book value of equity, operating cash flow, and dividend). Therefore, the efficiency of portfolios is assessed along three axes: profit ability, risk, and market multiples for each stock included in the portfolio. Results showed that the use of fundamental information embedded in market multiples can lead to improved portfolio performance, and that the benefits of using this information increase as the investment horizon elongates. Although unconstrained semi-variance portfolios and those restricted solely by an expected return constraint produce the best results at the end of three months, the advantages of using fundamental information become more apparent over six months. Specifically, the earnings-to-price ratio emerged as the most useful market multiple. Portfolios restricted by this metric outperform all other portfolio specifications for a six-month holding period. These results validate the idea that financial analysts' informational advantage requires time to reflect in stock prices and, therefore, should have the greatest benefit in a long-term investment context.

Additionally, results demonstrate that the benefits of optimizing for semi-variance are even greater during periods of increased market volatility. During the COVID-19 pandemic, semi-variance-minimizing portfolios exhibit superior risk-adjusted performance and yield positive abnormal returns over extended horizons. However, they also demonstrate continued superiority in the aftermath of the pandemic and therefore support the use of semi-variance as a reliable measure of risk across varying market conditions.

Overall, our results indicate that investors can improve their portfolio performance by optimizing portfolios relative to downside risks and/or incorporating select fundamental characteristics when investing sufficiently long enough to allow adequate time for these characteristics to have an effect on prices. This study provides empirical evidence for why semi-variance should be used instead of variance when measuring investment risk and demonstrates how to incorporate accounting information into a multi-criteria optimization problem. Future studies can build on this work by extending this type of optimization to alternative asset classes, international markets, alternative measures of downside risk, and/or other corporate characteristics that may add to portfolio performance.

Disclosure of Interest

The authors report there are no competing interests to declare.

Declaration of Funding

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Data Availability Statement

The data that support the findings of this study are available from London Stock Exchange Group (LSEG) Workspace with Datastream. Restrictions apply to the availability of these data, which were used under license for this study. Data are available from the authors with the permission of LSEG.

CRediT authorship contribution statement

A. Rutkowska-Miczka: Conceptualization, Methodology, Validation, Formal analysis, Investigation, Resources, Data curation, Writing—original draft preparation, Supervision, Project administration. **P. Kliber:** Methodology, Software, Validation, Formal analysis, Investigation, Resources, Writing—original draft preparation, Writing—review and editing. **K. Szydłowski:** Methodology, Validation, Formal analysis, Investigation, Resources, Writing—original draft preparation, Writing—review and editing. **M. Umutlu:** Methodology, Validation, Formal analysis, Investigation, Resources, Writing—original draft preparation, Writing—review and editing.

References

Ang, A., & Bekaert, G. (2007). Stock Return Predictability: Is it There? *The Review of Financial Studies*, 20(3), 651-707. https://doi.org/10.1093/rfs/hhl021

Baker, H.K., De Ridder, A., & Rasbrant, J. (2020). Investors and dividend yields. *The Quarterly Review of Economics and Finance*, 76, 386-395. https://doi.org/10.1016/j.qref.2019.09.004

Ballestero, E., Bravo, M., Perez-Gladish, B., Arenas-Parra, M., & Pla-Santamaria, D. (2012). Socially Responsible Investment: a Multicriteria Approach to Portfolio Selection Combining Ethical and Financial Objectives. *European Journal of Operational Research*, 216, 487–494.

Barbee Jr, W. C., Jeong, J. G., & Mukherji, S. (2008). Relations between portfolio returns and market multiples. *Global Finance Journal*, 19(1), 1-10.

Basu, S. (1977). Investment Performance of Common Stocks in Relation to their Price-Earnings Ratios: a Test of the Efficient Market Hypothesis. *Journal of Finance*, 32(3), 663–682.

Basu, S. (1983). The Relationship Between Earnings' Yield, Market Value and Return for NYSE Common Stocks. Further Evidence. *Journal of Financial Economics*, 12, 129–156.

Bawa, V.S. (1975). Optimal rules for ordering uncertain prospects. *Journal of Financial Economics*, 2(1), 95–121. https://doi.org/10.1016/0304-405X(75)90025-2

Bird, R., & Casavecchia, L. (2007). Sentiment and financial health indicators form value and growth stocks: The European experience. *European Journal of Finance*, 13(8), 769-793.

Breen, W. (1968). Low Price-Earnings Ratios and Industry Relatives. *Financial Analyst Journal*, 24(4), 125–127.

Briec, W., Kerstens, K., & Jokund, O. (2007). Mean-Variance-Skewness Portfolio Performance Gauging: A General Shortage Function and Dual Approach. *Management Science*, 53, 135–149.

Chan, K.C., & Chen, N-F. (1991). Structural and return characteristics of small and large firms. *Journal of Finance,* 46(4), 1476-1484.

Cakici, N., & Zaremba, A. (2025). Accounting vs technical information: What matters more for stock return predictability? *Journal of International Financial Markets, Institutions and Money*, *104*, 102207. https://doi.org/10.1016/j.intfin.2025.102207

Crawford, S.S., Gray, W.R., & Vogel, J. (2019). Why Do Enterprise Multiples Predict Expected Stock Returns? The Journal of Portfolio Management, 46(1), 123-138. DOI: 10.3905/jpm.2019.1.105

Damodaran, A. (2006). *Damodaran on valuation (2d ed.)*. John Wiley, New York.

Doumpos, M., & Zopounidis, C. (2014). *Multicriteria Analysis in Finance*. Springer, Heidelberg, New York.

Fabozzi, F., Focardi, S., & Jonas, C. (2007). Trends in Quantitative Equity Management: Survey Results. *Quantitative Finance*, 7, 115–122.

Fama, & French. (1992). The Cross-Section of Expected Stock Returns. *Journal of Finance*, 47(2), 427-465. https://dx.doi.org/10.1111/j.1540-6261.1992.tb04398.x

Fama, E.F., & French, K.R. (2015). A five-factor asset pricing model. *Journal of Financial Economics*, 116, 1–22.

Fishburn, P.C. (1977). Mean-Risk Analysis with Risk Associated with Below-Target Returns. *The American Economic Review*, 67(2), 116–126. https://doi.org/10.2307/1807225

Foo, T., & Eng, S. (2000). Asset Allocation in a Downside Risk Framework. *Journal of Real Estate. Portfolio Management*, 6(3), 213–223. https://doi.org/10.1080/10835547.2000.12089612

Galagedera, D., & Brooks, R.D. (2007), Is co-skewness a better measure of risk in the downside than downside beta? Evidence in emerging market data. *Journal of Multinational Financial Management*, 17, 214-230.

Garcia-Bernabeu, A., Hilario-Caballero, A., Tardella, F., & Pla-Santamaria, D. (2024). ESG integration in portfolio selection: A robust preference-based multicriteria approach. *Operations Research Perspectives*, *12*, 100305.

Goldfarb, D., & Idnani A. (1983). A numerically stable dual method for solving strictly convex quadratic programs. *Mathematical Programming*, 27, 1–33.

Harb, H., & Umutlu, M. (2024). Shock resistors or transmitters? Contagion across industries and countries during the COVID-19 pandemic and the global financial crisis. *Economics Letters*, 244, 112026.

Harlow, W.V., & Rao, R.K.S. (1989). Asset pricing in a generalized mean-lower partial moment framework: theory and evidence. *Journal of Financial and Quantitative Analysis*, 24, 285–311.

Jacobs, B.I., & Levy, K.N. (2013). Leverage aversion, efficient frontiers, and the efficient region. *Journal of Portfolio Management*, 39, 54–64.

Kahneman, D., & Tversky, A. (1979). Prospect theory: An analysis of decision under risk. *Econometrica*, 47, 263–291.

Kassimatis, K. (2021). Mean-variance versus utility maximization revisited: The case of constant relative risk aversion. *International Review of Financial Analysis*, 78, 101932. https://doi.org/10.1016/j.irfa.2021.101932

Klebaner, F., Landsman, Z., Makov, U., & Yao, J. (2017). Optimal portfolios with downside risk. *Quantitative Finance*, 17(3), 315–325.

Kolm, P.N., Tütüncü, R., & Fabozzi, F.J. (2014). 60 Years of Portfolio Optimization: Practical Challenges and Current Trends. *European Journal of Operational Research*, 234, 356–371.

Lo, A.W., Petrov, C., & Wierzbicki, M. (2003). Its 11pm—Do You Know Where Your Liquidity Is? The Mean-Variance-Liquidity Frontier. *Journal of Investment Management*, 1, 55–93.

Markowitz, H. (1952). Portfolio Selection. *Journal of Finance*, 7(1), 77-91. https://dx.doi.org/10.1111/j.1540-6261.1952.tb01525.x

Markowitz, H. (1959). *Portfolio selection: Efficient diversification of investments*. Wiley.

Markowitz, H. M., Starer, D., Fram, H., & Gerber, S. (2020). Avoiding the downside: A practical review of the critical line algorithm for mean–semivariance portfolio optimization. Handbook of applied investment research, 369-415.

Markowski, L. (2024). Conventional and downside CAPM with higher-order moments: Evidence from emerging markets. *Equilibrium : Quarterly Journal of Economics and Economic Policy*, 19(1), 93-138. https://doi.org/10.24136/eq.2043

Maier-Paape, S., Platen, A., & Zhu, Q.J. (2019). A General Framework for Portfolio Theory. Part III: Multi-Period Markets and Modular Approach. *Risks*, 7, 60. https://doi.org/10.3390/risks7020060.

Nakajima, J. (2020). Skew selection for factor stochastic volatility models. *Journal of Applied Statistics*, *47*(4), 582–601. https://doi.org/10.1080/02664763.2019.1646227

Nawrocki, D., & Viole, F. (2024). Estimation error and partial moments. *International Review of Financial Analysis*, *95*, .https://doi.org/10.1016/j.irfa.2024.103443.

Peiro, A. (1999). Skewness in financial returns. *Journal of Banking and Finance*, 23, 847–862.

Pla-Santamaria, D., & Bravo, M. (2013). Portfolio optimization based on downside risk: A mean-semivariance efficient frontier from Dow Jones blue chips. *Annals of Operations Research*, 205(1), 189–201. https://doi.org/10.1007/s10479-012-1243-x

Post, T., & van Viet, P. (2006). Downside risk and asset pricing. *Journal of Banking and Finance*, 30, 823–849.

Rosenberg, B., Reid, K., & Lanstein, R. (1985). Persuasive evidence of market inefficiency. *Journal of Portfolio Management*, 11, 9–16.

Rutkowska-Miczka, A., Kliber, P., & Szydlowski, K. (2024). Portfolio Choice Using Additional Information from Financial Statements-Evidence from the Frankfurt Stock Exchange. *Engineering Economics*, *35*(5), 507-520.

Rutkowska-Ziarko, A., & Garsztka, P. (2014). Diversification of risk of a fundamental portfolio based on semi-variance. Poznań University of Economics Review, 14, 80–96.

Rutkowska-Ziarko, A. (2013). Fundamental Portfolio Construction Based on Semi-Variance. *Olsztyn Economic Journal*, 8(2), 151–162.

Sharpe, W.F. (1963). A simplified model of portfolio analysis. *Management Science*, 9(2), 277–293. https://doi.org/10.1287/mnsc.9.2.277.

Sloan, R.G. (1996). Do stock prices fully reflect information in accruals and cash flows about future earnings? *The Accounting Review*, 71 (3), 289–315.

Sun, Q., & Yan, Y. (2003). Skewness persistence with optimal portfolio selection. *Journal of Banking and Finance*, 27, 1111-1121.

Tarczyńska-Luniewska, M. (2022). Assessment of portfolio investment effectiveness during the covid-19 pandemic - multivariate fundamental approach. *Procedia Computer Science*, *207*, 4348–4358. https://doi.org/10.1016/j.procs.2022.09.498

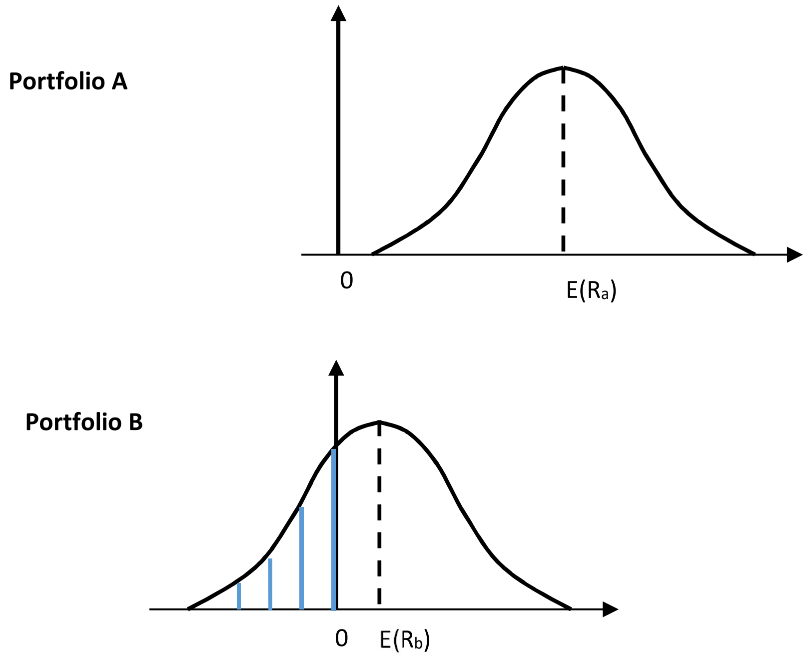
Tarczyński, W., & Tarczyńska-Łuniewska, M. (2018). The construction of fundamental portfolio with the use of multivariate approach. *Procedia Computer Science*, 126, 2085–2096. https://doi.org/10.1016/j.procS.2018.07.243.

Wu, Q., Liu, X., Qin, J., Zhou, L., Mardani, A., & Deveci, M. (2022). An integrated multi-criteria decision making and multi-objective optimization model for socially responsible portfolio selection. *Technological Forecasting and Social Change*, 184(August), 121977. https://doi.org/10.1016/j.techfore.2022.121977.

Vafaeikhah, M., Najafi, A. A., & Rezaei, F. (2025). A Multi-Objective Robust Optimization Model for the Portfolio Rebalancing Problem with Constant Proportion Portfolio Insurance Strategy: Evidence from the US Stock Market: *Computational Economics*, 1-42.

Xidonas, P., Mavrotas, G., & Psarras, J. (2010). Equity Portfolio Construction and Selection Using Multiobjective Mathematical Programming. *Journal of Global Optimization*, 47, 185–209.

Figure 1: Generic Return Distributions



Source: Authors' own elaboration

Figure 2. Iterative Portfolio Optimization Algorithm

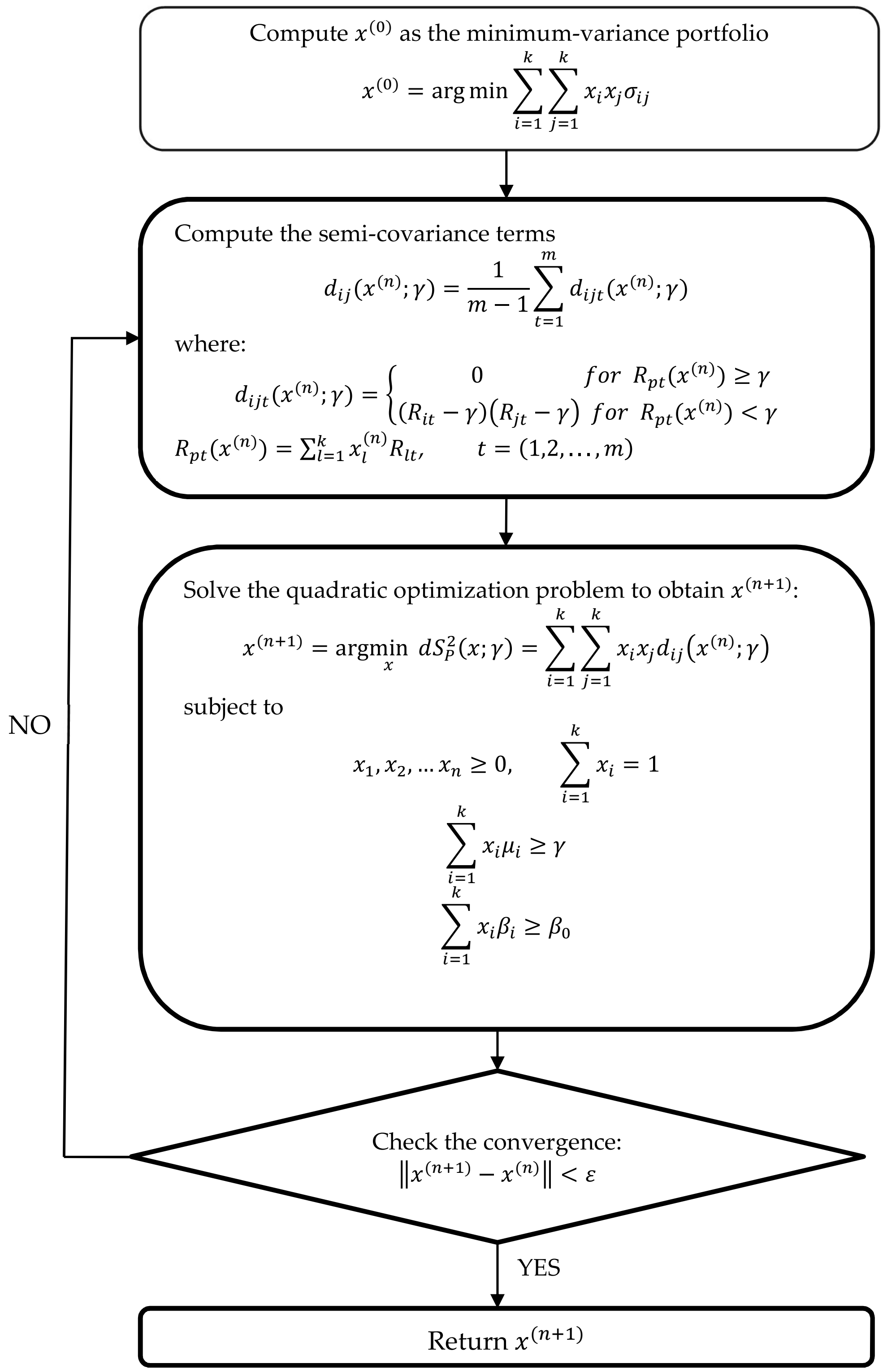


Table 1. Portfolio types considered in the study

Portfolio	Description
EW	A portfolio with an equal share of each asset
V	A portfolio that minimizes unconditional variance
V-E	A variance-minimizing portfolio subject to the requirement for mean return
V-E-EP	A variance-minimizing portfolio subject to requirements for mean return and EP
V-E-BVP	A variance-minimizing portfolio subject to requirements for mean return and BVP
V-E-SP	A variance-minimizing portfolio subject to requirements for mean return and SP
V-E-DY	A variance-minimizing portfolio subject to requirements for mean return and DY
V-E-CFP	A variance-minimizing portfolio subject to requirements for mean return and CFP
SV	A portfolio that minimizes unconditional semi-variance
SV-E	A semi-variance-minimizing portfolio subject to the requirement for mean return
SV-E-EP	A semi-variance-minimizing portfolio subject to requirements for mean return and EP
SV-E-BVP	A semi-variance-minimizing portfolio subject to requirements for mean return and BVP
SV-E-SP	A semi-variance-minimizing portfolio subject to requirements for mean return and SP
SV-E-DY	A semi-variance-minimizing portfolio subject to requirements for mean return and DY
SV-E-CFP	A semi-variance-minimizing portfolio subject to requirements for mean return and CFP

Table 2. Summary statistics for one-month portfolio returns

	Mean (%)	Median (%)	Std. (%)	Min (%)	Max (%)	S-Dev (%)	Skewness	Sharpe	Sortino
EW	0.93	1.22	4.13	-27.31	26.3	2.75	-0.98	0.23	0.34
V	0.25	0.25	3.95	-26.76	20.1	2.79	-0.75	0.06	0.09
V-E	0.29	0.30	3.93	-26.76	20.1	2.76	-0.77	0.07	0.10
V-E-EP	0.34	0.41	3.93	-26.76	20.1	2.76	-0.79	0.09	0.12
V-E-BVP	0.30	0.31	3.93	-26.68	20.1	2.76	-0.76	0.08	0.11
V-E-SP	0.27	0.29	3.89	-26.76	20.2	2.75	-0.85	0.07	0.10
V-E-DY	0.29	0.28	3.93	-26.76	20.0	2.76	-0.77	0.07	0.10
V-E-CFP	0.40	0.40	3.97	-26.76	20.1	2.73	-0.73	0.10	0.15
SV	0.44	0.61	4.09	-23.25	19.1	2.73	-0.29	0.11	0.16
SV-E	0.44	0.61	4.09	-23.25	19.1	2.73	-0.29	0.11	0.16
SV-E-EP	0.50	0.73	4.08	-23.23	19.1	2.72	-0.34	0.12	0.18
SV-E-BVP	0.46	0.57	4.06	-23.11	19.1	2.71	-0.31	0.11	0.17
SV-E-SP	0.40	0.59	3.99	-23.56	20.2	2.68	-0.33	0.10	0.15
SV-E-DY	0.39	0.51	3.96	-23.23	18.7	2.66	-0.33	0.10	0.14
SV-E-CFP	0.56	0.74	4.05	-23.23	19.0	2.66	-0.38	0.14	0.21

This table presents the mean, median, standard deviation, minimum, maximum, and skewness of realized returns over a one-month holding period. S-Dev denotes semi-deviation. The Sharpe ratio is calculated as the mean return divided by the standard deviation, while the Sortino ratio is the mean return divided by the semi-deviation. The abbreviations in the first column refer to different types of optimal portfolios, including one-, two-, and three-criteria portfolios. The one-criterion portfolios are minimum-risk portfolios constructed with respect to Variance (V) and Semi-variance (SV). The two-criterion portfolios, in addition to risk minimization, ensure the achievement of a predefined expected rate of return (E). The three-criterion portfolios include an additional objective: attaining a specified level of selected fundamental indicators (EP, BVP, SP, DY, CFP) for the entire portfolio, which correspond respectively to earnings-to-price, book value-to-price, sales-to-price, dividend yield, and cash flow-to-price ratios. A detailed description of all portfolio types used is presented in Table 1.

Table 3. Differences (row minus column) in median realized rates of return and Kruskal-Wallis test (p-value) for the one-month portfolios

	EW	V	V-E	V-E-EP	V-E-BVP	V-E-SP	V-E-DY	V-E-CFP	SV	SV-E	SV-E-EP	SV-E-BVP	SV-E-SP	SV-E-DY
V	**-0.0097**	-	-	-	-	-	-	-	-	-	-	-	-	-
	(0.0000)	-	-	-	-	-	-	-	-	-	-	-	-	-
V-E	**-0.0092**	0.0004	-	-	-	-	-	-	-	-	-	-	-	-
	(0.0000)	(0.7981)	-	-	-	-	-	-	-	-	-	-	-	-
V-E-EP	**-0.0081**	0.0016	0.0011	-	-	-	-	-	-	-	-	-	-	-
	(0.0000)	(0.4431)	(0.6023)	-	-	-	-	-	-	-	-	-	-	-
V-E-BVP	**-0.0090**	0.0006	0.0002	-0.0009	-	-	-	-	-	-	-	-	-	-
	(0.0000)	(0.7141)	(0.9161)	(0.6840)	-	-	-	-	-	-	-	-	-	-
V-E-SP	**-0.0093**	0.0004	0.0000	-0.0012	-0.0002	-	-	-	-	-	-	-	-	-
	(0.0000)	(0.8302)	(0.9666)	(0.5547)	(0.8795)	-	-	-	-	-	-	-	-	-
V-E-DY	**-0.0094**	0.0003	-0.0002	-0.0013	-0.0003	-0.0001	-	-	-	-	-	-	-	-
	(0.0000)	(0.7908)	(0.9957)	(0.6080)	(0.9235)	(0.9614)	-	-	-	-	-	-	-	-
V-E-CFP	**-0.0082**	0.0015	0.0011	-0.0001	0.0009	0.0011	0.0012	-	-	-	-	-	-	-
	(0.0000)	(0.2883)	(0.4146)	(0.7646)	(0.4804)	(0.3823)	(0.4170)	-	-	-	-	-	-	-
SV	**-0.0061**	0.0036	0.0032	0.0020	0.0030	0.0032	0.0033	0.0021	-	-	-	-	-	-
	(0.0000)	(0.1843)	(0.2799)	(0.5372)	(0.3282)	(0.2520)	(0.2854)	(0.7709)	-	-	-	-	-	-
SV-E	**-0.0061**	0.0036	0.0031	0.0020	0.0030	0.0032	0.0033	0.0021	0.0000	-	-	-	-	-
	(0.0000)	(0.1852)	(0.2804)	(0.5383)	(0.3293)	(0.2525)	(0.2861)	(0.7730)	(0.9974)	-	-	-	-	-
SV-E-EP	**-0.0049**	**0.0048**	**0.0043**	0.0032	0.0041	**0.0043**	**0.0045**	0.0033	0.0012	0.0012	-	-	-	-
	(0.0001)	(0.0524)	(0.0893)	(0.2170)	(0.1117)	(0.0794)	(0.0919)	(0.3659)	(0.5744)	(0.5727)	-	-	-	-
SV-E-BVP	**-0.0065**	0.0031	0.0027	0.0016	0.0025	0.0027	0.0028	0.0016	-0.0005	-0.0005	-0.0016	-	-	-
	(0.0000)	(0.1418)	(0.2191)	(0.4405)	(0.2622)	(0.2016)	(0.2233)	(0.6593)	(0.8765)	(0.8756)	(0.6814)	-	-	-
SV-E-SP	**-0.0063**	0.0034	0.0029	0.0018	0.0027	0.0030	0.0031	0.0019	-0.0002	-0.0002	-0.0014	0.0002	-	-
	(0.0000)	(0.3301)	(0.4657)	(0.8219)	(0.5345)	(0.4356)	(0.4723)	(0.9179)	(0.6697)	(0.6718)	(0.3172)	(0.5757)	-	-
SV-E-DY	**-0.0071**	0.0026	0.0022	0.0010	0.0020	0.0022	0.0023	0.0011	-0.0010	-0.0010	-0.0021	-0.0005	-0.0008	-
	(0.0000)	(0.3785)	(0.5238)	(0.8760)	(0.5929)	(0.4840)	(0.5298)	(0.8699)	(0.6587)	(0.6594)	(0.3056)	(0.5511)	(0.9907)	-
SV-E-CFP	**-0.0048**	**0.0049**	**0.0044**	**0.0033**	**0.0042**	**0.0045**	**0.0046**	0.0034	0.0013	0.0013	0.0001	0.0017	0.0015	0.0023
	(0.0005)	(0.0184)	(0.0335)	(0.0950)	(0.0434)	(0.0282)	(0.0347)	(0.1772)	(0.3136)	(0.3119)	(0.6449)	(0.3922)	(0.1540)	(0.1375)

This table reports the pairwise differences in median realized returns for the one-month portfolios. Each entry represents the difference between the median return of the portfolio listed in the row and that of the portfolio listed in the column. The p-values from the Kruskal–Wallis tests, which assess the equality of medians, are shown in parentheses. Bold values denote statistically significant differences. The abbreviations in the first column refer to different types of optimal portfolios, including one-, two-, and three-criteria portfolios. The one-criterion portfolios are minimum-risk portfolios constructed with respect to Variance (V) and Semi-variance (SV). The two-criterion portfolios, in addition to risk minimization, ensure the achievement of a predefined expected rate of return (E). The three-criterion portfolios include an additional objective: attaining a specified level of selected fundamental indicators (EP, BVP, SP, DY, CFP) for the entire portfolio, which correspond respectively to earnings-to-price, book value-to-price, sales-to-price, dividend yield, and cash flow-to-price ratios. A detailed description of all portfolio types used is presented in Table 1.

Table 4. Summary statistics for three-month portfolio returns

	Mean (%)	Median (%)	Std. (%)	Min (%)	Max (%)	S-Dev (%)	Skewness	Sharpe	Sortino
EW	2.99	3.19	6.11	-27.89	35.1	3.25	-0.41	0.49	0.92
V	2.41	2.24	5.74	-30.40	19.1	3.08	-0.55	0.42	0.78
V-E	2.42	2.24	5.73	-30.40	19.1	3.07	-0.56	0.42	0.79
V-E-EP	2.53	2.42	5.69	-30.40	19.1	3.03	-0.61	0.45	0.84
V-E-BVP	2.61	2.39	5.72	-30.40	19.1	3.02	-0.59	0.46	0.86
V-E-SP	2.51	2.34	5.78	-30.40	21.3	3.06	-0.53	0.43	0.82
V-E-DY	2.30	2.08	5.71	-30.40	18.2	3.08	-0.54	0.40	0.75
V-E-CFP	2.71	2.83	5.89	-30.40	19.1	3.09	-0.59	0.46	0.88
SV	4.18	4.70	6.61	-25.47	21.3	2.99	-0.43	0.63	1.40
SV-E	4.18	4.70	6.61	-25.47	21.3	2.99	-0.43	0.63	1.40
SV-E-EP	3.89	4.13	6.07	-25.64	22.6	2.71	-0.40	0.64	1.44
SV-E-BVP	3.91	4.12	6.13	-25.64	22.6	2.75	-0.40	0.64	1.42
SV-E-SP	3.88	4.07	6.06	-25.64	22.6	2.70	-0.40	0.64	1.44
SV-E-DY	3.25	3.45	5.50	-25.64	19.2	2.61	-0.51	0.59	1.25
SV-E-CFP	3.90	4.04	6.06	-25.64	22.6	2.69	-0.39	0.64	1.45

This table presents the mean, median, standard deviation, minimum, maximum, and skewness of realized returns over a three-month holding period. S-Dev denotes semi-deviation. The Sharpe ratio is calculated as the mean return divided by the standard deviation, while the Sortino ratio is the mean return divided by the semi-deviation. The abbreviations in the first column refer to different types of optimal portfolios, including one-, two-, and three-criteria portfolios. The one-criterion portfolios are minimum-risk portfolios constructed with respect to Variance (V) and Semi-variance (SV). The two-criterion portfolios, in addition to risk minimization, ensure the achievement of a predefined expected rate of return (E). The three-criterion portfolios include an additional objective: attaining a specified level of selected fundamental indicators (EP, BVP, SP, DY, CFP) for the entire portfolio, which correspond respectively to earnings-to-price, book value-to-price, sales-to-price, dividend yield, and cash flow-to-price ratios. A detailed description of all portfolio types used is presented in Table 1.

Table 5. Differences (row minus column) in median realized rates of return and Kruskal-Wallis test (p-value) for the three-month portfolios

	EW	V	V-E	V-E-EP	V-E-BVP	V-E-SP	V-E-DY	V-E-CFP	SV	SV-E	SV-E-EP	SV-E-BVP	SV-E-SP	SV-E-DY
V	**-0.0095**	-	-	-	-	-	-	-	-	-	-	-	-	-
	(0.0002)	-	-	-	-	-	-	-	-	-	-	-	-	-
V-E	**-0.0095**	0.0000	-	-	-	-	-	-	-	-	-	-	-	-
	(0.0003)	(0.9416)	-	-	-	-	-	-	-	-	-	-	-	-
V-E-EP	**-0.0077**	0.0018	0.0018	-	-	-	-	-	-	-	-	-	-	-
	(0.0026)	(0.4688)	(0.5165)	-	-	-	-	-	-	-	-	-	-	-
V-E-BVP	**-0.0081**	0.0015	0.0015	-0.0003	-	-	-	-	-	-	-	-	-	-
	(0.0069)	(0.2695)	(0.3038)	(0.7127)	-	-	-	-	-	-	-	-	-	-
V-E-SP	**-0.0086**	0.0010	0.0010	-0.0008	-0.0005	-	-	-	-	-	-	-	-	-
	(0.0012)	(0.6571)	(0.7129)	(0.7889)	(0.5241)	-	-	-	-	-	-	-	-	-
V-E-DY	**-0.0111**	-0.0016	-0.0016	-0.0034	-0.0030	-0.0025	-	-	-	-	-	-	-	-
	(0.0000)	(0.5596)	(0.5122)	(0.1966)	(0.0905)	(0.3080)	-	-	-	-	-	-	-	-
V-E-CFP	**-0.0037**	0.0059	0.0059	0.0041	0.0044	0.0049	**0.0074**	-	-	-	-	-	-	-
	(0.0510)	(0.1151)	(0.1332)	(0.3624)	(0.5980)	(0.2588)	(0.0322)	-	-	-	-	-	-	-
SV	**0.0151**	**0.0246**	**0.0246**	**0.0228**	**0.0231**	**0.0236**	**0.0262**	**0.0187**	-	-	-	-	-	-
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	-	-	-	-	-	-
SV-E	**0.0151**	**0.0246**	**0.0246**	**0.0228**	**0.0231**	**0.0236**	**0.0262**	**0.0187**	0.0000	-	-	-	-	-
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(1.0000)	-	-	-	-	-
SV-E-EP	**0.0094**	**0.0189**	**0.0189**	**0.0171**	**0.0175**	**0.0180**	**0.0205**	**0.0131**	-0.0057	-0.0057	-	-	-	-
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.1247)	(0.1247)	-	-	-	-
SV-E-BVP	**0.0092**	**0.0188**	**0.0188**	**0.0170**	**0.0173**	**0.0178**	**0.0203**	**0.0129**	-0.0058	-0.0058	-0.0002	-	-	-
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.1482)	(0.1482)	(0.9278)	-	-	-
SV-E-SP	**0.0088**	**0.0183**	**0.0183**	**0.0165**	**0.0168**	**0.0173**	**0.0199**	**0.0124**	-0.0063	-0.0063	-0.0006	-0.0005	-	-
	(0.0001)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.1080)	(0.1080)	(0.9432)	(0.8703)	-	-
SV-E-DY	0.0026	**0.0121**	**0.0121**	**0.0103**	**0.0106**	**0.0111**	**0.0137**	**0.0062**	**-0.0125**	**-0.0125**	**-0.0068**	**-0.0067**	**-0.0062**	-
	(0.3150)	(0.0000)	(0.0000)	(0.0001)	(0.0004)	(0.0000)	(0.0000)	(0.0055)	(0.0000)	(0.0000)	(0.0015)	(0.0011)	(0.0020)	-
SV-E-CFP	**0.0084**	**0.0179**	**0.0179**	**0.0161**	**0.0165**	**0.0170**	**0.0195**	**0.0121**	-0.0066	-0.0066	-0.0010	-0.0008	-0.0004	**0.0058**
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.1258)	(0.1258)	(0.9976)	(0.9274)	(0.9416)	(0.0015)

This table reports the pairwise differences in median realized returns for the three-month portfolios. Each entry represents the difference between the median return of the portfolio listed in the row and that of the portfolio listed in the column. The p-values from the Kruskal–Wallis tests, which assess the equality of medians, are shown in parentheses. Bold values denote statistically significant differences. The abbreviations in the first column refer to different types of optimal portfolios, including one-, two-, and three-criteria portfolios. The one-criterion portfolios are minimum-risk portfolios constructed with respect to Variance (V) and Semi-variance (SV). The two-criterion portfolios, in addition to risk minimization, ensure the achievement of a predefined expected rate of return (E). The three-criterion portfolios include an additional objective: attaining a specified level of selected fundamental indicators (EP, BVP, SP, DY, CFP) for the entire portfolio, which correspond respectively to earnings-to-price, book value-to-price, sales-to-price, dividend yield, and cash flow-to-price ratios. A detailed description of all portfolio types used is presented in Table 1.

Table 6. Summary statistics for six-month portfolio returns

	Mean (%)	Median (%)	Std. (%)	Min (%)	Max (%)	S-Dev (%)	Skewness	Sharpe	Sortino
EW	6.03	6.53	7.69	-22.76	42.5	2.94	-0.03	0.78	2.05
V	5.29	5.43	6.00	-25.94	31.5	2.33	-0.27	0.88	2.27
V-E	5.36	5.56	5.98	-25.94	31.5	2.33	-0.30	0.90	2.30
V-E-EP	5.58	6.04	6.03	-25.94	31.5	2.36	-0.40	0.93	2.36
V-E-BVP	5.44	5.80	5.94	-25.94	31.5	2.30	-0.34	0.92	2.37
V-E-SP	5.47	5.78	5.97	-25.94	32.5	2.31	-0.28	0.92	2.37
V-E-DY	5.20	5.47	5.91	-25.94	30.3	2.33	-0.35	0.88	2.23
V-E-CFP	5.59	6.15	5.90	-25.94	31.5	2.32	-0.41	0.95	2.40
SV	12.91	12.53	10.82	-15.62	43.4	2.03	0.09	1.19	6.35
SV-E	12.91	12.53	10.82	-15.62	43.4	2.03	0.09	1.19	6.35
SV-E-EP	12.88	12.54	10.63	-15.62	43.4	1.96	0.11	1.21	6.55
SV-E-BVP	12.59	12.33	10.77	-15.62	43.4	2.01	0.11	1.17	6.26
SV-E-SP	12.35	12.26	10.87	-16.18	43.4	2.17	0.10	1.14	5.68
SV-E-DY	8.92	9.70	7.61	-12.82	33.7	1.85	-0.02	1.17	4.81
SV-E-CFP	12.68	12.30	10.74	-15.62	43.4	2.08	0.11	1.18	6.09

This table presents the mean, median, standard deviation, minimum, maximum, and skewness of realized returns over a six-month holding period. S-Dev denotes semi-deviation. The Sharpe ratio is calculated as the mean return divided by the standard deviation, while the Sortino ratio is the mean return divided by the semi-deviation. The abbreviations in the first column refer to different types of optimal portfolios, including one-, two-, and three-criteria portfolios. The one-criterion portfolios are minimum-risk portfolios constructed with respect to Variance (V) and Semi-variance (SV). The two-criterion portfolios, in addition to risk minimization, ensure the achievement of a predefined expected rate of return (E). The three-criterion portfolios include an additional objective: attaining a specified level of selected fundamental indicators (EP, BVP, SP, DY, CFP) for the entire portfolio, which correspond respectively to earnings-to-price, book value-to-price, sales-to-price, dividend yield, and cash flow-to-price ratios. A detailed description of all portfolio types used is presented in Table 1.

Table 7. Differences (row minus column) in median realized rates of return and Kruskal-Wallis test (p-value) for the six-month portfolios

	EW	V	V-E	V-E-EP	V-E-BVP	V-E-SP	V-E-DY	V-E-CFP	SV	SV-E	SV-E-EP	SV-E-BVP	SV-E-SP	SV-E-DY
V	**-0.0111**	-	-	-	-	-	-	-	-	-	-	-	-	-
	(0.0002)	-	-	-	-	-	-	-	-	-	-	-	-	-
V-E	**-0.0097**	0.0013	-	-	-	-	-	-	-	-	-	-	-	-
	(0.0006)	(0.6667)	-	-	-	-	-	-	-	-	-	-	-	-
V-E-EP	**-0.0049**	**0.0062**	0.0048	-	-	-	-	-	-	-	-	-	-	-
	(0.0228)	(0.0565)	(0.1328)	-	-	-	-	-	-	-	-	-	-	-
V-E-BVP	**-0.0074**	0.0037	0.0024	-0.0024	-	-	-	-	-	-	-	-	-	-
	(0.0023)	(0.3268)	(0.5735)	(0.3379)	-	-	-	-	-	-	-	-	-	-
V-E-SP	**-0.0075**	0.0036	0.0022	-0.0026	-0.0001	-	-	-	-	-	-	-	-	-
	(0.0027)	(0.3200)	(0.5708)	(0.3299)	(0.9995)	-	-	-	-	-	-	-	-	-
V-E-DY	**-0.0106**	0.0005	-0.0009	**-0.0057**	-0.0032	-0.0031	-	-	-	-	-	-	-	-
	(0.0000)	(0.7444)	(0.4515)	**(0.0239)**	(0.1898)	(0.1832)	-	-	-	-	-	-	-	-
V-E-CFP	**-0.0039**	**0.0072**	0.0059	0.0011	0.0035	0.0036	**0.0067**	-	-	-	-	-	-	-
	(0.0212)	(0.0495)	(0.1193)	(0.9902)	(0.3225)	(0.3158)	(0.0200)	-	-	-	-	-	-	-
SV	**0.0600**	**0.0710**	**0.0697**	**0.0649**	**0.0673**	**0.0675**	**0.0706**	**0.0638**	-	-	-	-	-	-
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	-	-	-	-	-	-
SV-E	**0.0600**	**0.0710**	**0.0697**	**0.0649**	**0.0673**	**0.0675**	**0.0706**	**0.0638**	0.0000	-	-	-	-	-
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(1.0000)	-	-	-	-	-
SV-E-EP	**0.0601**	**0.0712**	**0.0698**	**0.0650**	**0.0674**	**0.0676**	**0.0707**	**0.0639**	0.0001	0.0001	-	-	-	-
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.9437)	(0.9437)	-	-	-	-
SV-E-BVP	**0.0579**	**0.0690**	**0.0677**	**0.0628**	**0.0653**	**0.0654**	**0.0685**	**0.0618**	-0.0020	-0.0020	-0.0022	-	-	-
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.4070)	(0.4070)	(0.4509)	-	-	-
SV-E-SP	**0.0572**	**0.0683**	**0.0670**	**0.0621**	**0.0646**	**0.0647**	**0.0678**	**0.0611**	-0.0027	-0.0027	-0.0029	-0.0007	-	-
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.1675)	(0.1675)	(0.1842)	(0.5678)	-	-
SV-E-DY	**0.0316**	**0.0427**	**0.0414**	**0.0366**	**0.0390**	**0.0391**	**0.0422**	**0.0355**	**-0.0283**	**-0.0283**	**-0.0284**	**-0.0263**	**-0.0256**	-
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	-
SV-E-CFP	**0.0577**	**0.0688**	**0.0674**	**0.0626**	**0.0651**	**0.0652**	**0.0683**	**0.0616**	-0.0023	-0.0023	-0.0024	-0.0002	0.0005	**0.0261**
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.4851)	(0.4851)	(0.5277)	(0.9056)	(0.4512)	(0.0000)

This table reports the pairwise differences in median realized returns for the six-month portfolios. Each entry represents the difference between the median return of the portfolio listed in the row and that of the portfolio listed in the column. The p-values from the Kruskal–Wallis tests, which assess the equality of medians, are shown in parentheses. Bold values denote statistically significant differences. The abbreviations in the first column refer to different types of optimal portfolios, including one-, two-, and three-criteria portfolios. The one-criterion portfolios are minimum-risk portfolios constructed with respect to Variance (V) and Semi-variance (SV). The two-criterion portfolios, in addition to risk minimization, ensure the achievement of a predefined expected rate of return (E). The three-criterion portfolios include an additional objective: attaining a specified level of selected fundamental indicators (EP, BVP, SP, DY, CFP) for the entire portfolio, which correspond respectively to earnings-to-price, book value-to-price, sales-to-price, dividend yield, and cash flow-to-price ratios. A detailed description of all portfolio types used is presented in Table 1.

Table 8. Alphas over one-, three-, and six-month holding periods

	One month		Three months		Six months	
	Alpha (%)	t-stat	Alpha (%)	t-stat	Alpha (%)	t-stat
EW	0.09	0.79	0.33	1.66	0.90	3.99
V	-0.44	-2.12	0.34	0.84	2.34	3.98
V-E	-0.42	-2.01	0.35	0.86	2.37	4.04
V-E-EP	-0.38	-1.89	0.44	1.11	2.25	4.03
V-E-BVP	-0.42	-2.05	0.48	1.20	2.35	4.08
V-E-SP	-0.43	-2.16	0.33	0.83	2.51	4.29
V-E-DY	-0.42	-2.00	0.25	0.62	2.29	3.92
V-E-CFP	-0.36	-1.81	0.56	1.39	2.35	4.30
SV	-0.30	-1.49	1.71	4.17	7.44	9.69
SV-E	-0.29	-1.49	1.71	4.17	7.44	9.69
SV-E-EP	-0.25	-1.25	1.26	3.92	7.55	9.94
SV-E-BVP	-0.30	-1.54	1.23	3.83	7.11	9.26
SV-E-SP	-0.34	-1.86	1.24	3.90	6.76	9.08
SV-E-DY	-0.32	-1.65	1.07	3.06	5.44	10.29
SV-E-CFP	-0.21	-1.17	1.27	3.98	7.25	9.61

This table reports the alphas from the Fama–French–Carhart six-factor model for optimal portfolios with single or multiple constraints over one-, three-, and six-month holding periods. The corresponding Newey and West (1987) t-statistics are adjusted for heteroskedasticity and autocorrelation. The abbreviations in the first column refer to different types of optimal portfolios, including one-, two-, and three-criteria portfolios. The one-criterion portfolios are minimum-risk portfolios constructed with respect to Variance (V) and Semi-variance (SV). The two-criterion portfolios, in addition to risk minimization, ensure the achievement of a predefined expected rate of return (E). The three-criterion portfolios include an additional objective: attaining a specified level of selected fundamental indicators (EP, BVP, SP, DY, CFP) for the entire portfolio, which correspond respectively to earnings-to-price, book value-to-price, sales-to-price, dividend yield, and cash flow-to-price ratios. A detailed description of all portfolio types used is presented in Table 1.

Table 9. One-month alphas on long-short portfolios

	EW	V	V-E	V-E-EP	V-E-BVP	V-E-SP	V-E-DY	V-E-CFP	SV	SV-E	SV-E-EP	SV-E-BVP	SV-E-SP	SV-E-DY
V	-0.0053	-	-	-	-	-	-	-	-	-	-	-	-	-
	(-3.86)													
V-E	-0.0051	0.0003	-	-	-	-	-	-	-	-	-	-	-	-
	(-3.66)	(0.88)												
V-E-EP	-0.0048	0.0006	0.0003	-	-	-	-	-	-	-	-	-	-	-
	(-3.66)	(1.15)	(0.76)											
V-E-BVP	-0.0052	0.0002	-0.0001	-0.0004	-	-	-	-	-	-	-	-	-	-
	(-3.78)	(0.62)	(-0.30)	(-0.80)										
V-E-SP	-0.0052	0.0001	-0.0001	-0.0004	-0.0001	-	-	-	-	-	-	-	-	-
	(-4.06)	(0.32)	(-0.40)	(-0.93)	(-0.16)									
V-E-DY	-0.0051	0.0003	0.0000	-0.0003	0.0001	0.0001	-	-	-	-	-	-	-	-
	(-3.66)	(0.94)	(0.14)	(-0.74)	(0.33)	(0.41)								
V-E-CFP	-0.0045	0.0008	0.0006	0.0002	0.0006	0.0007	0.0006	-	-	-	-	-	-	-
	(-3.78)	(1.30)	(0.99)	(0.55)	(1.03)	(1.47)	(0.97)							
SV	-0.0039	0.0015	0.0012	0.0009	0.0013	0.0013	0.0012	0.0006	-	-	-	-	-	-
	(-2.85)	(1.42)	(1.19)	(0.92)	(1.32)	(1.25)	(1.17)	(0.61)						
SV-E	-0.0039	0.0015	0.0012	0.0009	0.0013	0.0013	0.0012	0.0007	0.0000	-	-	-	-	-
	(-2.84)	(1.43)	(1.20)	(0.93)	(1.33)	(1.26)	(1.18)	(0.61)	(0.26)					
SV-E-EP	-0.0034	0.0019	0.0017	0.0014	0.0017	0.0018	0.0017	0.0011	0.0005	0.0005	-	-	-	-
	(-2.54)	(1.77)	(1.56)	(1.46)	(1.67)	(1.62)	(1.54)	(1.06)	(1.54)	(1.53)				
SV-E-BVP	-0.0039	0.0014	0.0012	0.0009	0.0012	0.0013	0.0012	0.0006	0.0000	0.0000	-0.0005	-	-	-
	(-2.95)	(1.36)	(1.13)	(0.86)	(1.28)	(1.19)	(1.11)	(0.56)	(-0.15)	(-0.17)	(-1.27)			
SV-E-SP	-0.0043	0.0010	0.0008	0.0005	0.0009	0.0009	0.0008	0.0002	-0.0004	-0.0004	-0.0009	-0.0004	-	-
	(-3.63)	(1.00)	(0.78)	(0.49)	(0.87)	(0.95)	(0.76)	(0.24)	(-0.93)	(-0.95)	(-1.74)	(-0.83)		
SV-E-DY	-0.0042	0.0012	0.0009	0.0006	0.0010	0.0010	0.0009	0.0004	-0.0003	-0.0003	-0.0008	-0.0003	0.0001	-
	(-3.29)	(1.36)	(1.11)	(0.78)	(1.23)	(1.16)	(1.09)	(0.41)	(-0.67)	(-0.68)	(-1.60)	(-0.56)	(0.22)	
SV-E-CFP	-0.0031	0.0023	0.0020	0.0017	0.0021	0.0021	0.0020	0.0015	0.0008	0.0008	0.0003	0.0008	0.0012	0.0011
	(-2.67)	(1.97)	(1.77)	(1.68)	(1.91)	(1.94)	(1.75)	(1.53)	(1.43)	(1.43)	(0.62)	(1.53)	(2.54)	(1.72)

The abbreviations in the first column refer to different types of optimal portfolios, including one-, two-, and three-criteria portfolios. The one-criterion portfolios are minimum-risk portfolios constructed with respect to Variance (V) and Semi-variance (SV). The two-criterion portfolios, in addition to risk minimization, ensure the achievement of a predefined expected rate of return (E). The three-criterion portfolios include an additional objective. The numbers in the main body of the table represent the one-month alphas from Fama-French-Carhart six-factor models on the long-short (row minus column) portfolio that goes long the row portfolio and shorts the column portfolio. The numbers in parentheses are the Newey and West (1987) t-statistics that are adjusted for heteroscedasticity and autocorrelation.

Table 10. Three-month alphas on long-short portfolios

	EW	V	V-E	V-E-EP	V-E-BVP	V-E-SP	V-E-DY	V-E-CFP	SV	SV-E	SV-E-EP	SV-E-BVP	SV-E-SP	SV-E-DY
V	0.0001	-	-	-	-	-	-	-	-	-	-	-	-	-
	(0.04)													
V-E	0.0002	0.0001	-	-	-	-	-	-	-	-	-	-	-	-
	(0.07)	(2.21)												
V-E-EP	0.0012	0.0010	0.0009	-	-	-	-	-	-	-	-	-	-	-
	(0.39)	(2.07)	(1.87)											
V-E-BVP	0.0015	0.0014	0.0013	0.0003	-	-	-	-	-	-	-	-	-	-
	(0.51)	(3.98)	(3.66)	(0.57)										
V-E-SP	0.0000	-0.0001	-0.0002	-0.0011	-0.0015	-	-	-	-	-	-	-	-	-
	(0.01)	(-0.26)	(-0.52)	(-1.95)	(-3.21)									
V-E-DY	-0.0008	-0.0009	-0.0010	-0.0019	-0.0022	-0.0008	-	-	-	-	-	-	-	-
	(-0.26)	(-2.16)	(-2.43)	(-3.02)	(-3.97)	(-1.34)								
V-E-CFP	0.0024	0.0023	0.0022	0.0012	0.0009	0.0024	0.0031	-	-	-	-	-	-	-
	(0.80)	(3.14)	(3.01)	(1.67)	(1.17)	(3.22)	(3.82)							
SV	0.0138	0.0137	0.0136	0.0127	0.0123	0.0138	0.0146	0.0114	-	-	-	-	-	-
	(3.97)	(4.37)	(4.34)	(4.13)	(4.09)	(4.50)	(4.50)	(3.82)						
SV-E	0.0138	0.0137	0.0136	0.0127	0.0123	0.0138	0.0146	0.0114	-	-	-	-	-	-
	(3.97)	(4.37)	(4.34)	(4.13)	(4.09)	(4.50)	(4.50)	(3.82)						
SV-E-EP	0.0093	0.0092	0.0091	0.0082	0.0078	0.0093	0.0101	0.0070	-0.0045	-0.0045	-	-	-	-
	(3.74)	(3.39)	(3.36)	(3.16)	(3.00)	(3.57)	(3.61)	(2.55)	(-2.85)	(-2.85)				
SV-E-BVP	0.0090	0.0089	0.0088	0.0079	0.0075	0.0090	0.0098	0.0066	-0.0048	-0.0048	-0.0003	-	-	-
	(3.61)	(3.13)	(3.09)	(2.89)	(2.76)	(3.29)	(3.35)	(2.33)	(-2.95)	(-2.95)	(-0.97)			
SV-E-SP	0.0091	0.0090	0.0089	0.0080	0.0076	0.0091	0.0099	0.0068	-0.0047	-0.0047	-0.0002	0.0001	-	-
	(3.69)	(3.31)	(3.28)	(3.06)	(2.92)	(3.49)	(3.53)	(2.48)	(-2.99)	(-2.99)	(-1.01)	(0.33)		
SV-E-DY	0.0074	0.0073	0.0072	0.0062	0.0059	0.0074	0.0081	0.0050	-0.0064	-0.0064	-0.0019	-0.0016	-0.0017	-
	(3.10)	(3.34)	(3.30)	(2.95)	(2.86)	(3.51)	(3.64)	(2.29)	(-3.65)	(-3.65)	(-1.80)	(-1.46)	(-1.64)	
SV-E-CFP	0.0094	0.0093	0.0092	0.0083	0.0079	0.0094	0.0102	0.0070	-0.0044	-0.0044	0.0001	0.0004	0.0003	0.0020
	(3.77)	(3.36)	(3.32)	(3.14)	(2.97)	(3.55)	(3.58)	(2.53)	(-2.63)	(-2.63)	(0.40)	(1.06)	(1.24)	(1.76)

The abbreviations in the first column refer to different types of optimal portfolios, including one-, two-, and three-criteria portfolios. The one-criterion portfolios are minimum-risk portfolios constructed with respect to Variance (V) and Semi-variance (SV). The two-criterion portfolios, in addition to risk minimization, ensure the achievement of a predefined expected rate of return (E). The three-criterion portfolios include an additional objective. The numbers in the main body of the table represent the three-month alphas from Fama-French-Carhart six-factor models on the long-short (row minus column) portfolio that goes long the row portfolio and shorts the column portfolio. The numbers in parentheses are the Newey and West (1987) t-statistics that are adjusted for heteroscedasticity and autocorrelation.

Table 11. Six-month alphas on long-short portfolios

	EW	V	V-E	V-E-EP	V-E-BVP	V-E-SP	V-E-DY	V-E-CFP	SV	SV-E	SV-E-EP	SV-E-BVP	SV-E-SP	SV-E-DY
V	0.0143	-	-	-	-	-	-	-	-	-	-	-	-	-
	(3.15)													
V-E	0.0146	0.0003	-	-	-	-	-	-	-	-	-	-	-	-
	(3.21)	(1.85)												
V-E-EP	0.0135	-0.0008	-0.0011	-	-	-	-	-	-	-	-	-	-	-
	(3.14)	(-0.87)	(-1.17)											
V-E-BVP	0.0144	0.0001	-0.0002	0.0009	-	-	-	-	-	-	-	-	-	-
	(3.24)	(0.15)	(-0.28)	(1.09)										
V-E-SP	0.0161	0.0018	0.0015	0.0026	0.0017	-	-	-	-	-	-	-	-	-
	(3.54)	(3.01)	(2.63)	(3.71)	(2.50)									
V-E-DY	0.0138	-0.0005	-0.0008	0.0003	-0.0006	-0.0023	-	-	-	-	-	-	-	-
	(3.04)	(-1.07)	(-1.96)	(0.35)	(-0.77)	(-3.22)								
V-E-CFP	0.0145	0.0001	-0.0001	0.0010	0.0000	-0.0016	0.0006	-	-	-	-	-	-	-
	(3.35)	(0.12)	(-0.12)	(1.22)	(0.04)	(-1.74)	(0.52)							
SV	0.0654	0.0510	0.0507	0.0519	0.0509	0.0492	0.0515	0.0509	-	-	-	-	-	-
	(8.72)	(6.95)	(6.92)	(7.47)	(7.22)	(6.89)	(6.97)	(7.68)						
SV-E	0.0654	0.0510	0.0507	0.0519	0.0509	0.0492	0.0515	0.0509	-	-	-	-	-	-
	(8.72)	(6.95)	(6.92)	(7.47)	(7.22)	(6.89)	(6.97)	(7.68)						
SV-E-EP	0.0665	0.0522	0.0519	0.0530	0.0521	0.0504	0.0527	0.0520	0.0012	0.0012	-	-	-	-
	(8.99)	(7.19)	(7.16)	(7.75)	(7.48)	(7.15)	(7.21)	(7.95)	(2.79)	(2.79)				
SV-E-BVP	0.0621	0.0477	0.0474	0.0486	0.0476	0.0459	0.0482	0.0476	-0.0033	-0.0033	-0.0045	-	-	-
	(8.33)	(6.54)	(6.51)	(7.01)	(6.80)	(6.45)	(6.56)	(7.18)	(-4.52)	(-4.52)	(-5.08)			
SV-E-SP	0.0586	0.0442	0.0440	0.0451	0.0441	0.0425	0.0447	0.0441	-0.0068	-0.0068	-0.0079	-0.0035	-	-
	(8.10)	(6.10)	(6.07)	(6.52)	(6.31)	(5.97)	(6.13)	(6.67)	(-3.77)	(-3.77)	(-4.07)	(-2.22)		
SV-E-DY	0.0454	0.0311	0.0308	0.0319	0.0310	0.0293	0.0316	0.0309	-0.0199	-0.0199	-0.0211	-0.0166	-0.0132	-
	(10.18)	(6.22)	(6.18)	(6.94)	(6.48)	(6.06)	(6.22)	(7.04)	(-4.80)	(-4.80)	(-5.09)	(-3.95)	(-3.33)	
SV-E-CFP	0.0635	0.0492	0.0489	0.0500	0.0491	0.0474	0.0496	0.0490	-0.0019	-0.0019	-0.0030	0.0014	0.0049	0.0181
	(8.58)	(6.74)	(6.71)	(7.25)	(6.99)	(6.67)	(6.76)	(7.46)	(-2.97)	(-2.97)	(-3.83)	(1.47)	(2.75)	(4.41)

The abbreviations in the first column refer to different types of optimal portfolios, including one-, two-, and three-criteria portfolios. The one-criterion portfolios are minimum-risk portfolios constructed with respect to Variance (V) and Semi-variance (SV). The two-criterion portfolios, in addition to risk minimization, ensure the achievement of a predefined expected rate of return (E). The three-criterion portfolios include an additional objective. The numbers in the main body of the table represent the six-month alphas from Fama-French-Carhart six-factor models on the long-short (row minus column) portfolio that goes long the row portfolio and shorts the column portfolio. The numbers in parentheses are the Newey and West (1987) t-statistics that are adjusted for heteroscedasticity and autocorrelation.

Table 12: Alphas in sub-periods over one-, three-, and six-month holding periods

	Panel A: One month			Panel B: Three months			Panel C: Six months		
	before	during	after	before	during	after	before	during	after
EW	0.2085	-0.2635	0.2879	0.0842	-0.5774	0.9540	0.1251	-1.8321	1.6788
	(1.51)	(-1.53)	(1.77)	(0.16)	(-2.11)	(3.49)	(0.18)	(-5.12)	(6.15)
V	-0.0715	-1.3544	0.0461	-0.9440	-2.4040	1.9003	2.3634	-3.1823	3.6365
	(-0.18)	(-4.61)	(0.19)	(-1.67)	(-3.71)	(4.46)	(1.29)	(-3.20)	(6.96)
V-E	-0.0715	-1.3571	0.0993	-0.9440	-2.4040	1.9118	2.3634	-3.1823	3.6221
	(-0.18)	(-4.63)	(0.41)	(-1.67)	(-3.71)	(4.51)	(1.29)	(-3.20)	(7.01)
V-E-EP	-0.0715	-1.2442	0.1100	-0.9440	-2.2073	2.0234	2.3634	-2.3102	3.3855
	(-0.18)	(-4.37)	(0.47)	(-1.67)	(-3.41)	(4.85)	(1.29)	(-2.26)	(6.98)
V-E-BVP	-0.0746	-1.3221	0.0667	-0.9440	-2.2068	2.0619	2.3634	-2.8242	3.3558
	(-0.19)	(-4.53)	(0.28)	(-1.67)	(-3.46)	(4.92)	(1.29)	(-2.77)	(6.28)
V-E-SP	-0.0715	-1.2922	0.0125	-0.9440	-2.4500	1.9614	2.3634	-2.7764	3.8438
	(-0.18)	(-4.82)	(0.05)	(-1.67)	(-4.25)	(4.70)	(1.29)	(-2.74)	(7.52)
V-E-DY	-0.0625	-1.3582	0.1004	-0.9439	-2.3889	1.7564	1.7070	-2.9961	3.4808
	(-0.16)	(-4.63)	(0.42)	(-1.66)	(-3.63)	(4.10)	(1.00)	(-2.96)	(6.68)
V-E-CFP	-0.0714	-1.0177	0.0319	-0.9440	-1.8818	2.2233	2.3634	-2.1709	3.2595
	(-0.18)	(-3.51)	(0.13)	(-1.67)	(-2.69)	(5.34)	(1.29)	(-2.06)	(6.97)
SV	0.4621	-1.1699	0.0672	-0.6163	3.3659	2.5266	1.3322	12.5970	7.1207
	(1.28)	(-3.32)	(0.30)	(-0.56)	(3.79)	(5.96)	(0.82)	(6.51)	(8.41)
SV-E	0.4621	-1.1733	0.0696	-0.6163	3.3659	2.5266	1.3322	12.5970	7.1207
	(1.28)	(-3.33)	(0.31)	(-0.56)	(3.79)	(5.96)	(0.82)	(6.51)	(8.41)
SV-E-EP	0.4654	-1.0007	0.0982	-0.1028	1.8696	1.8048	1.3322	12.7508	7.2378
	(1.29)	(-2.88)	(0.43)	(-0.12)	(2.88)	(4.45)	(0.82)	(6.68)	(8.78)
SV-E-BVP	0.4640	-1.0759	0.0016	-0.0991	1.9081	1.6706	1.3322	12.4648	6.5524
	(1.28)	(-3.19)	(0.01)	(-0.12)	(2.94)	(4.17)	(0.82)	(6.53)	(7.68)
SV-E-SP	0.4654	-1.1571	-0.03821	-0.0991	1.8795	1.7727	1.3322	12.7554	6.4140
	(1.29)	(-4.10)	(-0.17)	(-0.12)	(2.90)	(4.45)	(0.82)	(6.68)	(8.14)
SV-E-DY	0.4952	-1.2065	0.0614	0.0574	0.6272	2.0305	-0.0133	2.5519	6.1932
	(1.43)	(-3.64)	(0.26)	(0.07)	(1.04)	(4.65)	(-0.01)	(3.17)	(9.44)
SV-E-CFP	0.4654	-0.6305	-0.0085	-0.0991	1.8076	1.8480	1.3322	12.5970	7.0895
	(1.29)	(-1.97)	(-0.04)	(-0.12)	(2.80)	(4.62)	(0.82)	(6.51)	(8.54)

This table reports the alphas from the Fama–French–Carhart six-factor model for optimal portfolios subject to single or multiple constraints over three-month holding periods. The corresponding Newey and West (1987) t-statistics are adjusted for heteroskedasticity and autocorrelation. The analysis is conducted across three distinct subperiods: the period before COVID, the COVID period, and the period after COVID.

Online Appendix For

Three-Criteria Portfolio Selection Model Incorporating Semi-variance and Market Multiples

Table A1. The results for the Levene test (p-values)

	EW	V	V-E	V-E-EP	V-E-BVP	V-E-SP	V-E-DY	V-E-CFP	SV	SV-E	SV-E-EP	SV-E-BVP	SV-E-SP	SV-E-DY
Panel A: One-month portfolios														
V	0.8137	-	-	-	-	-	-	-	-	-	-	-	-	-
V-E	0.6600	0.8309	-	-	-	-	-	-	-	-	-	-	-	-
V-E-EP	0.6903	0.8650	0.9651	-	-	-	-	-	-	-	-	-	-	-
V-E-BVP	0.7476	0.9286	0.9009	0.9356	-	-	-	-	-	-	-	-	-	-
V-E-SP	0.4029	0.5304	0.6796	0.6475	0.5900	-	-	-	-	-	-	-	-	-
V-E-DY	0.6670	0.8389	0.9918	0.9734	0.9091	0.6719	-	-	-	-	-	-	-	-
V-E-CFP	0.8945	0.9149	0.7492	0.7823	0.8444	0.4640	0.7570	-	-	-	-	-	-	-
SV	**0.0618**	**0.0278**	**0.0157**	**0.0176**	**0.0217**	**0.0045**	**0.0161**	**0.0371**	-	-	-	-	-	-
SV-E	**0.0609**	**0.0273**	**0.0154**	**0.0173**	**0.0213**	**0.0044**	**0.0159**	**0.0365**	0.9946	-	-	-	-	-
SV-E-EP	**0.0853**	**0.0407**	**0.0238**	**0.0265**	**0.0323**	**0.0073**	**0.0244**	**0.0534**	0.8796	0.8743	-	-	-	-
SV-E-BVP	**0.0627**	**0.0280**	**0.0158**	**0.0177**	**0.0219**	**0.0045**	**0.0162**	**0.0375**	0.9833	0.9779	0.8952	-	-	-
SV-E-SP	0.4016	0.2601	0.1797	0.1940	0.2228	**0.0778**	0.1829	0.3106	0.2756	0.2727	0.3492	0.2810	-	-
SV-E-DY	0.3976	0.2556	0.1755	0.1898	0.2184	**0.0749**	0.1787	0.3060	0.2705	0.2676	0.3441	0.2759	0.9995	-
SV-E-CFP	**0.0810**	**0.0379**	**0.0219**	**0.0245**	**0.0300**	**0.0066**	**0.0225**	**0.0501**	0.8858	0.8805	0.9925	0.9016	0.3397	0.3344
Panel B: Three-month portfolios														
V	0.3681	-	-	-	-	-	-	-	-	-	-	-	-	-
V-E	0.3603	0.9873	-	-	-	-	-	-	-	-	-	-	-	-
V-E-EP	0.2638	0.8208	0.8333	-	-	-	-	-	-	-	-	-	-	-
V-E-BVP	0.1860	0.6488	0.6602	0.8156	-	-	-	-	-	-	-	-	-	-
V-E-SP	0.6034	0.6867	0.6750	0.5274	0.3925	-	-	-	-	-	-	-	-	-
V-E-DY	0.3490	0.9722	0.9849	0.8473	0.6724	0.6599	-	-	-	-	-	-	-	-
V-E-CFP	0.5395	0.1070	0.1036	**0.0646**	**0.0398**	0.2281	**0.0981**	-	-	-	-	-	-	-
SV	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	-	-	-	-	-	-
SV-E	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	1.0000	-	-	-	-	-
SV-E-EP	**0.0537**	**0.0027**	**0.0026**	**0.0012**	**0.0006**	**0.0095**	**0.0023**	0.1592	**0.0013**	**0.0013**	-	-	-	-
SV-E-BVP	**0.0313**	**0.0012**	**0.0012**	**0.0005**	**0.0003**	**0.0047**	**0.0011**	**0.0993**	**0.0030**	**0.0030**	0.8047	-	-	-
SV-E-SP	**0.0601**	**0.0032**	**0.0030**	**0.0014**	**0.0007**	**0.0110**	**0.0027**	0.1755	**0.0011**	**0.0011**	0.9561	0.7623	-	-
SV-E-DY	0.1088	0.4712	0.4814	0.6241	0.8081	0.2553	0.4915	**0.0167**	**0.0000**	**0.0000**	**0.0001**	**0.0001**	**0.0002**	-
SV-E-CFP	**0.0654**	**0.0036**	**0.0035**	**0.0016**	**0.0008**	**0.0123**	**0.0031**	0.1886	**0.0009**	**0.0009**	0.9251	0.7331	0.9689	**0.0002**

Panel C: Six-month portfolios

	EW	V	V-E	V-E-EP	V-E-BVP	V-E-SP	V-E-DY	V-E-CFP	SV	SV-E	SV-E-EP	SV-E-BVP	SV-E-SP	SV-E-DY
V	**0.0000**	-	-	-	-	-	-	-	-	-	-	-	-	-
V-E	**0.0000**	0.7871	-	-	-	-	-	-	-	-	-	-	-	-
V-E-EP	**0.0000**	0.9795	0.7689	-	-	-	-	-	-	-	-	-	-	-
V-E-BVP	**0.0000**	0.6397	0.8441	0.6237	-	-	-	-	-	-	-	-	-	-
V-E-SP	**0.0000**	0.6300	0.8324	0.6142	0.9872	-	-	-	-	-	-	-	-	-
V-E-DY	**0.0000**	0.6960	0.9064	0.6790	0.9357	0.9232	-	-	-	-	-	-	-	-
V-E-CFP	**0.0000**	0.3580	0.5168	0.3480	0.6496	0.6628	0.5909	-	-	-	-	-	-	-
SV	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	-	-	-	-	-	-
SV-E	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	1.0000	-	-	-	-	-
SV-E-EP	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	0.5154	0.5154	-	-	-	-
SV-E-BVP	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	0.8764	0.8764	0.4159	-	-	-
SV-E-SP	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	0.4044	0.4044	0.1334	0.4939	-	-
SV-E-DY	0.8256	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	-
SV-E-CFP	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	0.9132	0.9132	0.4422	0.9622	0.4630	**0.0000**

This table reports the p-values from Levene’s test, which evaluates whether return variances are equal across the row and column portfolios. Results for the one-, three-, and six-month horizons are presented in Panels A, B, and C, respectively. Bold values indicate statistically significant differences. A detailed description of all portfolio types used is presented in Table 1.